Computer Modeling for the Complex Response Analysis of Nonstandard Structural Dynamics Problems

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Abstract: Over the past several decades, two intriguing classes of problems, having a wide range of applications in engineering, have been of interest to many researchers: (1) coupled dynamics of a distributed parameter system traversed by one or more moving oscillators; and (2) transient dynamic analysis of axially moving media (and associated phenomena of parametric resonances). Bridge vehicle interaction falls into the first class of problems, and the analysis of flexible appendages deployed from a satellite or a spacecraft is typical of the second class. Mathematically, these two problems are dual to each other, and they often are highly nonlinear in nature and typically involve large overall motion in space with complex effects of convective inertia terms in their governing equations of dynamic equilibrium. The “nonstandard” analytical nature of these problems stems from the fact that we are dealing with one or more of the following peculiarities: (1) variable problem domain; (2) varying spatial distribution of forces over the time duration of the analysis; and/or (3) changing location and type of constraints. Many researchers are trying to formulate the response of these problems, each having a different approach, but applicable only to certain specific details. Moreover, few researchers have concluded that these problems are beyond the scope of the present commercial finite-element (FE) software packages. However, we believe that if the nature and details of these problems are studied properly and carefully, it is immediately possible to simulate these problems in available commercial FE programs. An added advantage would also be the avoidance of many unrealistic simplifying assumptions that are often introduced to reduce the mathematical complexity; e.g., neglecting possible separation (after periods of prior contacts) in beam-moving vehicle problems, assuming linear behavior of suspension systems, and restriction to beam configuration only, among many others. For demonstration, we use ABAQUS in a large number of test cases to be presented. The results are compared with those presented in literatures.

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Introduction

Motivation

This work is concerned with two spatial classes of dynamics; i.e., (1) a primary (continuous or distributed-parameters) system traversed by a set of moving discrete sub-(secondary-) systems, as typical in bridge vehicle interaction (BVI); and (2) the problem of axially moving media (AMM). Each of these problems has a long standing history in mechanics and has become especially important in recent years for many applications, ranging from transportation systems in civil engineering to advanced processes of deployment and retrieval in space exploration industries. The dynamic response quantities, such as deflections, stresses, etc., are always higher than those in the static case and thus require accurate analysis of forces and stresses for reliable design and service life prediction. Unlike standard dynamic problems, where the forces (e.g., applied load, inertia, and elastic interaction force) are fixed in their spatial locations, these nonstandard dynamic problems involve forces that are changing their spatial location with time, leading to singular terms in mass, stiffness, and damping matrices of standard finite-element analysis (FEA) approach, and highly coupled nonlinear system of differential equations due to interaction between system and moving subsystem. Despite physical differences, the essence of these problems is the same, and hence, dual to each other.

The available extensive literature on this subject brings out many solution approaches that have been developed over the decades. Most of the formulations are problem specific and often consider idealized cases only, such as a simply supported beam, continuous contact between system and subsystem, and linear geometric deformations. A glimpse of different approaches for BVI problems can be found in Yu and Chan (2007). Moreover, each formulation has a special numerical scheme, and thus require its independent implementation which may not be applicable to other similar problems. However, there is notable paucity in the available literature in the use of commercial, large-scale, FEA codes. Our main motivation here is to fill such a gap. For a greater benefit to the practicing engineers, we do not intend to show “bias” toward the use of either explicit or implicit dynamics, therefore we selectively report a number of results for each of the options (ABAQUS/Explicit versus ABAQUS/Standard) for demonstration purposes only.

Scope and Formulation

We provide a solution using the ABAQUS program for a number of BVI and AMM problems to compare with the extensive avail-
able literature. To this end, the primary systems can have any configuration (i.e., cable, beam, plate, and shell configurations) and we account for intermittent impact conditions (contact and separation, as well as the possibility of having a series of moving secondary oscillators). Furthermore, we account for the effects of geometric nonlinearities when appropriate (e.g., pretensioned cable, parametric resonance phenomena, etc.). Finally, we provide dynamic response quantities related to both primary and/or secondary oscillators.

For a background on the mathematical formulations, we refer to the typical works of Akin and Mofid (1989), Yang and Yau (1997), Biondi and Muscolino (2005), and Sofi and Muscolino (2007) for BVI; and Vu-Quoc and Li (1995), and Behdinan and Tabarrok (1997a,b) for AMM problems. Note that the complexity here stems from the appearance of singular time-dependent terms for the interaction forces (gravitational, inertia, elastic) that are spatially varying due to the motion of secondary oscillators. Computationally, this leads to the need for varying topology in forming the “finite-element” operators (inertia, stiffness, load vectors, etc.) rendering the data structure markedly different from conventional FEA approaches.

The key of our proposed methodology lies in the modeling of the actual physics of the problem through contact interaction capabilities of large scale FE software packages and large overall sliding. In our modeling, the system and subsystems are two independent sets of objects and the contact/interaction provides a mode of communication for different response quantities (Fig. 1). In our treatments using ABAQUS, we completely bypass the difficulty by directly accounting for the spatial variations through ABAQUS interaction-effects capabilities. In this approach, the use of large sliding is essential, and all the problems are thus treated nonlinearly (for detailed information, see Saleeb and Kumar 2009; Kumar and Saleeb 2009).

Numerical Examples

To illustrate the validity of our methodology, six numerical simulation examples are presented in this paper. The examples are selected from literature on the basis of their engineering applicability and complexity involved in their mathematical model. All finite-element simulations are performed in ABAQUS, treating each problem as a nonlinear dynamic problem, involving small or large deformation mechanics (depending on the targeted comparisons with literature and/or the investigated phenomena of interest), frictionless tangential behavior between system and subsystem, and “hard” pressure overclosure relationship for contact definition, unless specifically mentioned. Furthermore, the results are presented for final converged mesh after performing mesh convergence analysis.

Dynamic Analysis of Suspended Cable Carrying Two Oscillating Subsystems with Varying Speed

Although the dynamic analysis of continuum cable, carrying moving subsystem, has been considered as a difficult problem due to nonlinear geometry and coupling between cable and subsystem, it has the same features and complexity involved as other BVI problems discussed in the literature. The schematic of the problem is shown in Fig. 2. A circular cross section for the cable was chosen in order to satisfy cable definition.

The cable was modeled with 50 Euler–Bernoulli beams (B23), whereas the oscillators were modeled as a spring—point mass—dashpot. The analysis was performed in two steps, accounting for fully nonlinear geometry and deformations; a static step to get an initial catenary shape by applying a uniform line load of magnitude equaling weight per unit length of the cable, followed by a dynamic step to analyze the response of cable due to the motion of two moving oscillating subsystems. The midspan vertical deflection of cable obtained at the end of the static step was 11.7275 m, after accounting for fully nonlinear deformation. In the dynamic analysis step, the spring mass subsystems were moved with constant velocity, \(v=5 \, \text{m/s} \) for the first 90 s and then a constant deceleration, \(a=-0.5 \, \text{m/s}^2\), so that the subsystems were brought to rest at \(t=100 \, \text{s}\). The motion of oscillators was defined by displacement amplitude function.

The time-histories of midspan vertical displacement, the vertical displacement at the contact point between the cable and the second oscillator, and the slope at midspan, are shown in Fig. 3. The results obtained from ABAQUS/Standard simulation are in

Fig. 1. Overall strategy for modeling of BVI and AMM problems in standard FEA packages.
good agreement with those presented in the reference (Fig. 7 in Sofi and Muscolino 2007). As opposed to continuous contact assumption in reference, we considered the more realistic case of contact where the oscillators were allowed to separate from the contact surface of the cable. Note that ABAQUS/Standard was able to capture the small amplitude local vibration of cable resulting from impact due to contact between oscillators and cable. The time histories of vertical acceleration of two oscillators are shown in Fig. 4. There is a sudden jump in the vertical acceleration of first oscillator when the second oscillator starts moving on cable due instantaneous impact force exerted on the cable by a second oscillator, since at the time of its entrance the cable was oscillating. Note that the vertical acceleration of oscillators depicts the comfort of the rider.

Dynamic Response of Plate under Moving Distributed Mass

To demonstrate the generality and capabilities of present day finite-element software packages, a rectangular plate, simply supported at two edges, subjected to a moving distributed mass (Fig. 5) is selected from the specialized work done by Wu (2006).

Similar to the above reference, the plate was meshed with 60 shell elements of Type S4 (10×6 divisions along A and B, respectively, in Fig. 5). The distributed mass (labeled M in Fig. 5) was modeled with 64 shell element of 0.025×0.025 element size giving a total area of 0.04 m² for distributed mass. Note that ABAQUS recommends using refined mesh for slave surfaces. The thickness of distributed mass was taken as 0.001 m and elas-
ticy modulus taken as ten times higher than that of the plate. The total mass of the distributed mass was 6.5 kg. The Raleigh damping coefficients for the plate were calculated using first two natural frequencies of vibration of the plate giving an equivalent damping ratio, $\xi$, equal to 0.005.

Unlike the previous example in this paper, the linear pressure overclosure relationship for surface to surface interaction was defined with contact stiffness $10^{15}$, such that it would give exactly the same amount of midpoint static deflection if the distributed mass was placed at midpoint of the plate and “hard” pressure overclosure relationship was defined. An equivalent instantaneous pressure load was defined over the distributed mass surface to account for the weight of the distributed mass. The center of distributed mass was moved linearly from $x=0.1$ to 0.9 in 0.4 s along the center line of plate at a constant speed of 2 m s$^{-1}$. The time-history of midpoint deflection of the plate is shown in Fig. 6, which is in good agreement with that reported in Wu (2006) (Fig. 5 for the case of $d=0.1$ m, $d=0.4$ m) thus, validating our methodology. Also, the contour of maximum in-plane principal stress distribution at time $t=0.2$ s, when the distributed mass was at the midspan of the plate, is shown in Fig. 7. The maximum value of stress (1.613 MPa) in the contour plot is appearing at the midspan of the plate. For a larger value of distributed mass, maximum stress can reach the yield stress of material. The formulation reported by Wu (2006), which is limited to linear isotropic elastic material, would require a revision in order to deal with such situations, whereas the problem can still be analyzed in ABAQUS with their available material models without any modification in the proposed methodology.

**Dynamic Analysis of Curved Panel Subjected to Point Mass**

The most general case that needs to be analyzed in BVI problems has to be motion of traversing secondary subsystem in two-dimensional space. With this point of view, the dynamic response of a curved panel subjected to a concentrated mass moving in a circumferential direction (Fig. 8), selected from a journal paper by de Faria (2004), is being considered here (all data are given in a global coordinate system). To our knowledge, only the works reported by de Faria (2004) and de Faria and Oguamanam (2005) involve dynamic analysis of curved shells subjected to moving mass/load.

The curved panel was modeled with shell elements (30 elements along an axial direction, and 60 elements along a circumferential direction). In Fig. 8, dark edges represent fixed boundary conditions; i.e., edges parallel to the $z$ axis are fixed, whereas circumferential edges are free. The moving mass was modeled as a point mass with predefined displacement in a global coordinate along the circumferential center line of the panel. The motion of mass was obtained by transforming the motion of mass in the local coordinate system into a global coordinate system as per the following expression:

$$u_x = R \left\{ \sin \left( \Omega t \frac{\pi}{12} \right) + \sin \frac{\pi}{12} \right\}$$

where $u_x$ = displacement of moving mass along the $x$ axis.

The time history response at the midpoint of the panel is shown in Fig. 9 (displacements are plotted as a function of angle calculated at respective times to compare them with the reference solution). The result from ABAQUS/Standard analysis shows consistency with the reference solution [Fig. 6 in de Faria (2004)]. Note that ABAQUS/Standard was able to capture small vibration at the panel’s midpoint, which is not present in the reference solution. The small amplitude local vibration at the midpoint of the panel is attributed to the impact due to the instantaneous contact and separation between the moving mass and the panel’s surface.

**Reverse Spaghetti Problem with Transverse Tip Loading**

Deployment from a rigid channel of a highly flexible beam of undeformed length, $L=10$ and with shear deformation is considered here (Vu-Quoc and Li 1995). The “cantilever” beam ($A=20$, $I_x=I_y=1$, $I_z=0$, $\rho=0.1$, $E=500$, $v=0$) starts from its

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**Fig. 4.** Time history of vertical acceleration of two oscillators

**Fig. 5.** Schematic of plate subjected to moving distributed mass

**Fig. 6.** Time history of midpoint vertical deflection of plate subjected to moving distributed mass
zero initial length outside the channel, i.e., the whole beam is inside the channel and undergoes exponential sliding motion to its undeformed length as follows:

\[ u = 10 - 10e^{-10x} \]  

A time varying transverse load, \( F \), is acting on the free end for the analysis period of 40 s as

\[ F(t) = \begin{cases} 
30t & \text{for } t \in [0,0.1] \\
1 & \text{for } t \in [0.1,40] 
\end{cases} \]  

Once the deployment operation is complete, the fully extended flexible beam will eventually undergo some form of free flight in space and the description “spaghetti” was “coined” for this class of problem.

For discretization, 100 “2D Timoshenko beam” elements (B21) were used to model the beam. To restrain all the degree of freedoms (DOFs) of the beam inside the channel, the separation between the beam and rigid channel was made zero. The rigid channel was moved relative to the beam and an exponential linearly distributed axial force was applied in order to account for the inertia effect. The purpose of this was to make analysis easier on ABAQUS, since element level calculations would not be performed for rigid parts.

The time history trace of tip vertical deflection, obtained from geometric nonlinear analysis, is shown in Fig. 10. This is in agreement with the reference solution (Fig. 14 in Vu-Quoc and Li 1995). Note that ABAQUS was able to capture the vibration of the beam tip due to the impulsive load for time period \([0, 0.1]\) along with the vibration of the whole beam. The slight differences between ABAQUS/Standard and ABAQUS/Explicit are due to the differences in the contact algorithms employed (see “Interactions,” ABAQUS Analysis Users Manual 2004). Also, note that the number of points to plot the results from ABAQUS/Standard is 472, whereas that in results from ABAQUS/Explicit is 4,001.

**Reverse Spaghetti Problem: Repositional Motion**

Another reverse spaghetti problem with repositional motion (Behdinan and Tabarrok 1997a,b) is considered here. The total length of beam \( L \) is 1.05 m \((A=4.3434 \times 10^{-5} \text{ m}^2, I_{xx}=I_{yy}=1.059 \times 10^{-11} \text{ m}^4, I_{xy}=0, \rho=3,144.39 \text{ kg/m}^3, E=6.896 \times 10^{10} \text{ N/m}^2, v=0)\). With the initial length of 0.35 m outside the channel, the beam is extruding from the channel in 1.2 s with \( x \) displacement of the tip varying as

\[ u(t) = \frac{0.7}{1.2} \left( t - \frac{1.2}{2\pi} \sin \left( \frac{2\pi t}{1.2} \right) \right) \]  

To simulate this problem in ABAQUS/Explicit, 100 two-dimensional Timoshenko beam (B21) elements were used to

**Fig. 7.** Contour plot of max. in-plane principal stress (N m⁻²) at time \( t=0.2 \) s

**Fig. 8.** Schematic of curved panel subjected to moving mass

**Fig. 9.** Time history of response of midpoint of curved panel
model the beam. Unlike reference conditions, a separation of $10^{-3}$ m between the rigid channel surface and the beam surface was invoked in order to allow some motion to the beam elements inside the channel. A transverse force of 0.75 N was applied statically at the tip of the beam for the first 1 s to match the initial vertical deflection of approximately 0.02 m as presented in the reference solution [Fig. 4 in Behdinan and Tabarrok (1997)]. For the next 1.2 s, rigid channels were moved according to the above defined displacement function $u(t)$ relative to the beam and a linearly distributed axial load of total magnitude equaling inertia force due to axial motion, over the beam length, in the direction of the beam’s translation was applied to account for the inertia effect. Geometric nonlinearity was accounted for in this analysis.

The time history trace of vertical deflection at the beam tip is shown in Fig. 11. The slight deviation from the reference solution [Fig. 4 in Behdinan and Tabarrok (1997)] is due to the fact that the initial deflection of the beam tip was not given in the reference. Therefore, an approximate force was applied for 1 s in the initial step to achieve the desired approximate deflection. However, the results are in good agreement, showing similar orders of magnitude and shapes.

### Parametric Resonance of Sliding Beam

A cantilever beam problem (Fig. 12) subjected to sinusoidal sliding motion, $6 \cdot \sin(2.8 \pi t)$, is selected to analyze parametric resonance (Vu-Quoc and Li 1995). The material and cross-sectional properties ($A=0.001223$, $I_{xx}=I_{yy}=1.631 \times 10^{-7}$, $I_{xy}=0$, $\rho=1,000$, 

$E=1.6761 \times 10^{12}$, $\nu=0.4748$) are modified in order to match the ratios given in Vu-Quoc and Li (1995), Eq. (5.26).

The problem was solved in ABAQUS/Standard and ABAQUS/Explicit with two approaches: first, the beam was moved while rigid channels were kept at rest and second, when channels were moved relative to the beam. In all the analysis cases, the beam was discretized by 100 Timoshenko beam elements (B21). Unlike the reference, the initial geometric perturbation was applied while modeling the beam due to limitation of ABAQUS. The cantilever beam member, in the initial configuration, was modeled as a straight segment inside the channel [coordinates $(0, 0)$–$(0, 6)$] and a slightly inclined segment outside the channel [coordinates $(6, 0)$–$(-22, -0.001)$]. The channel’s walls were at 0.001 units away from the beam axis, in order to model a more realistic case, and an initial transverse velocity of −0.1 was assigned to the tip of the beam’s free end. A linear pressure overclosure relationship was chosen for contact normal behavior with contact stiffness of $2.04984 \times 10^{6}$.

The time history traces of tip vertical deflection are shown in Fig. 13. ABAQUS/Standard completed both runs successfully for a specified analysis time of 20. Both approaches show consistency in the case of ABAQUS/Standard, though the run time for moving the rigid channel approach was less. ABAQUS/Explicit run was aborted at time equals to 13.84 in the case of a sliding beam approach, whereas the sliding channel run failed after time equal to 18.06. Also, significant differences can be seen, which were expected due the differences in contact formulations in ABAQUS/Standard and ABAQUS/Explicit. From these results, it is clear that ABAQUS/Standard is capable of solving the sliding beam problem using either approach. This problem needs to be investigated further in ABAQUS/Explicit.

### Summary and Conclusions

From the viewpoint of practical applicability, the current status of the available literature on BVI and AMM problems is found to be incomplete in several respects. First, the reported works remain largely of a very specialized nature (requiring high-level mathematics not typically within the conventional training of practicing engineers; e.g., advanced perturbation techniques, modified mode superposition schemes, dynamic greens function, etc., see Behdinan et al. 1998a,b, 1997a,b), with different researchers considering one or the other of these two problems classes. Almost always, there is the restriction to the case of beams or cables, and “unrealistic” assumptions are sometimes made to simplify the analyses (e.g., continued/persistent contact, or only the at-rest conditions for the primary system, irrespective of the number of traversing secondary oscillators in BVI problems). Second, contradicting conclusions were also reported in regard to the commonly utilized limiting cases of moving mass and moving load in BVI problems with different initial conditions for the primary and secondary dynamical systems (Pesterev et al. 2003).
problems to clearly demonstrate the power of commercial FEA codes (using the ABAQUS program exclusively here). In this, we bypass completely any complications resulting from the change in data structure of standard FEA analysis; i.e., by realistically accounting for all interaction forces (be it of any type such as gravitational, inertia, or elastic in BVI, or of the constraint/Lagrange multipliers type in AMM problems) using the large sliding and intermittent impulse/impact capabilities in ABAQUS. Consequently, we have been able to consider a rather impressive range of BVI and AMM problems, ranging from simpler beam/cable type to plates and shells, accounting for large displacement effects and many parametric resonance phenomena.

**Fig. 13.** Comparison of tip vertical deflection using two approaches: (a) ABAQUS/Standard; (b) ABAQUS/Explicit

Third, there has been a notable absence of any applications involving large-scale FEA codes. In fact, these were sometimes dismissed (either explicitly or implicitly) at the outset as inapplicable to the complex situations in BVI problems (Biondi and Muscolino 2005; Wu 2006). For a case to the point, we summarize here the explicit statement made by Biondi and Muscolino (2005) in conjunction with a solution for a BVI problem and referring to a standard commercial FEA code ADINA (Biondi et al. 2004), as a means for a solution instead of the newly developed techniques by these authors to solve the problem. These new and highly specialized methods were referred to as conventional series expansion (CSE), mode-acceleration method (MAM), and dynamic correction method (DCM) (differing in their treatment of the bending and shear forces calculations in the BVI analysis). For the standard case of an oscillating force at a fixed location on a girder, accurate solution from ADINA provided a base for assessing the quality of CSE, MAM, and DCM methods. However, when the motion of the oscillator on the girder was considered next, no solution from any standard FEA code was provided for comparison; the reason being the “poor accuracy” anticipated by the authors from these codes. Similarly, Wu (2006) had to avoid the use of any standard finite-element code, and instead created his own specialized program (accounting for the freshly derived stiffness, mass, damping, and load arrays) to analyze the rectangular plate under moving distributed mass.

Finally, there is a lack of useful guidelines as to which (if any) of the described schemes is appropriate for treating other equivalent classes of problems.

In an attempt to improve the above situation, we have presented the solutions of a rather extensive set of BVI and AMM

### References