Traversing Tight Tunnels—Implementing an Adaptive Concertina Gait in a Biomimetic Snake Robot

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ABSTRACT

Snakes move through cluttered habitats and tight spaces with extraordinary ease, and consequently snake robots are a popular design for such situations. The remarkable locomotor performance of snakes is due in part to their diversity of locomotor modes for addressing a range of environmental challenges. Concertina locomotion consists of alternating periods of static anchoring and movement along the snake's body, and is used to negotiate narrow spaces such as tunnels or bare branches. However, concertina locomotion has rarely been implemented in snake robots. In this paper, the anchor formation process in live snakes during concertina locomotion is quantified and used to implement concertina locomotion in a snake robot, with automatic detection of the width of the tunnel walls to modulate the waveform. This allows effective concertina locomotion in tunnels of unknown width, without prior knowledge of tunnel geometry, expanding the range of gaits in snake robots.

INTRODUCTION

The elongate, limbless body plan is extremely common throughout nature, in both vertebrate and invertebrates alike (Pechenik, J. A. 2005; Pough, F. H. et al. 2002), and has evolved independently numerous times (Gans, C 1975). Indeed, within lizards (Order Squamata) alone there are over two dozen independent evolutions of limblessness or functional limblessness (Wiens, J. J. et al. 2006). Snakes are by far the most successful limbless taxon, with a near-global distribution, nearly 3000 species, and a wide range of ecological niches and habitats (Pough, F. H. et al. 2004). Limb loss typically occurs in animals which are either fossorial burrowers or which inhabit dense vegetation (Gans, C 1975; Wiens, J. J. et al. 2006), with the loss of limbs enabling them to move more freely through cluttered environments.

Unlike terrestrial limbed locomotion, in which force-generating substrate interactions are largely restricted to the ends of the appendages, much or all of the body of a limbless organism is in contact with the substrate during locomotion (Gray, J. 1946). Consequently, limbless locomotion is strongly dependent upon the environment, and many limbless vertebrates can switch between multiple discrete modes of locomotion (Gans, Carl 1986; Jayne, B C 1986). Unlike limbed organisms, which change gait with speed (from walk to run to gallop) (Hildebrand, M. 1985), limbless vertebrates select different locomotor modes based on their environment (Gray, J., and Lissmann, H. W. 1950; Jayne, B C 1986). Snakes show the greatest diversity of locomotor modes, with four distinct modes (Gray, J. 1946; Jayne, B C 1986; Lissmann, H. W. 1950; Mosauer, W. 1932), though each mode may have subtle variations (Astley, H. C., and Jayne, B. C. 2007; Jayne, B C 1986), may grade into each other (Jayne, B C 1986), and snakes may even use different modes at different points along their body at the same time (pers. obs.).

The most familiar of these modes is lateral undulation, more colloquially called "slithering", in which the snake generates posteriorly-propagating horizontal bends of undulation along the

body (Gray, J., and Lissmann, H. W. 1950; Jayne, B C 1986; Mosauer, W. 1932). These bends press against obstructions in the environment (e.g. rocks, twigs, grass) forming points of contact which generate opposing reaction forces to propel the snake forward (Gray, J., and Lissmann, H. W. 1950). As a consequence of adapting to the environment encountered, these bends may be highly irregular in size and shape (Astley, H. C., and Jayne, B. C. 2009; Gray, J., and Lissmann, H. W. 1950; Jayne, B C 1986). During lateral undulation, all points on the snake are moving with the same velocity, and largely follow the path of points ahead of them, with minimal slip (Jayne, B C 1986; Sharpe, S. S. et al. 2014). Sidewinding is an uncommon mode, seen in only a handful of species for traversing granular media, but shares the property of posteriorly-propagating undulations with the addition of second, offset, vertical wave which lifts and lowers the body (Jayne, B C 1986; Marvi, H. et al. 2014; Mosauer, W. 1932). Consequently, each point on the body is cyclically lifted up, moved forward, and then placed back on the ground, in regions of stasis and movement which propagate down the body, allowing traversal of sandy environments with no slip (Gans, C, and Kim, H. 1992; Jayne, B C 1986; Marvi, H. et al. 2014). During concertina locomotion (Fig. 1), snakes anchor themselves to the substrate with bends of the body (whether by friction with the ground, lateral pressure against tunnel walls, or medial gripping on branches) then release the anterior anchor bends and move that portion of the body forward to form new anterior anchor bends, which allows them to release the posterior anchor bends and pull them forward (Gray, J. 1946; Jayne, B C 1986; Jayne, Bruce C, and Davis, J. D. 1991; Mosauer, W. 1932). This mode is slower than the others and metabolically much more expensive but allows traversal of tight tunnels and narrow branches which are otherwise inaccessible (Jayne, B C 1986; Jayne, Bruce C, and Davis, J. D. 1991; Walton, M. et al. 1990). Finally, rectilinear locomotion does not rely on any axial movement at all, or even skeletal movement; this mode is entirely produced by motion of the skin (Lissmann, H. W. 1950; Marvi, Hamidreza et al. 2013; Mosauer, W. 1932). Each segment of belly skin is lifted up and moved forward then placed down and pulled backward, and slight timing offsets result in posteriorly propagating waves of motion (Lissmann, H. W. 1950; Marvi, Hamidreza et al. 2013; Mosauer, W. 1932).
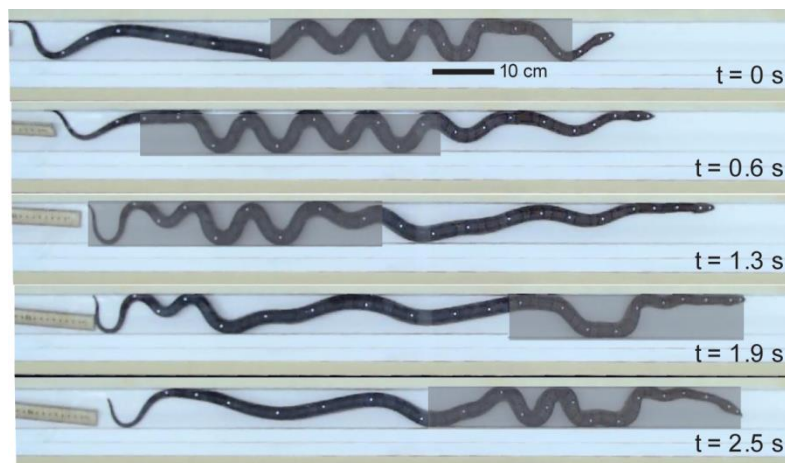

Figure 1 – A corn snake performing concertina locomotion in a 5.9 cm width tunnel as part of this experiment. White dots are motion capture markers. Grey boxes show regions of static contact.

Snake robots have attracted attention since the 1970's (Hirose, S. 1993) for their potential to replicate the ability of snakes to negotiate cluttered, confined, and otherwise inaccessible areas, with potential uses ranging from machinery repair to surgery to search and rescue to

extraterrestrial exploration (Hopkins, J. K. et al. 2009). While duplicating the elongate body form of snakes gives these robots a low cross-section for a given mass (necessary for moving through holes and tight confines), a snake robot needs to be capable of many or all of the locomotion modes seen in real snakes to be able to cope with diverse environments it may encounter. However, to date, most snake robots are only capable of one or two modes of locomotion. The most common is lateral undulation, either with the addition of passive wheels to create large frictional anisotropy for smooth-surface movement or without wheels (Hopkins, J. K. et al. 2009). Sidewinding, concertina, and rectilinear have been implemented in a smaller number of snake robots, with varying levels of fidelity to the original snake behavior (Astley, H. C. et al. 2015; Barazandeh, F. et al. 2007; Burdick, J. W. et al. 1993; Gong, C. et al. 2015; Marvi, H. et al. 2014; Marvi, Hamidreza et al. 2011, 2013; Shan, Y., and Koren, Y. 1993; Virgala, I. et al. 2014).

Concertina locomotion is a particularly important mode of locomotion for traversing the tight spaces which are frequently invoked as future locomotor environments for snake robots, particularly tunnels (either natural or manufactured). The largely parallel walls of tunnels or pipes effectively prevent the generation of anteriorly-directed reaction forces, preventing lateral undulation unless obstacles (Astley, H. C., and Jayne, B. C. 2009) or bends in the tunnel (Gray, J., and Lissmann, H. W. 1950) are present. Sidewinding is suitable only for open spaces due to the wide area of influence for the snake (Gray, J. 1946), and while rectilinear is ideal for moving through tunnels, this mode is also very slow (Lissmann, H. W. 1950; Marvi, Hamidreza et al. 2013) and requires an extensively actuated skin. Few snake robots have attempted concertina locomotion in tunnels, and most are highly restricted in use, allowing only a single mode of locomotion (Barazandeh, F. et al. 2007; Marvi, Hamidreza et al. 2011; Shan, Y., and Koren, Y. 1993; Trebuňa, F. et al. 2015), having a highly limited range of diameters (Barazandeh, F. et al. 2007; Trebuňa, F. et al. 2015), only performing flat-surface concertina (Shan, Y., and Koren, Y. 1993), or are "model based", in that they do no consist of a series of repeating identical motors (Marvi, Hamidreza et al. 2011); none are capable of other modes of locomotion.

In this paper, biomimetic principles are used to translate the behavior of living snakes to a robot, in order to implement tunnel concertina locomotion in a "generalist" snake robot (which is also capable of lateral undulation and sidewinding) with no modifications to the snake robot external morphology. This method seeks to apply the benefits of millions of years of snake behavioral evolution to the robot, and has been used previously to substantially improve the performance of sidewinding snake robots on inclines (Marvi, H. et al. 2014) and when turning (Astley, H. C. et al. 2015). The concertina movement of corn snakes (*Pantherophis guttatus*) moving in tunnels of two diameters is analyzed to devise an algorithm for tunnel concertina locomotion then determine the most essential characteristics of this mode of locomotion to create a bioinspired concertina locomotion algorithm. Furthermore, this algorithm is capable of adapting to a range of tunnel diameters without user input, greatly expanding its range of usefulness.

METHODS

Study Animals: Four adult, wild-caught corn snakes (*Pantherophis guttatus*) with masses of (445 ± 120 grams) and snout-vent lengths of (95.1 ± 7.9 cm) were used for this study, performed in accordance with University of Akron IACUC protocol 16-08-16-ASD. Animals were housed in a "rack style" caging system with enclosures of 58 x 40 x 15 (L x W x H) furnished with aspen shavings, a hiding spot, and a water bowl with *ad libitum* water. Heat was provided by

heat tape (basking area: 27°C, ambient temperature: 24°C), and snakes were fed one adult mouse every 14 days.

Snake Experiments: A 1.8 meter long tunnel with a rectangular cross-section was constructed of expanded PVC for the floor and walls, with additional snug-fitting expanded PVC inserts to reduce tunnel diameter. The tunnel was covered by a sheet of clear acrylic to prevent snakes from leaving the tunnel during experiments, with a tunnel height of 4.6 cm, and a small cardboard box was placed at the far end of the tunnel to provide snakes with a goal. Snakes were all tested at body temperatures of 24°C (within the range of field active temperatures for this genus (Brattstrom, B. H. 1965)), and were given a minimum of 5 minutes to rest between trials, as concertina locomotion is metabolically expensive and snakes show low endurance during this mode (Jayne, Bruce C, and Davis, J. D. 1991; Walton, M. et al. 1990). Snakes were presented with two tunnel widths, 5.9 and 7.9 cm; wider tunnel widths prompted ineffective lateral undulation until exhaustion (Astley, H. C., and Jayne, B. C. 2007). During trials, snakes were encouraged to move with gentle contact on the tail (Fig. 1). Three trials were conducted at each width for each individual, with multiple cycles of locomotion visible per trial.

Data were collected using an Optitrack Flex13 4-camera IR motion capture system, which tracked the movement of flat, circular (3.2 mm diameter) reflective stickers on the mid-dorsal line of the snake at 120 frames per second. These markers were manually placed on the head and at approximately 5 cm intervals along the anterior of the snake's midline, with the spacing increasing to 10 cm posteriorly, as the initial anterior anchor was considered the most important, with a total of between 15-20 markers per snake; the tail was not marked due to failure of the markers to reliably adhere. The XYZ coordinates of each of these points in meters (relative to a ground plane object during calibration) were exported after each trial to a CSV file.

Snake Data Analysis: Marker data were imported into a custom MATLAB script, which sorted the points into an m x 3 x n array, where m is the number of points and n is the number of frames, with the other axis representing the x, y, and z axes. An automated process used distance calculations to resolve point identity discrepancies and to order the points from head to tail, after which a cubic smoothing spline for each point over time was used to smooth out errors and fill in gaps smaller than 20 frames ($1/6^{th}$ second) with splines being computed independently along each axis and for each point. A second, interpolating cubic spline was used across all point at a given time in a given axis to upsample the snake body to 500 points for easier computation. Following this, for each point and each time, the total velocity was computed for each point (forward velocity yielded similar results) and the local body curvature (inverse of radius of curvature) across a 20-point window centered on the current point (excluding the first and last 10 points) was calculated; curvature is used in preference because the value drops to zero as a body segment straightens, rather than rising rapidly to infinity. Negative versus positive curvature was determined by the sign of the cross product of vectors extending from the current point to 10 points anterior and from 10 points posterior to the current point. Points with a total velocity of less than $2/3^{rd}$ the mean velocity were considered static. Heat maps of velocity and curvature over time and body length for the period when all markers were visible were inspected to determine the behavioral algorithm for tunnel concertina in the snake robot.

Robot Overview: The snake robot consisted of a series of 24 Dyamixel XL-320 actuator (ROBOTIS Inc.) arranged with alternating horizontal and vertical movement axes to allow both vertical and horizontal movement (12 actuators in each direction); actuators were joined with 3d-printed ABS brackets and hot glue. Dynamixel actuators have several features which differ from standard commercial servomotors, including the ability to "daisy-chain" up to six motors

together (controlling them with a single 3-wire input), half duplex asynchronous serial communication up to 1 Mbps, and, most notably, the ability to both dynamically control numerous aspects of the actuator (goal position, goal speed, maximum torque, PID gain coefficients) and get feedback from the servo (position, speed, torque, temperature). Actuators are controlled using an OpenCM 9.04C controller, a proprietary controller broadly similar to an Arduino Pro, with an Arduino-like C++ based IDE programming language. This snake robot had previously been programmed to perform sidewinding and lateral undulation, and the concertina algorithm was added to the existing code. Modes were selected using a rotary switch connected to an analog input. Only concertina mode was used in these experiments. The fully constructed snake robot has a mass of 550 grams, a length of 90 cm, and a square cross-section of 3.4 cm. All motors had PID control, with I and D gains set of zero and the P gain set to 32.

Robot Experiments: To test the effectiveness of this algorithm, the robot was placed in two tunnels with widths of 7.3 cm and 14 cm, with frictional walls of foam rubber shelf liner to compensate for the very low friction between the ABS snake robot brackets and expanded PVC wall. Preliminary testing with 9.5 cm tunnel width showed that it was insufficiently wide to trigger the three-motor anchoring configuration, producing the same two-motor behavior at 7.3 cm but with larger motor deflections, and so this width was not included. Overhead video of the robot trial was recorded using a Logitech C920 webcam at 1080p, and calibrated with a meter-stick.

RESULTS

Snake results: All snakes, when presented with the two tunnel widths above, performed tunnel concertina locomotion for all cycles (Fig. 1); at higher tunnel widths, concertina cycles were intermittently punctuated by attempts at lateral undulation with extremely high slip and no forward progress. Snakes moved at an average speed of 5.0 ± 1.6 cm/s, with a period of 3.6 ± 0.8 seconds. As in prior studies (Astley, H. C., and Jayne, B. C. 2007; Jayne, B C 1986; Jayne, Bruce C, and Davis, J. D. 1991), each point on the snake's body alternated between movement and stasis (Fig. 2A), and bends in the body did not propagate posteriorly (Fig. 2B). Regions of movement and stasis in the body propagated posteriorly down the snake, though not always at constant speed (which would be a constant-slope region in Fig 2A). In many cycles, some fraction of the most anterior segments began moving simultaneously, followed by a posterior propagation, and ending in near-simultaneous movement of a fraction of the most posterior points (as seen in the first cycle of Fig 2A). At any time, an average of $56.0 \pm 3.8\%$ of the body was moving, though this varied greatly within a cycle (Fig. 2A).

Body curvature was greatest in static regions and lower during movement (Figs 1, 2B), with changes in curvature tending to occur at the transition between static and moving. Posterior propagation of the moving region was associated with a posteriorly propagating decrease in curvature magnitude (Fig 2B). Additionally, the curvature during static periods was not repeatable for a given point along the body; a single point could be bent to the right in one cycle, straight the next, and bent to the left in the next, or any variation of that order (Fig 2B). This was particularly notable in the anterior of the snake, where the anterior anchoring bends for a new cycle would be formed, while posterior bends tended to be more repeatable (Fig 2B). The irregularity of these bends suggests that the snake is forming each anterior contact without prior assumptions about tunnel width, but is instead judging tunnel width based on feedback from the body.
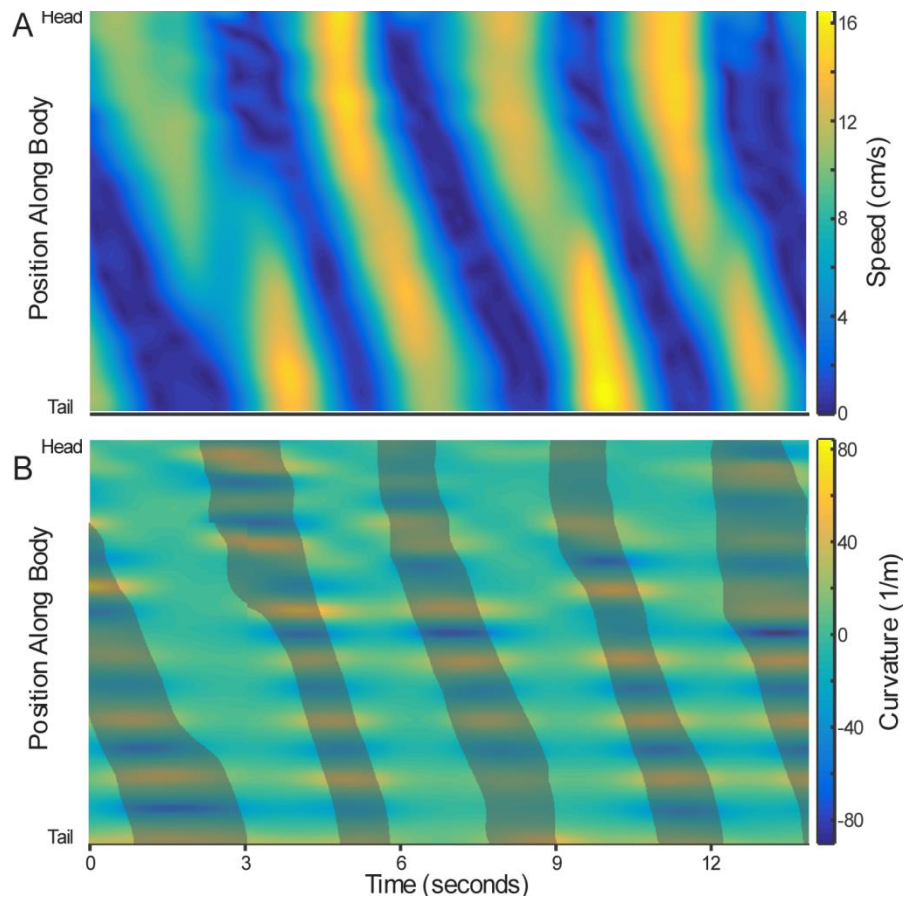
Figure 2 – Heatmaps showing velocity (A) and curvature (B) along the snake's body (vertical axis) over time (horizontal axis) for the same trial as in Figure 1. Shaded regions in B show static contact. A horizontal row shows changes over time for a point, while a column shows values for all points along the snake at one time. Posterior propagation of moving and static regions is visible as a slope from top left to bottom right.

Robot Algorithm: These results formed the basis of a robot concertina algorithm. Although total replication would be ideal, the limitations of robot design forced a significant departure on the issue of regularity: while corn snakes typically have over 220 body vertebrae, the snake robot has only 12 horizontal motors. Consequently, some level or regularity had to be imposed upon the waveform of the snake robot in order to ensure that 12 motors would produce reliable and effective motion.

The overall control algorithm includes 4 repeating stages, implemented as separate functions in the code, with only a few variables passed between them. These stages are "Anterior Straightening", "Anterior Anchoring", "Posterior Straightening", and "Posterior Anchor".

During the Anterior Extension phase, the first 6 motors are straightened, regardless of their initial position (Fig. 3). If the robot has not performed a cycle of concertina previously, the entire robot is straightened, to eliminate any effects of previous lateral undulation or sidewinding postures and to facilitate introduction of the robot into the tunnel.

Anterior Anchoring begins by imposing an ever increasing set of opposite curvatures to the first two motors, such that one deflects to the left and the other to the right, translating the inert head segment and first vertical motor sideways without net rotation (Fig. 3). After each movement command of an additional 1.2° of deflection and an 80 millisecond delay, position

feedback was used to compute the position error for each motor, the difference between commanded and achieved position. The P gain functions like a Hookean spring, the larger the position error, the greater the voltage applied and consequently the greater the torque. If the sides of the robot contact the walls of the tunnel, the motors will be able to move no further, causing an increasing position error and an increased lateral bracing force, similar to the joint locking mechanism of (Barazandeh, F. et al. 2007) but requiring active energy expenditure. At a position error of 5.8° (chosen to balance force applied to the tunnel walls versus the possibility of actuator overload), the anchoring movement ceases and returns a value of 2 from the function, to indicate the number of motors involved (see below).

If the position error never reaches this level, as in a larger tunnel, the two-motor implementation terminates when each motor reaches 75.4°, and a second implementation takes over, in which the first motor remains in position while the second motor straightens and the third bends, with the same rates and parameters as previously used in the 2-motor anchoring protocol, translating the anterior-most segment further laterally (Fig. 4). As before, if position error exceeds the threshold of 5.8°, the anchoring movement ceases and returns a value of 3 from the function, indicating a three-motor anchor. If the position error again reaches 75.4°, the tunnel is too wide for the snake robot in this current algorithm, and a value of zero is returned from the function, terminating locomotion.
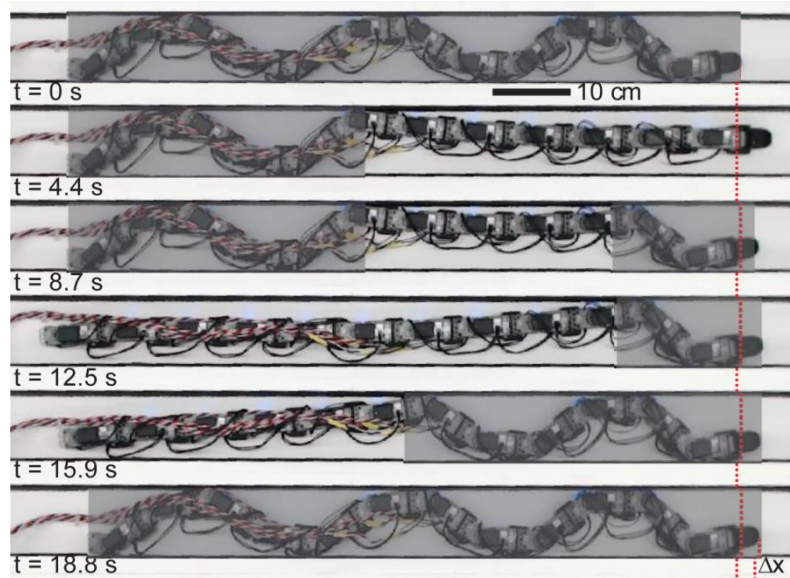

Figure 3 – The snake robot performing concertina locomotion in a 7.3 cm width tunnel in accordance with the commanded algorithm. Shaded grey regions indicate static contact, and the red dashed lines indicate starting and ending position. Δx was 2.3 cm for this cycle.

Once the Anterior Anchoring phase is complete, the Posterior Straightening phase starts, in which the posterior 6 motors of the snakebot are all simultaneously straightened (Fig. 3). Then, during the Posterior Anchoring phase, the commanded position of the motors involved in the anterior anchoring (whether 2 or 3) are sent to the posterior motors in sets of 2 or 3, each mirroring the previous set, one bend at a time (Fig. 3). At the conclusion of the Posterior Anchoring phase, the entire snake robot is in static contact with the walls, and the cycle can begin again (Fig. 3). Because of the total number of motors (12), anterior straightening in postures derived from two-motor anchoring would allow the anchor to be formed in the same

direction as previously (Fig. 3) but, in 3-motor configurations, the straightening would require that the anterior anchoring phase alternate sides (Fig. 4); this ability was added to the code via global "side" variable of positive or negative one, which was multiplied by the increment of movement in anterior anchoring (Fig. 4).

Robot Results: Implementation of this algorithm in the 7.3 cm width tunnel produced effective concertina locomotion in the snake robot, using two motors for the anterior anchor point (Fig. 3). Displacement was 2.3 cm in the sample cycle depicted (Fig. 3); other cycles were similar. In the 14 cm wide tunnel, the snake robot was unable to contact the walls using only two motors for anchoring but was able to make contact with the three motor anterior anchoring method (Fig. 4), and this method was able to produce reliable concertina locomotion with a displacement of 5.9 cm per cycle (Fig. 4). During the anterior straightening phase with three motors, the posterior motors did not have sufficient torque to direct the straightened anterior portion of the robot parallel to the wall, thus a small "nudge" from the experimenter was required – the addition of I gain in future implementations may remedy this shortcoming.
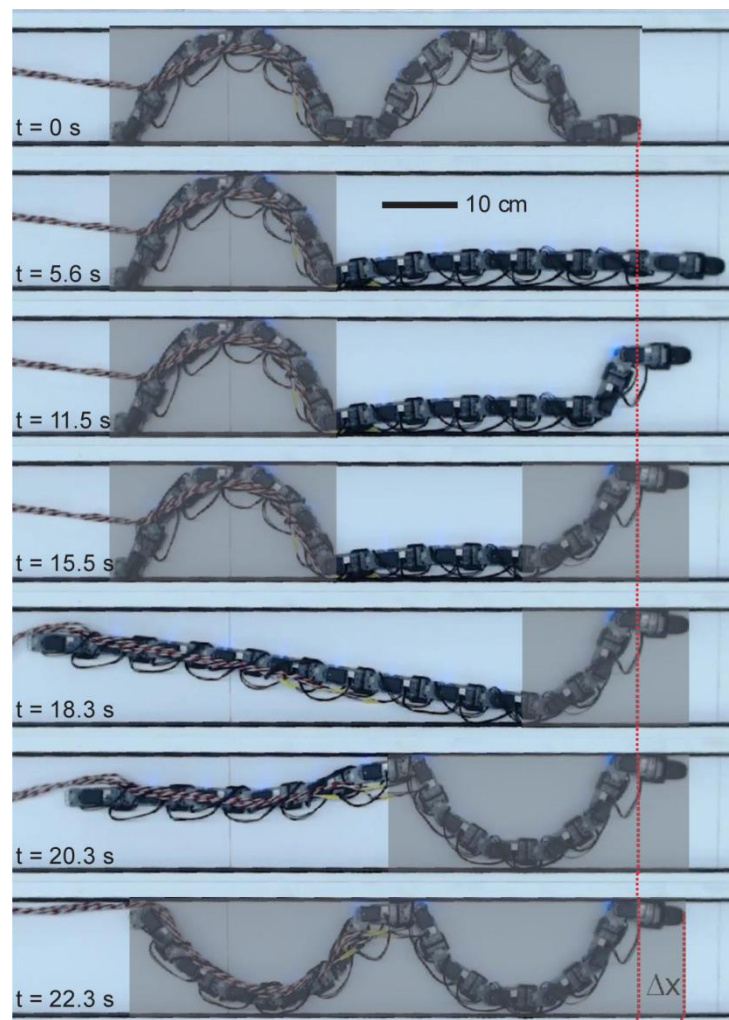

Figure 4 – The snake robot performing concertina locomotion in a 14 cm width tunnel in accordance with the commanded algorithm. Shaded grey regions indicate static contact, and the red dashed lines indicate starting and ending position. Δx was 2.3 cm for this cycle.

DISCUSSION

The ability of the robot to dynamically adjust to tunnel width, and to do so for each cycle, without user input or any higher-level sensory information, is a significant benefit to the robot's ability to move in the real world. If a robot's locomotion is sensitive to conditions and perturbations, it may require either user guidance, which limits autonomy, or higher level sensory systems, which require additional instrumentation and computation. In contrast, by adapting its locomotion to a wide range of tunnel widths, this robot requires no additional input to move effectively, reducing weight, power, and computational needs.

This algorithm was also implemented in a "generalist" snake robot, one explicitly constructed for sidewinding studies (hence the incorporation of vertical motors), with no special modifications for concertina locomotion such as locking motors (Barazandeh, F. et al. 2007) or anisotropic frictional skin (Marvi, Hamidreza et al. 2013). These additions would doubtless increase the performance of the robot by reducing energy consumption and improving surface interactions but this shows that the algorithm is robust enough to achieve concertina locomotion even in the absence of such improvements, requiring only sufficient friction with the environment. Although the actuators used in this robot are quite sophisticated and capable of extensive control and feedback, the actuators were configured for purely proportional control, as in commercial servomotors, and this algorithm uses only position feedback, a feature available on a growing fraction of commercially available servomotors.

However, this algorithm is not without limitations, most notably, changes in tunnel diameter will cause failures of the posterior anchors. The anterior anchoring phase determines the tunnel width, and all subsequent bends are based upon the positions of those first two or three actuators. Consequently, if the tunnel narrows, the anterior anchor will detect this, but the posterior anchors will be at insufficient widths to contact the wider walls in the posterior part of the tunnel, preventing anchoring during the subsequent Anterior Straightening phase and possibly leading to high slip. Alternatively, if the tunnel widens, the anterior anchor will detect this too, but the posterior anchors will be positioned at too great a width, possibly causing motor overload or damage due to excessive force in interaction with the walls. Work is ongoing to use the feedback mechanisms during the Anterior Anchoring phase to counteract this but is not yet complete. Additional future improvements could include the ability to negotiate turns in pipes (or to switch to lateral undulation, if possible (Gray, J., and Lissmann, H. W. 1950)), as well as increases in speed, torque, and number of actuators, to more effectively replicate a living snake.

CONCLUSIONS

In this study, the concertina locomotion of snakes was quantified and used to devise a bio-inspired adaptive concertina control algorithm. This algorithm was applied to a "generalist" snake robot, allowing it to successfully perform concertina locomotion at a range of tunnel diameters by dynamically adapting its posture to the tunnel width, with no prior information on tunnel width. Concertina locomotion is specifically used by snakes for movement in confined spaces, and consequently, this algorithm (and future improvements thereof) can be useful for snake robots in a wide range of settings, such as moving through pipes to repair machinery, exploiting gaps in the rubble to locate survivors of building collapses, and even exploring rock piles on other worlds.

ACKNOWLEDGEMENTS

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