# Characterize Dynamic Dilemma Zone and Minimize its Effect at Signalized Intersections 

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## PREFACE

Dilemma zone at signalized intersection has been recognized as a major potential causing rearend and right-angle crashes, and has been widely studied by researches since it was initially proposed as the GHM model in 1960. However, concepts conventionally defined to represent the yellow phase dilemma lack integrity. This research conducts a comprehensive literature review with attempt to clarify the interrelationship among dilemma zone, option zone, and indecision (decision) zone, and to develop a heuristic framework to present the contributing factors in dilemma zone modeling. A new method for modelling the locations and lengths of the dilemma zone using video-capture techniques and vehicle trajectory data is presented in this report. First, dilemma zone is mathematically modeled based on the GHM model. Then, field-observed trajectory data extracted by the video-capturing-based approach are used to calibrate the contributing factors involved in the dilemma zone model. The high accuracy of the time-based trajectory data has significantly enhanced the accuracy of the calibrated dilemma zone models. Two sets of trajectories are explored for calibrating the dilemma zone contributing factors. One is concerned with maximum yellow-onset safe passing distance and minimum yellow-onset stopping distance. The other is concerned with $X^{\text {th }}$ percentile yellow-onset passing distance and $(100-X)^{\text {th }}$ percentile stopping distance for the prevailing travel behaviors. The latter alternative actually precludes "too conservative" and "too aggressive" maneuvers in response to yellow indications.

One critically important result is the dilemma zone look-up charts that are developed based on the calibrated dilemma zone models. Such charts provide a convenient tool to identify the locations and lengths of dilemma zones for any speed and yellow duration conditions. Additionally, impact of arrival type and vehicle types are also explored. Results reveal that traffic in a good progression (Arrival Type $\geq 4$ ) has a further option zone. It is also discovered that the length of option zone decreases as the vehicle size increases, while the downstream boundary of option zone is further from the stop line as the vehicle size increases. In overall, this project aims to conduct a preliminary research for providing a proof of concept about the modeling of dynamic dilemma zones, and validating the feasibility of the methodology for calibrating the dilemma zone model using trajectory data. The methodology used in this study establishes a solid basis for future research of the optimum signal detection placement and related dilemma zone protection problems with consideration of multi-speed protection.

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## CHAPTER 1:

## PROBLEM STATEMENT

The report "National Agenda for Intersection Safety" (USDOT, 2002) quotes that in the year 2000, more than 2.8 million intersection related crashes occurred, which amounts to 44 percent of all reported crashes (USDOT, 2002). In Ohio, intersection crashes account for 24 percent of the fatalities and 37 percent of the disabling injuries (ODOT, 2006). The National Highway Traffic Safety Administration has estimated that aggressive drivers cause two-thirds of all fatal crashes and are responsible for nearly 35 percent of all crashes (ACEP, 2006). Among all possible factors contributing to the traffic-signal-related crashes, intersection dilemma zone is one of major causes and critical issues that have not been fully solved yet.

According to the ITE handbooks (ITE, 1982; 1999), a dilemma zone is a (length) range within which a vehicle approaching an intersection during its yellow phase can neither safely clear the intersection, nor stop comfortably at the stop line. With the existence of dilemma zone, the drivers are actually exposed to a potentially hazardous condition in which a rear end accident may occur if the front vehicle stops abruptly during the yellow period. An angle accident may occur if the driver attempts to cross the intersection at the onset of the red interval (ODOT \& FHWA, 2005). To minimize the safety problems caused by the dilemma zone, protection strategies, such as detection-based control systems, are implemented at high-speed intersections to clear vehicles out of the dilemma zone before the onset of the yellow indication. Therefore, the accurate and exact location of dilemma zone is of great importance for those dilemma zone protection systems. However, the range and location of a dilemma zone is dynamically featured because of variations in the vehicle approach speed, driving behaviors, vehicle break performance, intersection geometry, and duration of the yellow interval. The standard practice of using the average driver data with traditional methods for computing the dilemma zone is hard to reflect the dynamic features of the dilemma zone.

Recent study conducted by Maryland DOT (MDOT, 2004; 2006) indicates that systematically modeling dynamic dilemma zones is quite difficult without accurate trajectory data. Traditional traffic counting methods that were used in data collection in old studies are difficult to obtain the trajectory data that describe the dynamic natures of dilemma zone related maneuvers. The trajectory data applied in MDOT's study (MDOT, 2006) describe the times when an individual vehicle passes fixed reference lines perpendicular to the roadway (termed as "fixed-spatial-point trajectory data" in this report). In order to accurately reflect vehicles' speed and acceleration/deceleration changes in responding to yellow indication, it could be another alternative to use the data that describe the track of a moving vehicle over a small time interval (termed as "time-based trajectory data"). Time-based trajectory data can well relate the instant speed at any time step ( 30 steps per second maximum). This feature makes it valuable to obtain the data from which the interrelations between driving maneuvers (e.g., speed, acceleration, stop/pass decision), durations of yellow interval, and distributions of dilemma zone ranges can be well represented. This research uses VEVID (Vehicle Video-Capture $\underline{\text { Data Collector) (Wei et }}$
al., 2000; 2005), which is a software program developed by the author and can extract timebased trajectory data from digital videos, to study the dynamic nature of the dilemma zone.

Another important concept, option zone, is defined as a zone within which at the onset of yellow indication, the driver could choose either to clear the intersection before the end of the yellow interval or stop at the stop line. According to the review of literature, option zone has less been studied and in particular distinguished from the so called dilemma zone. The option zone commonly exists at high speed intersections. Existence of the option zone also has great potential to cause drivers' hesitation about either to stop or pass the intersection during the yellow interval, and it is also one of the contributing factors to rear-end and right-angle collisions at high speed intersections. This research project puts much effort on identifying the existence of dilemma zone, option zone and their inherent relationship.

## CHAPTER 2:

## GOAL AND OBJECTIVES

The goal of the project is to conduct a preliminary research for providing a proof of concept about the methodology for extracting dilemma zone vehicle trajectory data and quantitatively modeling locations and lengths of the dilemma zone under various approaching speeds and yellow durations. A case study at a high speed signalized intersection is conducted to demonstrate the feasibility of applying the trajectory data to investigating the dynamic features of dilemma zone and calibrating the dilemma zone model. The mythology used in this research will provide a solid basis for preparing further research on the analysis of more locations as well as studying the detectors layout issue for dilemma zone protection. The specific objectives are:

1) To conduct comprehensive literature review covering topics about concurrent researches on characterizing the dilemma zone;
2) To conduct field data collection through videotaping and traffic counting techniques. And to extract vehicle trajectory data related to the dilemma zone;
3) To develop a model that accurately addresses the location and length of the dilemma zone. And to calibrate the model with observed trajectory data; and
4) To tentatively develop dilemma zone look-up charts to provide accurate dilemma zone locations for helping designing the loops layout for dilemma zone protection.

## CHAPTER 3:

## RELATED WORK AND LITERATURE REVIEW

### 3.1 Existing Definitions of Dilemma and Option Zones

The concept of dilemma zone was initially proposed by Gazis, Herman and Maradudin (1960), which is usually referred to as the GHM model by the acronyms of the authors' names. A dilemma zone is defined by the authors as a zone within which a driver can neither bring his/her car to a stop safely nor go through the intersection before the signal turns red. The concept of dilemma zone is illustrated by Figure 1.


Figure 1: Formation of a Dilemma Zone
In Figure $1, X_{c}$ is referred to as the critical distance or the minimum (possible) stopping distance from the stop line. At a closer distance from the stop line than $X_{c}$, a vehicle cannot safely stop before the stop line. $X_{0}$ is the maximum distance a vehicle can travel during the entire yellow interval and clear the intersection before the end of yellow interval. Thus, $X_{0}$ is usually referred to as the maximum yellow passing distance from the stop line. When $X_{\mathrm{c}}>X_{0}$, the vehicle physically located somewhere between $X_{\mathrm{c}}$ and $X_{0}$ is actually within a "dilemma situation", in which the vehicle can neither safely stop before the stop line and nor safely pass the intersection during the yellow interval. The physical zone between $X_{\mathrm{c}}$ and $X_{0}$ when $X_{\mathrm{c}}>X_{0}$ is the dilemma zone. In this situation, the word "dilemma" exactly represents such a circumstance, although the driver might not be aware of it. According to GHM model, $X_{\mathrm{c}}$ and $X_{0}$ can be represented by Equations (1) and (2) (Gazis 1960), respectively.

$$
\begin{align*}
& X_{c}=V_{0} \delta_{2}+\frac{V_{0}^{2}}{2 a_{2}}  \tag{1}\\
& X_{0}=V_{0} \tau-W+\frac{1}{2} a_{1}\left(\tau-\delta_{1}\right)^{2} \tag{2}
\end{align*}
$$

Where, $V_{0}=$ the vehicle's approach speed (ft/s);
$\delta_{2}=$ the driver's perception-reaction time for stopping (s);
$a_{2}=$ the maximum vehicle's deceleration rate ( $\mathrm{ft}^{2} / \mathrm{s}$ );
$\delta_{1}=$ the driver's perception-reaction time for running (s);
$a_{1}=$ the constant vehicle's acceleration rate $\left(\mathrm{ft}^{2} / \mathrm{s}\right)$;
$\tau \quad=$ the duration of yellow interval (s);
$\mathrm{W}=$ the summation of intersection width and the length of vehicle.
When $X_{0}>X_{\mathrm{c}}$, i.e., as the maximum yellow passing distance is greater than the minimum stopping distance, the vehicle within the "zone" between $X_{\mathrm{c}}$ and $X_{0}$ at the onset of the yellow indication faces two options: either to pass the intersection during the yellow time or to slow down and stop before the stop line. The "zone" between $X_{\mathrm{c}}$ and $X_{0}$ (when $X_{0}>X_{\mathrm{c}}$ ) is termed as the option zone, as shown by Figure 2.


Figure 2: Formation of an Option Zone

Therefore, an option zone is defined as a zone within which at the onset of yellow indication, the driver can either come to a stop safely or proceed through the intersection before the end of the yellow interval. The word "option" means that the driver's final decision of whether to pass or to stop is optional. Whatever passing or stopping is chosen, he/she could finally make it.

The dilemma zone is also modeled by probabilistic approaches based on probability of drivers' decision to stop in response to the yellow indication. Zegeer (1977) defined a dilemma zone as "the road segment where more than $10 \%$ and less than $90 \%$ of the drivers would choose to stop." The upstream boundary of the dilemma zone is the distance beyond which more than 90 percent drivers would stop if presented with a yellow indication. Sheffi and Mahmassani (1981) used speed and distance from the stop line to estimate this probability of stopping. Dilemma zone curves (probability of stopping vs. distance from stop line) were developed to determine the boundaries of dilemma zones at various speeds.

El-Shawarby et al. (2006) summarized the above two definitions of the dilemma zones from perspectives of stopping distance and drivers' choice of stopping, respectively, which likely caused somewhat confusion to the researchers. Typically, those two definitions are referred to as the one initially defined by GHM model (Gazis et al. 1960) and the probabilistic dilemma zone definition (Zegeer, 1977).

Parsonson (1992) indicated in his research report that the probabilistic definition of the dilemma zone is actually about an option zone --- a length of an approach in advance of an intersection where an individual driver may experience indecisiveness upon seeing the indication of the yellow signal. And the calculation of the boundaries of the option zone follows " $10 \%$ to $90 \%$ " rule based on Zegeer's study (1977). According to Parsonson's definition, this kind of option zone is also interpreted as "indecision zone" or "decision zone". Si et al. (2007) followed Parsonson's definition of the option zone. They stated that the dilemma zone and option zone are fundamentally different issues, although the boundaries of the dilemma zone and the option zone may overlap to a certain extent. The dilemma zone can be eliminated by appropriate yellow and red clearance times, whereas the option zone always exist as a result of varied travel decisionmaking choices of stop or go behaviors.

Urbanik and Coonce (2007) recently conducted a comprehensive literature review on the definitions of dilemma zone, with intention to clarify the "the dilemma" with dilemma zones. They believe that there is a lack of rigor with regard to defining terminology and the documenting of assumptions when discussing dilemma zones. They termed the dilemma zone, which was originally defined and formulated by Gazis et al. (1960) as the Type I dilemma zone, and the other one initially defined by Zegger (1977) as the Type II dilemma zone. They also indicated that Type I dilemma zone could be eliminated when yellow interval is long enough. And driver's exposure to the Type II dilemma zone can be minimized by applying the detectionbased dilemma zone protection system.

### 3.2 Driver's Response to Yellow Indication

Driving behavior in response to the yellow signal has been recognized as one of contributors to the dynamic natures of dilemma zones. Olson and Rothery (1962) continued Gazis et al.'s study, seeking possible behavioral trends in this decision-making problem at the onset of yellow indication. Their research came to a significant conclusion that driver's behavior does not seem to change as a function of different yellow interval durations. Liu, et al (1996) investigated the incompatibilities of the yellow-light phase duration and traffic ordinances, a problem raised from the GHM Model. They also made a significant progress in uncovering the complex interrelationships between dilemma zone, driver response, and the yellow interval duration.

El-Shawarby et al (2006) conducted an experiment to study driver's behavior during the yellow interval. 60 drivers with various ages and sex were hired to drive a test vehicle at a test roadway system. Real-time speeds and distances from stop line were collected through a communication and computer system. They observed driver's stopping at five predetermined distances, and made a diagram representing the relationship between the probability of stopping and the distance from the stop line. By identifying the locations where $10 \%$ and $90 \%$ drivers would choose to stop, rough location of the option zone was estimated. The research results
indicated that at the speed of 45 mph , the dilemma zone lies between around 108 ft to 253 ft from stop line. Also, male drivers are less likely to stop when compared to female drivers. Old drivers are more likely to stop, while younger drivers are approximately $20 \%$ more likely to attempt to run yellow compared to older drivers. The research conducted by Shinar and Compton (2004) also reached a similar conclusion. Based on observations of more than 2000 drivers' responses to the yellow indication, they found male drivers are more aggressive than female drivers, and senior drivers are less likely to take aggressive action than young drivers. Maryland DOT (2006) comprehensively studied driver's behavior over the yellow intervals by using fixed spatial-point trajectory data. In this study, driver types were defined based on aggressiveness. Their research results were also in accordance with Shinar and Compton's (2004) conclusions.

Papaioannou (2006) conducted a similar study in Greece. Practical vehicle data were collected at a T intersection. Yellow onset speeds were obtained using radar guns, while the yellow onset distances from the stop line were approximately determined by means of a scale drawn on the roadway pavement with markings every 5 meters. Only the platoon leading vehicles and the first following vehicles were included into the sampling data. Given a constant maximum deceleration rate and a minimum drivers' reaction time, length of the dilemma zone or option zone for each vehicle was calculated by using the GHM model with the yellow-onset speed as an input. Thus, spatial relationship between the location of dilemma/option zone and the position of vehicle at the onset of yellow interval was established. Drivers were then classified into three groups by their aggressiveness, namely, aggressive, normal and conservative. The results indicated that a large percentage of vehicles are within dilemma zone rather than option zone. And the percentage of aggressive drivers among all the drivers is as high as more than $50 \%$.

A key issue that is related to the driver's behavior during the yellow interval is driver's perception-reaction time (PRT), which directly influences the location of a dilemma zone base on GHM model. PRT is the time interval from the onset of the yellow indication to the instant when the brake pedal is applied (Rakha et. al, 2007). Usually, PRT data are recorded as the time elapsed from the onset of the yellow indication until the brake light is observed. Previous study (Taoka, 1989) has demonstrated that the 85th-percentile PRT falls into the range of $1.5-1.9 \mathrm{~s}$ at low speed intersection approaches, and is shorter at high speed intersection approaches (greater than 40 mph ), with the 85 th-percentile PRTs in the range of 1.1-1.3 s. Chang et al.'s (1985) study results revealed that speed effectively influences the median PRT, which converges to 0.9 s at speeds equal to or greater than 45 mph . Caird et al. (2005) found through 77 drivers' driving behavior by using a driving simulator that the yellow onset distance from stop line also influences the PRT, which is ranging from 0.86 s for drivers closest to the intersection stop line to 1.03 s for drivers farthest from it. The recent research on PRT conducted by Rakha et al (2007) summarized that at a high speed intersection approach ( 45 mph ) the average PRT from 351 observed samples is ranging from 0.3 s to 1.7 s , with a mean equals to 0.742 s , a median of 0.700 s , and a standard deviation of 0.189 s . Maryland DOT (2006) indicated in their research report that average PRT is in the range between 0.93 s and 1.16 s .

### 3.3 Impact of Yellow Duration on Dilemma/Option Zone

The impact of yellow duration on dilemma and option zones has been studied in previous efforts. Saito et al. (1990) conducted a research to study the characteristics of dilemma zones and option
zones. Video-taping techniques were utilized in their research to collect the speed, distance, driver's PRT and deceleration rate of vehicles at the onset of yellow interval. Only the first stopped and the last passing vehicles during the yellow intervals were studied in their research. The result revealed that as the duration of yellow interval increases, the rate of vehicles in the dilemma zone decreases while the rate of vehicles in the option zone increases; and the size of the dilemma zone decreases while the size of the option zone increases. Their research also indicated that drivers within the dilemma zone and option zone are forced to make decisions about whether to pass or to stop during a very short time period.

Koll et al.'s (2004) research also indicated that prolonging the yellow interval will not improve the intersection safety, because it will create longer option zones and drivers within option zone will still experience uncertainties about whether to pass or to stop, which may contribute to the rear-end accidents.

### 3.4 Dilemma Zone Protection

Although the dilemma and option zone could not be fully eliminated due to varying and unpredictable driver's behaviors, safety problems caused by the yellow phase dilemma could be reduced by taking strategic countermeasures, which are usually referred to as dilemma zone protection.

Ohio DOT (2005) conducted a field testing of implementation of dilemma zone protection and signal coordination at closely-spaced high-speed intersections. Traditional traffic counting methods were applied to collect data related to three types of conflicting vehicles, namely, running red light, stopping abruptly and accelerating through yellow times. Those three types of conflicts were used to identify vehicles that experience dilemma zone problems. The "conflict percentage," which was calculated as the "total conflict volume" over "total volume," was considered as a good measure of effectiveness for dilemma zone protection in the study. The results revealed that accelerating through yellow was the major conflict for all intersections of study. To address the location of the dilemma zone, clearance distance assuming acceleration equals to $10 \mathrm{ft} / \mathrm{s}^{2}$ were theoretically identified. The detector placement was based on this assumption. The study indicated that different intersection has its unique best extension of green, and there is no one "universal" rule for dilemma zone protection of extending the green time.

Moon et al. $(2002,2003)$ developed and field-tested an in-vehicle dilemma zone warning system, which provides real-time visual and audible warnings for drivers before the onset of yellow indications. McCoy and Pesti (2003) also proposed a detection and advance warning flasher system to inform drivers of stopping before the onset of yellow indications.

Another sophisticated detection-based dilemma zone protection system, i.e., detectioncontrol system (D-CS), was developed by Zimmerman et al. (2003; 2004; 2005; 2007). It uses dual loops in a speed-trap configuration installed about 1000 ft upstream of the intersection to detect the presences of approaching vehicles and to measure the speed of individual vehicles, so as to provide appropriate green extensions. It differs from the traditional multiple advance detector system because it employs a computerized algorithm that uses vehicle speed and length
information to predict when each vehicle arrives in its dilemma zone, where a fixed dilemma zone range is assumed. It attempts to identify the best time to end the major-road through phase based on the number of vehicles in the dilemma zone, the number of trucks in the dilemma zone, and the waiting time of vehicles in conflicting phases. The result of the field implementation showed that D-CS has significantly improved both operations and safety of the concerned intersection.

## CHAPTER 4: METHODOLOGY

### 4.1 New Understanding of Dilemma Zone

According to the literature, the location of the dilemma zone can be expressed as a function of critical (or minimum) safe brake stopping distance $\boldsymbol{X}_{\mathbf{c}}$ and maximum safe yellow passing distance $\boldsymbol{X}_{\mathbf{0}}$. Two scenarios are refereed to the yellow phase dilemma: a dilemma zone exists as $X_{\mathrm{c}}>X_{0}$, or an option zone exists as $X_{\mathrm{c}}<X_{0}$.

Based on the mathematical expressions of $X_{\mathrm{c}}$ and $X_{0}$ (Equations (1) and (2)), it is not hard to recognize that the location of a dilemma zone depends on the following factors: approaching speed at the onset of yellow indication, driver's perception-reaction time (i.e., PRT), maximum deceleration for safe stopping or acceleration for safe passing, yellow duration, as well as intersection width and vehicle's length. In this research, the intersection width and the vehicle's length are not considered in calculating $X_{0}$. It is assumed that once a vehicle passes the stop line before the end of yellow interval, it is regarded as a yellow passing vehicle. Based on field observations, when a driver perceives the yellow indication, he/she does not consider whether he/she could clear the intersection completely during the yellow interval. Actually, his/her concern is whether he/she could pass the stop line before the onset of red indication.

To better prepare the research tasks and field data collection and analysis, hierarchy of contributing factors analysis for dilemma zone modeling is developed as shown by Figure 3. Contributing factors which can be observed in field are also illustrated by Figure 3. However, because of variations in drivers' characteristics (such as age, sex, and driving aggressiveness), vehicles' characteristics (such as allowable maximum deceleration), and impact of speed limit as well as intersection size, some parameters such as speeds, PRTs, maximum accelerations or decelerations vary with drivers, vehicles, and driving environments in reality. While these factors (e.g., maximum deceleration for safe stopping or acceleration for safe passing) can be quantitatively assumed with infrastructure design experience, it's actually uncertain of the range of the factor values that are really matching with real-world travel behaviors. In other words, field observations should be conducted to identify the most appropriate range of the associated parameter values that can be effectively applied into dilemma zone modeling and protection.

As a result of dynamic human traveling and driving behaviors as well as diverse maneuver characteristics of different vehicles, the dilemma zone is actually featured with "dynamic" characteristics. Referring to each individual driver, values of $X_{c}$ and $X_{0}$ may be different and the length or range of the dilemma zone varies; however, statistical methods can be used to identify the distribution of the dilemma zone from sampling vehicle trajectory data under scenarios of possible yellow durations and speeds for the intersection approach. The dynamics are therefore featured through statistically quantified distributions of the dilemma zones in terms of location and length for different speeds. Concept of percentile (e.g., $95^{\text {th }}$ percentile yellow
passing distance) can be used to define an $\underline{X}^{\text {th }}$-percentile dilemma zone under the prevailing traffic conditions.


Figure 3: Hierarchy of Contributing Factors Analysis for Dilemma Zone Modeling

As discussed earlier, dilemma and option zone are a result of theoretically calculating $X_{\text {c }}$ and $X_{0}$. If the calculation results in $X_{\mathrm{c}}>X_{0}$ given a yellow duration $\tau$ and an approaching speed $V_{0}$, the physical zone between $X_{\mathrm{c}}$ and $X_{0}$ is traditionally defined as the dilemma zone (Figure 1). A vehicle located within such a zone at the onset of the yellow indication can neither safely stop before the stop line, nor safely pass the stop line during the yellow interval. However, vehicles within in such a defined dilemma zone are quite difficult to be observed or identified in realworld observations. We can only use the observed locations of vehicles at the onset of yellow
indications, which actually stopped before and/or passed the stop line during the yellow period, to statically analyze possible distributions of $X_{\mathrm{c}}$ and $X_{0}$ (as illustrated by Figure 3), and calculate the length of the dilemma zone with the values of $X_{\mathrm{c}}$ and $X_{0}$.

On the other hand, if the calculation results in $X_{\mathrm{c}}<X_{0}$ given a yellow duration $\tau$ and an approaching speed $V_{0}$, the physical zone between $X_{\mathrm{c}}$ and $X_{0}$ is traditionally defined as the option zone (Figure 2). A vehicle located within the option zone at the onset of the yellow indication faces two choices: either passing the intersection or slowing down and stopping before the stop line. Unlike dilemma zone, vehicles falling into the option zone are observable with ease by observed trajectory data, as shown by Figure 4.


Figure 4: Modeling Boundaries of Option Zones using Observed Trajectory Data

According to the literature review, an indecision zone is defined as a length of a roadway in advance of the intersection where a driver may experience indecisiveness upon seeing the indication of the yellow signal (Parsonson, 1992). Usually it is modeled as the road segment where more than $10 \%$ and less than $90 \%$ of the drivers would choose to stop, as shown by Figure 5. Indecision zone is a concept that measures the indecisiveness of drivers in a
probabilistic and statistical way. To some extent, indecision zone should be a segment within the option zone, and most parts of them should overlap each other.


Figure 5: Illustration of Relationship between Option Zone and Indecision Zone

The existence of dilemma zone (when $X_{\mathrm{c}}>X_{0}$ ) provides a risky chance for any vehicles happening to be located within this zone to run red, because the yellow duration is insufficient for the vehicle to safely pass the stop line while not sufficient distance for the vehicle to comfortably stop before the stop line. It is in essential a Risky Zone (RZ). Therefore, the RZ should be eliminated at all possible. Although not so risky as the RZ, the option zone (OZ) potentially leads the driver's hesitation in the decision-making process to decide pass or stop. The OZ also creates a dilemma situation to some extent. Whatever RZ or OZ exists, drivers are forced to make a judgment and decision about whether to pass or to stop within a very short period of yellow time. Any hesitation during that time period could potentially result in a rearend accident if the front vehicle stops abruptly during the yellow interval or an angle accident if the driver attempts to cross the intersection at the onset of the red interval. Therefore, the Dilemma Zone is understood to be a general concept and is composed of Risky Zone (RZ) and Option Zone (OZ), which are referred to as dilemma zone and option zone discussed in the previous sections of the report, respectively.

Regarding the above analysis, dilemma zone protection aims to minimize the effect of dilemma zones. According to the literature review and above discussions, the RZ is hard to be fully eliminated by a longer yellow interval because of dynamic driving behaviors. The OZ could be even harder to be removed because longer yellow interval yields longer OZ. Nevertheless, the dilemma zone protection with appropriate detection functionality can be helpful to reducing the potentials for vehicles to be trapped in the dilemma zone. Dilemma zone modeling through analysis of observed trajectory data could help reveal true locations of dilemma zones and
provide basis for developing the methods for optimum placement of detectors and yellow durations. The hierarchy of contributing factors to dilemma zone and their associations with observable parameters is illustrated by Figure 3 to provide a navigator for developing field data collection and dilemma zone modeling work, as described below.

### 4.2 Extraction of Dilemma Zone Vehicular Trajectory Data

A vehicle trajectory describes the vehicle's path over a period of time, as shown by Figure 6. Other parameters, such as velocity, acceleration/deceleration and headway, can be simply derived from vehicle trajectory data. A low-cost method using video-capturing technology for extracting vehicle trajectory data over the yellow interval was developed by using VEVID. This method consists of five basic steps.


Figure 6: Illustration of Vehicle Trajectory
First, one approach of the mainline street of a signalized intersection is videotaped from an elevated position from which the vehicle responding traffic lights can be fully viewed. After the video is digitized, the screen-measured distance between a vehicle's position and the stop line could be obtained from rectangular coordinates. However, for the sake of perspective effect and the angle between the line of sight of the camcorder and the surface of pavement, screenmeasured distance does not represent the real world distance. In order to convert screenmeasured distance into real world distance, reference points must be set up in field from stop line to the position of the camcorder at a fixed space like 15 or 20 ft along curbs of both sides and with no interruption to traffic. With the help of those reference points, screen distance from the vehicle position to the stop line can be converted to real world distance (Wei et al., 2005). In field, a chalk is used to mark those points on curbs on both sides of the approach or median. Then, a surveyor steps on each mark for a short while (e.g., 5 seconds) and all these actions are recorded by video camcorder. After back to office, the marked reference points are established into the database of VEVID by identifying the surveyor's feet locations when he steps on the marks in the video, as shown by Figure 7. In this way, traffic is neither interfered nor disturbed during the survey.

Second, video is digitized using video-capturing equipment, segments containing yellow interval for each cycle, including 5 seconds before and after the yellow indication, are exported to an AVI digital video file at a frame rate of 30 fps (frame $/ \mathrm{sec}$ ), which can guarantee the accuracy of identifying the exact time of the onset of yellow.

Third, those video segments are registered in VEVID by assigning established reference points to each video segment.

Fourth, in each frame, real world distance from the targeted vehicle to stop line can be obtained by simply clicking the mouse over the touching point between the rear tire (or front tire) and pavement. VEVID can record every distance generated by every click. With the distance, speed can be derived by dividing the distance interval between two consecutive frames by the time interval. (e.g., time interval between two consecutive frames at frame rate of 30 fps is $1 / 30$ second). Acceleration/deceleration can be derived by dividing the speed difference between two consecutive frames by the time interval.


Figure 7: Illustration of Setting up Field Reference Points and Camcorders

Fifth, VEVID can generate an output data file containing vehicle's trajectory over the entire yellow interval, which includes distance, speed, and acceleration/deceleration changing profiles. The output file can be imported into common analytical tools such as Microsoft Excel or Microsoft Access. Figure 8 shows the interface of the upgraded version of VEVID for extracting vehicular trajectory data over yellow intervals. The whole procedure of the trajectory data extraction is illustrated by a flowchart as shown by Figure 9.


Figure 8: Extracting Vehicular Trajectory Data over Yellow Interval by Using VEVID


Figure 9: Illustration of Procedure of Vehicular Trajectory Data Extraction

### 4.3 Case Study Site for Colleting Dilemma Zone Trajectory Data

The intersection of OH-4 and Seward Rd, Fairfield, Ohio was selected as the case study site (see Figure 10), where the speed limit is 50 mph and the yellow interval is 4.5 seconds on the $\mathrm{OH}-4$ approaches. In order to guarantee the full view coverage of the dilemma zone, two camcorders were placed on the south side of the eastbound $\mathrm{OH}-4$ at 300 and 500 ft from stop line, respectively. They were synchronized before videotaping, and 6.5 -hour period of traffic operation was videotaped at this location.


Figure 10: Site of Data Collection and Case Study (OH-4 \& Seward Rd)
Trajectory data of vehicles over the yellow intervals were extracted using VEVID with the user's interface showing in the lower-left corner of Figure 10. During each yellow interval, every running yellow and running red vehicle was targeted for extracting trajectory data. For those stopped vehicles, only the first vehicle of a platoon in each lane was targeted for trajectory data. It's because the decision made by the drivers of the following vehicle and other later vehicles is totally affected by the manoeuvre of the first stopped vehicle in the platoon. Those drivers usually can do nothing but slow down and stop following the first queuing vehicle. Therefore, those vehicles do not directly contribute to the formation of dilemma zone or option zone and are therefore not included in the samples. Totally 679 vehicle samples were obtained. Vehicle category is recorded for each sample vehicle. Traffic volumes and signal timings of each cycle were also counted and recorded through replaying the videos.

### 4.4 Modeling Dynamic Dilemma Zones

Based on GHM model (Gazis et al, 1960) and principle of kinetics, the minimum stopping distance $X_{\mathrm{c}}$ and the maximum yellow passing distance $X_{0}$ given the vehicle approaching speed $V_{0}$ can be expressed in the following equations, respectively.

$$
\begin{align*}
& X_{c}\left(V_{0}\right)=V_{0} \delta_{\text {stop }}+\frac{V_{0}^{2}}{2 a_{\text {stop }}}  \tag{3}\\
& X_{0}\left(V_{0}\right)=V_{0} \tau+\frac{1}{2} a_{\text {pass }}\left(\tau-\delta_{\text {pass }}\right)^{2} \tag{4}
\end{align*}
$$

Where, $V_{0}=$ Vehicle approaching speed (ft/s);
$X_{c}\left(V_{0}\right)=$ Minimum stopping distance from stop line given speed $V_{0}(\mathrm{ft}) ;$
$X_{0}\left(V_{0}\right)=$ Maximum yellow passing distance from stop line given speed $V_{0}(\mathrm{ft})$;
$\delta_{\text {stop }} \quad=$ Driver's minimum PRT for safe stopping (s);
$a_{\text {stop }} \quad=$ Vehicle's maximum deceleration rate for safe stopping $\left(\mathrm{ft}^{2} / \mathrm{s}\right)$;
$\delta_{\text {pass }}=$ Driver's minimum PRT for safe passing ( s );
$a_{\text {pass }}=$ Vehicle maximum acceleration rate for safe passing $\left(\mathrm{ft}^{2} / \mathrm{s}\right)$;
$\tau \quad=$ Duration of the yellow interval (s).
As discussed in Section 4.1, the intersection width and vehicle's length are not taken into the account in calculating $X_{0}$. It is assumed that once a vehicle passes the stop line before the end of yellow interval, it is regarded as a yellow passing vehicle. This assumption is from the driver's perspective and is in accordance with the field observation. When a driver perceives the yellow indication, he/she does not consider whether he/she could clear the intersection completely during the yellow interval. Actually, his/her concern is whether he/she could pass the stop line before the onset of red indication.

The length of the RZ can be modeled by Equation (5), when $X_{\mathrm{c}}>X_{0}$, while the length of the OZ can be modeled by Equation (6), when $X_{0}>X_{\mathrm{c}}$.

$$
\begin{align*}
& L_{R Z}\left(V_{0}\right)=X_{c}\left(V_{0}\right)-X_{0}\left(V_{0}\right)=V_{0} \delta_{\text {stop }}+\frac{V_{0}^{2}}{2 a_{\text {stop }}}-\left[V_{0} \tau+\frac{1}{2} a_{\text {pass }}\left(\tau-\delta_{\text {pass }}\right)^{2}\right]  \tag{5}\\
& L_{O Z}\left(V_{0}\right)=X_{0}\left(V_{0}\right)-X_{c}\left(V_{0}\right)=V_{0} \tau+\frac{1}{2} a_{\text {pass }}\left(\tau-\delta_{\text {pass }}\right)^{2}-\left(V_{0} \delta_{\text {stop }}+\frac{V_{0}^{2}}{2 a_{\text {stop }}}\right) \tag{6}
\end{align*}
$$

From these equations, it is not hard to find that values of $X_{\mathrm{c}}$ and $X_{0}$ are greatly dependent on the values of the contributing factors, $a_{\text {stop }}, a_{\text {pass }}, \delta_{\text {pass }}$, and $\delta_{\text {stop }}$, which are used to represent the driving maneuvers over the yellow interval. To complete the modeling of the location and length of the dilemma zone, appropriate values of these contributing factors need to be calibrated by using observed vehicle trajectories (e.g. observed $X_{\mathrm{c}}$ and $X_{0}$ ). However, before the calibration process, the observed $X_{\mathrm{c}}$ and $X_{0}$ could be statistically analyzed by applying $X^{\text {th }}$ percentile concept (e.g. $85^{\text {th }}$ or $95^{\text {th }}$ percentile $X_{0}$ ). Then, the $\mathrm{X}^{\text {th }}$ percentile $X_{\mathrm{c}}$ and $X_{0}$ are used as observed values to calibrate those contributing factors. From the engineering viewpoints, the percentile concepts are usually employed to represent the ranges of $X_{\mathrm{c}}$ and $X_{0}$ at a considerably confident level and thus extremely conservative or aggressive driving maneuvers are precluded. Accordingly, locations and lengths of the RZ and OZ based on this rational are assumed to be applicable to the prevailing traffic conditions.

# CHAPTER 5: SAMPLE DATA ANALYSIS AND RESULTS 

### 5.1 Calibrating the Dilemma Zone Model using Observed Trajectory Data

As discussed in Chapter 4, the locations and lengths of the dilemma zone can be identified by the locations of $X_{0}$ and $X_{\mathrm{c}}$. Based on Equations (3) and (4), the value of $X_{0}$ is determined by the duration of yellow interval $(\tau)$, the vehicle approaching speed $\left(V_{0}\right)$, the minimum PRT for passing ( $\delta_{\text {pass }}$ ), and the maximum passing acceleration ( $a_{\text {pass }}$ ), while the value of $X_{\mathrm{c}}$ has nothing to do with the duration of yellow interval $\tau$, but is associated with the vehicle approaching speed $\left(V_{0}\right)$, the minimum PRT for stopping ( $\delta_{\text {stop }}$ ), and the maximum stopping deceleration $\left(a_{\text {stop }}\right)$. Therefore, the key contributing factors in modeling the dilemma zone are $a_{s t o p}, a_{\text {pass }}, \delta_{\text {pass }}$, and $\delta_{\text {stop }}$. And determining their values plays a critical role in application of the models.

It is assumed that values of the maximum stopping deceleration ( $a_{\text {stop }}$ ), maximum passing acceleration ( $a_{\text {pass }}$ ), minimum PRT for passing ( $\delta_{\text {pass }}$ ) and minimum PRT for stopping ( $\delta_{\text {stop }}$ ) vary with different vehicle approaching speeds. Based on the common sense, we assume vehicles with a higher speed need higher deceleration to stop than vehicles with a lower speed, while vehicles with a lower speed need higher acceleration to pass the stop line before the end of yellow interval than vehicles with a higher speed. We also assume that drivers of stopped vehicles with a higher speed use more PRT to make a stop decision than those drivers with a lower speed. And drivers of the passing vehicles with a higher speed use less PRT to make a passing decision than those drivers with a lower speed. These assumptions are based for calibrating the values of $a_{\text {stop }}, a_{\text {pass }}, \delta_{\text {pass }}$, and $\delta_{\text {stop }}$ with observed trajectory data.

In order to prepare for the calibration, observed trajectory data are plotted on a coordinate system with the yellow onset speeds on the vertical axis and the yellow onset distances from the stop line on the horizontal axis (See Figure 11). Totally, 679 trajectory samples were observed at $\mathrm{OH}-4$ and Seward Road and were classified into passing, stopped, and running red vehicles.

Figure 12 shows the locations and speeds of observed stopping vehicles at yellow onsets. The locations are referred to as the stopping distance in response to the yellow indication. The profile of the minimum yellow-onset stopping distances is regressively modelled using the observed minimum stopping distances under various speeds, as illustrated in Figure 12. Similarly, Figure 13 shows the locations of observed passing vehicles at yellow onsets, which are referred to as the yellow-onset passing distance in response to yellow indications. The profile of the maximum yellow-onset passing distances is regressively modelled using the observed maximum passing distances under different speed ranges. Both profiles will be used as the observed $X_{\mathrm{c}}$ and $X_{0}$ values for calibrating the dilemma zone model.


Figure 11: Observed Vehicle Trajectories at the Onset of Yellow Indication


Figure 12: Profile of Observed Minimum Yellow-Onset Stopping Distance


Figure 13: Profile of Observed Maximum Yellow-Onset Passing Distance

To determine the values of the contributing factors, namely, $a_{\text {stop }}, a_{\text {pass }}, \delta_{\text {pass }}$, and $\delta_{\text {stop }}$, a process of trial-and-fit method is employed for the calibration. Appropriate values of the contributing factors are obtained through fitting the theoretically calculated $X_{\mathrm{c}}$ and $X_{0}$ values (based upon Equations (3) and (4)) to the observed $X_{\mathrm{c}}$ and $X_{0}$ values for various speed inputs from 24 mph to 50 mph . The calibration process are illustrated by Figure 14, where the goodness-of-fit analysis shows that the correlation coefficient $\mathrm{R}^{2}$ is 0.9976 for the modelled and observed $X_{\mathrm{c}}$, while the number is 0.9995 for modelled and observed $X_{0}$. Both the $\mathrm{R}^{2}$ values indicate a good fitting. Through the calibration, values of $a_{\text {stop }}, a_{\text {pass }}, \delta_{\text {pass }}$, and $\delta_{\text {stop }}$ for various speed inputs are finally obtained and shown in Table 1.

As mentioned in Section 4.3, $\mathrm{X}^{\text {th }}$ Percentile observed $X_{\mathrm{c}}$ and $X_{0}$ need to be used in this calibration process in order to consider the prevailing maneuvers. If the $95^{\text {th }}$ percentile observed $X_{0}$ and the $10^{\text {th }}$ percentile observed $X_{\mathrm{c}}$ are used as the calibration reference values, drivers of the stopped vehicles with yellow-onset distances $<10^{\text {th }}$ percentile $X_{c}$ are viewed as "extremely conservative" drivers. Meanwhile, drivers of the passing vehicles with yellow-onset distances > $95^{\text {th }}$ percentile $X_{0}$ are viewed as "extremely aggressive" drivers. Both "extreme" driving maneuvers will be precluded in the model calibration. To prepare for the calibration, the profiles of the $95^{\text {th }}$ percentile $X_{0}$ and the $10^{\text {th }}$ percentile $X_{\mathrm{c}}$ should be identified in advance, which requires the following three steps.


Figure 14: Calibration by Maximum $X_{c}$ and Minimum $X_{0}$ as Reference Values

Table 1: Obtained Parameters Values (Based on Maximum $\boldsymbol{X}_{\mathrm{c}} \&$ Minimum $\boldsymbol{X}_{\mathbf{0}}$ ) ( $\tau=4.5 \mathrm{~s}$ )

| Speed $V_{0}$ <br> $(\mathrm{mph})$ | $\delta_{\text {stop }}(\mathrm{s})$ | $a_{\text {stop }}\left(\mathrm{ft} / \mathrm{s}^{2}\right)$ | $\delta_{\text {pass }}(\mathrm{s})$ | $a_{\text {pass }}\left(\mathrm{ft} / \mathrm{s}^{2}\right)$ | $X_{\text {c-Calc }}(\mathrm{ft})$ | $X_{\text {c-Obs }}(\mathrm{ft})$ | $X_{0 \text {-Calc }}(\mathrm{ft})$ | $X_{0 \text {-Obs }}(\mathrm{ft})$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 24 | 0.4 | 4.3 | 0.68 | 11.1 | 158 | 161 | 239 | 238 |
| 26 | 0.415 | 5 | 0.65 | 9.9 | 161 | 162 | 245 | 244 |
| 28 | 0.43 | 5.8 | 0.62 | 9 | 163 | 164 | 253 | 251 |
| 30 | 0.445 | 6.6 | 0.59 | 7.9 | 166 | 166 | 258 | 257 |
| 32 | 0.46 | 7.5 | 0.56 | 6.9 | 168 | 168 | 265 | 263 |
| 34 | 0.475 | 8.4 | 0.53 | 5.8 | 172 | 171 | 270 | 269 |
| 36 | 0.49 | 9.4 | 0.5 | 4.9 | 174 | 175 | 277 | 276 |
| 38 | 0.505 | 10.3 | 0.47 | 4 | 179 | 179 | 283 | 282 |
| 40 | 0.52 | 11.2 | 0.44 | 2.9 | 184 | 184 | 288 | 288 |
| 42 | 0.535 | 12.1 | 0.41 | 2 | 190 | 189 | 294 | 294 |
| 44 | 0.55 | 13 | 0.38 | 1.3 | 196 | 195 | 301 | 301 |
| 46 | 0.565 | 13.9 | 0.35 | 0.5 | 202 | 201 | 308 | 307 |
| 48 | 0.58 | 14.8 | 0.32 | -0.4 | 208 | 208 | 313 | 313 |
| 50 | 0.595 | 15.7 | 0.29 | -1.2 | 215 | 216 | 319 | 319 |

Step 1: Determine the $95^{\text {th }}$ percentile $X_{0}$ by identifying the cumulative $95 \% X_{0}$ for each of the three speed classifications from 20 mph to 50 mph at an interval of 10 mph . As an example, Figure 15 illustrates how to identify the $95^{\text {th }}$ percentile $X_{0}$ from all the observed $\mathrm{X}_{0} \mathrm{~s}$ within the speed classification of $30-40 \mathrm{mph}$.


Figure 15: Determination of the $\mathbf{9 5}{ }^{\text {th }}$ Percentile $X_{0}$ for Speed Classification of 30-40 $\mathbf{~ m p h}$

Step 2: Determine the $10^{\text {th }}$ percentile $X_{\mathrm{c}}$ by identifying the cumulative $10 \% X_{\mathrm{c}}$ for each of the three speed classifications, and an example of the speed classification of $30-40 \mathrm{mph}$ is illustrated by Figures 16. If $X_{0}>X_{C}$, those $X_{\mathrm{c}} \mathrm{S}$ within the range $\left[\mathrm{X}_{\mathrm{C}}, \mathrm{X}_{0}\right.$ ] are considered as the sample for calculating the $10^{\text {th }}$ percentile $X_{\mathrm{c}}$. Samples are those $X_{\mathrm{c}} \mathrm{s}$ with values smaller than the furthest $\mathrm{X}_{0}$ within that speed classification. Figure 16 illustrates how to identify the sampling range. $X_{\mathrm{c}} \mathrm{s}$ of those stopped vehicles within the highlighted rectangle area constitute the sample. If $\mathrm{X}_{0} \leq \mathrm{X}_{\mathrm{C}}$, the second shortest $X_{\mathrm{c}}$ among all observed $X_{\mathrm{c}} \mathrm{S}$ will be viewed as the applicable values of $X_{C}$.

After Step 1 and Step 2, the $10^{\text {th }}$ percentile $X_{\mathrm{c}}$ and the $95^{\text {th }}$ percentile $X_{0}$ for each speed classification are identified and illustrated by Figure 17.

Step 3: Use the mid-point speed of each speed classification (e.g. use 35 mph for the speed classification of $30-40 \mathrm{mph}$ ) as independent variable x , and the corresponding $10^{\text {th }}$ percentile $X_{\mathrm{c}}$ and $95^{\text {th }}$ percentile $X_{0}$ as dependant variables y , respectively, profiles that represent the observed $10^{\text {th }}$ percentile $X_{\mathrm{c}}$ and the $95^{\text {th }}$ percentile $X_{0}$ under continuous speed inputs are obtained through regression analysis, as illustrated by Figure 18.


Figure 16: Sampling Range for $\mathbf{1 0}^{\text {th }}$ Percentile $X_{c}$ for Speed Class of 30-40 mph

Figure 19 shows the model calibration process of using the $10^{\text {th }}$ percentile $X_{\mathrm{c}}$ and the $95^{\text {th }}$ percentile $X_{0}$ as reference values (observed values). After the calibration, the calibrated values of $a_{\text {stop }}, a_{\text {pass }}, \delta_{\text {pass }}$, and $\delta_{\text {stop }}$ at various speeds are shown in Table 2.


Figure 17: Identified $10{ }^{\text {th }}$ Percentile $X_{\mathrm{c}}$ and $95^{\text {th }}$ Percentile $X_{\mathbf{0}}$ for Each Speed Classification


Figure 18: Profiles of $10^{\text {th }}$ Percentile $X_{c}$ and $95^{\text {th }}$ Percentile $X_{0}$


Figure 19: Calibration by the $10^{\text {th }}$ Percentile $X_{c}$ and the $95^{\text {th }}$ Percentile $X_{0}$ as Reference Values

Table 2: Values of Contributing Factors based on the $10^{\text {th }}$ percentile $\boldsymbol{X}_{\mathrm{c}}$ and the $\mathbf{9 5}{ }^{\text {th }}$ percentile $X_{0}(\tau=4.5 \mathrm{~s})$

| Speed $V_{0}$ <br> $(\mathrm{mph})$ | $\delta_{\text {stop }}(\mathrm{s})$ | $a_{\text {stop }}\left(\mathrm{ft} / \mathrm{s}^{2}\right)$ | $\delta_{\text {pass }}(\mathrm{s})$ | $a_{\text {pass }}\left(\mathrm{ft} / \mathrm{s}^{2}\right)$ | $X_{\text {c-Calc }}(\mathrm{ft})$ | $X_{\text {c--bs }}(\mathrm{ft})$ | $X_{0 \text {-Calc }}(\mathrm{ft})$ | $X_{0 \text {-obs }}(\mathrm{ft})$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 24 | 0.5 | 4.17 | 0.6 | 2.5 | 165 | 165 | 177 | 177 |
| 26 | 0.515 | 4.85 | 0.585 | 1.9 | 171 | 171 | 186 | 185 |
| 28 | 0.53 | 5.55 | 0.57 | 1.3 | 175 | 175 | 195 | 194 |
| 30 | 0.545 | 6.28 | 0.555 | 0.6 | 178 | 178 | 203 | 202 |
| 32 | 0.56 | 7 | 0.54 | 0.1 | 184 | 184 | 212 | 211 |
| 34 | 0.575 | 7.8 | 0.525 | -0.6 | 188 | 188 | 220 | 219 |
| 36 | 0.59 | 8.57 | 0.51 | -1.1 | 195 | 195 | 229 | 228 |
| 38 | 0.605 | 9.36 | 0.495 | -1.6 | 201 | 201 | 238 | 236 |
| 40 | 0.62 | 10.16 | 0.48 | -2.1 | 207 | 207 | 247 | 245 |
| 42 | 0.635 | 11 | 0.465 | -2.7 | 212 | 212 | 255 | 253 |
| 44 | 0.65 | 11.8 | 0.45 | -3.3 | 218 | 218 | 263 | 262 |
| 46 | 0.665 | 12.65 | 0.435 | -3.8 | 225 | 225 | 272 | 270 |
| 48 | 0.68 | 13.44 | 0.42 | -4.4 | 233 | 233 | 280 | 279 |
| 50 | 0.695 | 14.26 | 0.405 | -4.9 | 240 | 240 | 289 | 287 |

### 5.2 Development of the Dilemma Zone Look-up Charts

A dilemma zone look-up chart can be developed based on Equations (3) and (4) and the values of contributing factors $a_{\text {stop }}, a_{\text {pass }}, \delta_{\text {pass }}$, and $\delta_{\text {stop }}$ which are obtained from the model calibration process discussed in Section 5.1. The lookup table tells you whether RZ or OZ exists, and what the location and length of the zone are, for a specific speed and in response to a specific duration of yellow interval. Figure 20 is a dilemma zone look-up chart developed by using the parameter values in Table 2, which considers the majority of drivers.


Figure 20: Dilemma Zone Look-up Chart for Majority of Drivers

In Figure 20, there is only one $X_{\mathrm{c}}$ profile, because the value of $X_{\mathrm{c}}$ does not change with the yellow durations. And the feasibility of putting those $X_{0}$ profiles together is based on the research result that change of yellow duration does not affect the driving behavior (Olson and Rothery, 1962). In this figure, it is not hard to identify that when the yellow duration set to 4 seconds, the RZ is almost eliminated for higher speed inputs and the absolute value $\left|X_{\mathrm{c}}-X_{0}\right|$ is meanwhile minimized.

Figure 21 is a dilemma zone look-up chart that considers all possible driving behaviors, including extremely aggressive and extremely conservative behaviors, which is developed by using the parameter values in Table 1. In this figure, when the yellow duration is set to 3 seconds, the absolute value $\left|X_{\mathrm{c}^{-}} X_{0}\right|$ is minimized.


Figure 21: Dilemma Zone Look-up Chart for All Driving Behaviors

From both Figure 20 and Figure 21, it can be identified that the length of OZ becomes longer as the yellow duration increases, while the length of RZ becomes longer as the yellow duration decreases. Both of the results are in accordance with other researchers' results (Saito et al, 1990; Koll et al, 2004). It also can be identified that in the case of 4.5 -second yellow duration, there is always OZ and no RZ in the speed range from 24 to 50 mph . Therefore, it can be concluded that OZ dominates this case study site (OH-4 at Seward EB with 4.5 -second yellow duration).

### 5.3 Distributions of Option Zones under Different Vehicle Arrival Types

Besides modeling the dilemma zone, some discovering studies are also conducted in this project, including identifying the relationship between option zone and vehicle arrival types, and option zone distribution for various vehicle categories. In this section, how arrival type influences the length of option zone is studied.

Vehicle Arrival Type (TRB, 2000) describes the quality of progression at a definite approach. Base on Highway Capacity Manual 2000 (TRB, 2000), vehicle arrival type of each cycle can be estimated from Platoon Ration, which is calculated as:

$$
\begin{equation*}
R_{p}=\frac{P}{(g / c)} \tag{7}
\end{equation*}
$$

Where, $R_{p}=$ platoon ratio;
$\mathrm{P}=$ proportion of vehicles arriving on green;
$g=$ effective green time, s ;
$c=$ cycle length, s .
The relation between platoon ratio and arrival type is described in Table 6.
Table 3: Description of Vehicle Arrival Types (TRB, 2000)

| Arrival Type | Range of Platoon Ration, $R_{p}$ | Progression Quality |
| :---: | :---: | :---: |
| 1 | $\leq 0.50$ | Very Poor |
| 2 | $>0.5-0.85$ | Unfavorable |
| 3 | $>0.85-1.15$ | Random Arrivals |
| 4 | $>1.15-1.50$ | Favorable |
| 5 | $>1.50-2.00$ | Highly Favorable |
| 6 | $\geq 2.00$ | Exceptional |

Figure 22 illustrates the arrival type distribution resulted from totally 267 observed signal cycles. It is apparent that Arrival Type 4 dominates the arrivals and the progression is favorable to the operation as assessed by Table 3.


Figure 22: Distributions of Vehicle Arrival Types at Eastbound OH-4 at Seward

For the arrival types, we classify them into two categories, which are good progression and poor progression. Good progression refers to Arrival Types $\geq 4$, while poor progression refers to Arrival Types $\leq 3$. Comparisons are made between the option zone locations under the
two arrival type categories, where the length of the option zone is determined by the observed yellow onset trajectories as described by Figure 4. Results indicate that traffic in a good progression (Arrival Type $\geq 4$ ) has a further option boundary and longer option zone length than traffic in a poor progression (Arrival Type $\leq 3$ ). The results are in accordance with the findings of other researchers. Van der Horst and Wilmink (1986) found that the platoons formed by progression can increase the frequency of red-light-running, which is a kind of consequence caused by the effect of option zone. In a good progression, the drivers' desire to stay within the platoon makes them less willing to stop upon seeing the yellow indication, which causes a longer option zone. Figures 23 and 24 illustrate the option zone locations and lengths under different arrival type categories for the speed classifications of $30-40 \mathrm{mph}$ and $40-50 \mathrm{mph}$, respectively.


Figure 23: Option Zones under Different Arrival Types (Speed Classification 30-40 mph)


Figure 24: Option Zone under Different Arrival Types (Speed Classification 40-50 mph)

### 5.4 Distributions of Option Zones for Different Vehicle Categories

The distribution of the option zones are found out varying with different categories of vehicles. In this analysis, the locations and lengths of the option zones are identified by the same method as described in Section 5.3. As illustrated by Figure 25, the result indicates that Car has the longest length of option zone, while Truck has the shortest length. It shows that the length of option zone decreases as vehicle size increases, while the downstream boundary of option zone is further from the stop line as vehicle size increases. These findings reveal that the formation of the option zone is highly related to the maneuverability of vehicles. Vehicles with a more flexible maneuverability have a longer option zone.


Figure 25: Option Zone Distributions by Vehicle Categories (Speed Range 20-50 mph)

## CHAPTER 6:

CONCLUSIONS AND FUTURE RESEARCH

### 6.1 Conclusions

Since the concept of dilemma zone was proposed in 1960 (Gazis et al, 1960), it has been widely studied by researchers. However, terms conventionally defined to represent the yellow phase dilemma lack integrity, and are somewhat confusing in concepts. This research conducts a comprehensive literature review attempting to clarify the relationship among the dilemma zone, option zone, and indecision (decision) zone. A hierarchy diagram (Figure 3) is developed to help better understand the distinctions among different concepts and clearly identify the contributing factors to be targeted in the study. Some relevant key findings are summarized as follows:

- Dilemma Zone (DZ) is a general concept representing a range (length) of roadway, vehicles within which at the onset of yellow indication may experience yellow dilemma problem.
- There are two types of dilemma zone. The Risky Zone (RZ) is formed when $X_{\mathrm{c}}>X_{0}$. Vehicles within the RZ at the onset of yellow indications can neither manage to pass the stop line before the end of the yellow interval, nor safely stop before the stop line. Regardless passing or stopping decision is made by the driver, it is risky. The Option Zone (OZ) is formed when $X_{0}>X_{\mathrm{c}}$. Vehicles within the OZ at the onset of yellow indications can choose either to pass the stop line before the end of yellow interval, or to safely stop before the stop line. The driver has two options to choose.
- Indecision Zone (IZ) measures yellow dilemma indecisiveness from the probabilistic perspective. It overlaps with the OZ to some extent.

In this project, the dynamic dilemma zone is mathematically modeled, and its location and length are determined by the minimum safe stopping distance $X_{\mathrm{c}}$ and the maximum safe passing distance $X_{0}$, which can be represented by Equations (3) and (4), respectively. $X^{\text {th }}$ percentile observed $X_{\mathrm{c}}$ and $X_{0}$ derived from the vehicle trajectory data are used to calibrate the dilemma zone contributing factors $a_{\text {stop }}, a_{\text {pass }}, \delta_{\text {pass }}$, and $\delta_{\text {stop }}$. Dilemma zone look-up charts are developed from the calibrated dilemma zone models and they provide a convenient way to identify locations and lengths of dilemma zones for any speed and yellow duration inputs. Unlike traditional dilemma zone model with constant values of contributing factors (i.e. $a_{\text {stop }}, a_{\text {pass }}, \delta_{\text {pass }}$, and $\delta_{\text {stop }}$ ), the method presented in this research uses a more realistic assumption that values of those parameters vary with vehicle approaching speeds. Those values are determined through the calibration process using real-world observed trajectory data. Results from the dilemma zone look-up chart prove that as yellow duration increases, the length of RZ decreases while the length of the OZ increases.

Besides the modeling work, relationship between vehicle arrival types and option zone locations, as well as option zone distributions with various vehicle categories are explored in this project. Key findings are summarized as follows.

- Traffic in a good progression (Arrival Type $\geq 4$ ) has a further option zone boundary and longer option zone length than the traffic in a poor progression (Arrival Type $\leq 3$ ).
- The length of option zone decreases as vehicle size increases, while the downstream boundary of option zone becomes further from the stop line as the vehicle size increases.


### 6.2 Discussion and Future Research

As a preliminary case study for providing a proof-of-concept of the methodology for modelling the dynamic dilemma zones, this project validated the feasibilities of using trajectory-based $X^{t h}$ percentile $X_{\mathrm{c}}$ and $X_{0}$ profiles to calibrate dilemma zone contributing factors and develop dilemma zone look-up chart. The methodology used in this preliminary study has been proved to be workable and it establishes a solid basis for future study of optimal detection placement and related dilemma zone protection problem with the consideration of multiple speed ranges. However, there are still some aspects that need to be improved in future research.

First, future research will be expanded to more intersections to cover more categories of study site, such as

- High-Speed Mainline vs. Large-Volume Side Street;
- Mainline vs. Ramp/T Intersection; and
- High-Speed Mainline vs. Low-Speed Side Street.

Each category needs at least two study intersections for developing its specific dilemma zone look-up chart.

Second, more sample data are required at each study site. Equation (8) calculates the required sample size for conduct the statistical analysis.

$$
\begin{equation*}
\mathrm{N}=\left(t_{\alpha / 2} \frac{\delta}{\mu \varepsilon}\right)^{2} \tag{8}
\end{equation*}
$$

Where, $\mu$ and $\delta$ are the mean and standard deviation of the performance measures; $\varepsilon$ is the allowable error specified as a fraction of the mean $\mu$; and $t_{\alpha / 2}$ is the critical value of the $t$ distribution at the confidence interval of 1- $\alpha$.

Based on the means and standard deviations of $X_{0}$ and $X_{\mathrm{c}}$ for each speed classification we obtained through the observation, and by taking the allowable error as 0.05 and the confidence interval as $95 \%, 900$ observations of $X_{0}$ and 380 observations of $X_{\mathrm{c}}$ are calculated through Equation (8) as the required sample sizes for getting robust statistical results.

Third, a complete trajectory over the entire yellow interval is to be extracted for each vehicle, so that more accurate value ranges of the contributing factors $a_{\text {stop }}, a_{\text {pass }}, \delta_{\text {pass }}$, and $\delta_{\text {stop }}$ will be derived from trajectory data.

Fourth, the following assumptions are made in this project: (1) a vehicle with a higher speed needs more PRT to make a stop decision and higher deceleration to stop; (2) a vehicle with a lower speed needs more PRT to make a passing decision and higher passing acceleration to pass the stop line. These assumptions need to be further proved with the practical data.

Fifth, the optimal selection of the percentile values of $X_{0}$ and $X_{c}$ for developing the dilemma zone look-up charts needs to be further investigated in future research.

Sixth, the impact of vehicle arrival type on the location and length of the option zone needs to be further investigated and the results obtained in this project that high arrival types yields longer option zone needs to be validated by more observations at more study sites.

The most significant contribution of the dilemma zone look-up charts developed by the observed trajectory dada is that accurate dilemma zone locations and lengths under specific speeds could be easily obtained and it provides basis for the optimized deployment of loops for multi-speed dilemma zone protection at high speed intersections.

In future research, multiple loops would be placed based on the boundaries of dilemma zones in order to provide multi-speed protection at high speed intersections. For each speed range (e.g. 30-40 mph), a loop detector would be placed around the corresponding upstream boundary of dilemma zone for this speed range. For example, if three speed ranges are considered for dilemma zone protection, totally three dilemma zone detectors would be placed at this approach, as illustrated by Figure 26. Proper unit extension (UE) and passage time (PT) will be determined by the specific protected speed of the loops and the location of the nearest point of conflict.

This loops layout design would be evaluated and calibrated in a microscopic simulation environment (e.g. VISSIM) in order to achieve a balance between dilemma zone protection performance (evaluated by vehicle numbers in dilemma zone, max-out frequency and gap-out frequency) and operational efficiently (evaluated by overall delay). Results of this loops layout design will also be compared to other popular layout designs, such as Single-Detector configuration, Beirele configuration, SDITE configuration, and Bonneson configuration.


Figure 26: Illustration of Relations between Modeled DZ and Loops Placement

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