Three-dimensional deformation response of a NiTi shape memory helical-coil actuator during thermomechanical cycling: experimentally validated numerical model

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Three-dimensional deformation response of a NiTi shape memory helical-coil actuator during thermomechanical cycling: experimentally validated numerical model

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Received 31 March 2016, revised 1 July 2016
Accepted for publication 26 July 2016
Published 24 August 2016

Abstract
Shape memory alloy (SMA) actuators often operate under a complex state of stress for an extended number of thermomechanical cycles in many aerospace and engineering applications. Hence, it becomes important to account for multi-axial stress states and deformation characteristics (which evolve with thermomechanical cycling) when calibrating any SMA model for implementation in large-scale simulation of actuators. To this end, the present work is focused on the experimental validation of an SMA model calibrated for the transient and cyclic evolutionary behavior of shape memory Ni\textsubscript{49.9}Ti\textsubscript{50.1}, for the actuation of axially loaded helical-coil springs. The approach requires both experimental and computational aspects to appropriately assess the thermomechanical response of these multi-dimensional structures. As such, an instrumented and controlled experimental setup was assembled to obtain temperature, torque, degree of twist and extension, while controlling end constraints during heating and cooling of an SMA spring under a constant externally applied axial load. The computational component assesses the capabilities of a general, multi-axial, SMA material-modeling framework, calibrated for Ni\textsubscript{49.9}Ti\textsubscript{50.1} with regard to its usefulness in the simulation of SMA helical-coil spring actuators. Axial extension, being the primary response, was examined on an axially-loaded spring with multiple active coils. Two different conditions of end boundary constraint were investigated in both the numerical simulations as well as the validation experiments: Case (1) where the loading end is \textit{restrained} against twist (and the resulting torque measured as the secondary response) and Case (2) where the loading end is \textit{free} to twist (and the degree of twist measured as the secondary response). The present study focuses on the \textit{transient and evolutionary} response associated with the initial isothermal loading and the subsequent thermal cycles under applied constant axial load. The experimental results for the helical-coil actuator under two different boundary conditions are found to be \textit{within error} to their counterparts in the numerical simulations. The numerical simulation and the experimental validation demonstrate similar transient and evolutionary behavior in the deformation response under the complex, inhomogeneous, multi-axial stress-state and large deformations of the helical-coil actuator. This response, although substantially different in magnitude, exhibited similar evolutionary characteristics to the simple, uniaxial, homogeneous, stress-state of the isobaric tensile tests results used for the model calibration. There was no significant difference in the axial displacement (primary response) magnitudes observed between Cases (1) and (2) for
the number of cycles investigated here. The simulated secondary responses of the two cases evolved in a similar manner when compared to the experimental validation of the respective cases.

Keywords: experimental validation, NiTi, multi-axial, shape memory alloy, thermal cycling, helical-coil actuators, springs

(Some figures may appear in colour only in the online journal)

1. Introduction

Shape memory alloy (SMA) actuators are solid-state, thermal devices that produce useful actuation by way of conversion of thermal energy to mechanical energy (Otsuka and Wayman 1998). Generally, when used as actuators, SMAs are required to operate for a large number of thermomechanical cycles under complex states of stress. Due to their ability to produce large work outputs in compact spaces SMA actuators are becoming an attractive alternative to conventional actuators in many fields of application, such as aerospace, automotive, electrical household appliances, biomedical, etc. SMA actuators can be designed in various ways to exploit characteristic SMA features, such as, one-way shape memory effect (OWSME), two-way shape memory effect (TWSME), etc.

A number of SMA applications utilize the stress-free OWSME, by which martensite (the low temperature phase) is isothermally deformed (i.e., a loading step to convert self-accommodated martensite to detwinned martensite, followed by unloading after which detwinned martensite variants remain), then this deformed configuration is heated in a stress-free condition until it recovers its initial configuration as the material transforms to austenite (the high temperature phase) (Wen et al 1994, Erbtoeszer et al 2000, Miller and Lagoudas 2000, Predki et al 2008, Hartl and Lagoudas 2007, Stebner et al 2008, Auricchio et al 2014, Reedlunn et al 2014). In particular, when using complex geometries such as springs or torque tubes, operational conditions during actuation generally involve multi-axial stress states accompanied by large deformations and rotations. For example, springs experience torsional and flexural deformations in the wire when the coils are stretched axially, developing significant values for all six- components of the stress tensor (Cook and Young 1985, Saleeb et al 2013c). Furthermore, as alluded to above, each of these actuators may be required to provide stable actuation over a large number of thermal cycles during their service life. When operating in complex forms irrecoverable strain could potentially change the geometry and hence the stress state of that form as the SMA evolves. Such behavior has previously been observed during thermomechanical cycling of NiTiPdPt helical-coil actuators (Nicholson et al 2014).

In view of the aforementioned points, it becomes important to develop a general material modeling framework that can effectively cope with the complexities of cyclic, hysteretic SMA behavior under evolving three-dimensional states of stress and deformation. An essential step before implementing such a modeling framework for the practical design of actuators (in any configuration or form) is to validate the model by comparison with experimental results. Towards this goal, the general SMA modeling strategy using a multi-mechanism formulation, as described in Saleeb et al (2013a, 2013b, 2013c), was experimentally validated here against the test results of a 55NiTi helical-coil actuator. The utilized SMA model has been successfully implemented in the calibration of biomedical grade superelastic NiTi alloys test results and the numerical simulation of the surgical

2. Summary of the calibrated SMA model for Ni49.5Ti50.1

As originally formulated, the general, three-dimensional, multimechanism-based SMA model developed by Saleeb et al (2011) has targeted the inclusion of a large number of important response characteristics of different classes of SMA materials. Herein, for convenience, a summary of the model formulation and its parameterization procedure is given in appendix A. For the purpose of the numerical simulations reported in section 3, this SMA modeling framework was implemented as a UMAT subroutine in ABAQUS® standard commercial finite element analysis software (ABAQUS 2012).

The model was utilized to calibrate the details of the transient and evolutionary strain response of 55NiTi (49.9 at% Ni and At of 113 °C) during thermal cycling between a LCT of 30 °C and UCT of 165 °C, under isobaric tensile test conditions with different bias-stresses in the range of 10 to 300 MPa. These experiments were conducted on cylindrical, tensile specimens having a gauge diameter of 3.81 mm and a gauge length of 25.4 mm (Padula et al 2014).

In brief, calibration of the model for 55NiTi resulted in the material parameters shown in tables 1–3 (see also appendix A.2 for the specific definition of the material parameters). For this purpose, and following the parameterization procedure outlined in appendix A.2, the 25 material parameters in the model were subdivided into two groups; i.e., a fixed set of 17 material parameters and remaining set of eight functionally dependent parameters (in terms of temperature and stress state). Furthermore, since the experimental results for the present 55NiTi material only included tensile data, the two parameters c and d accounting for tension-compression-shear asymmetry (ATC) in the model (see appendix A.2) are not activated here. As a result, only 16 fixed parameters (see table 1) and seven temperature/stress-state dependent parameters (see tables 2 and 3) were involved. For further elaboration on the quality of the model prediction and its usefulness in large-scale simulation of actuator behavior, the readers are referred to our earlier work in Saleeb et al (2013b, 2013c) and Owusu-Danquah et al (2015), respectively. For our purpose here, it is sufficient to display only the representative comparison (shown in figure 1) between the model result and test data for the isobaric test conducted at an engineering bias-stress of 80 MPa for the first ten thermal cycles between LCT of 30 °C and UCT of 165 °C. The model was able to predict well the experimental results both qualitatively as well as quantitatively for: (1) the transient

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### Table 1. SMA model fixed material parameters.

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Units</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>E</td>
<td>MPa</td>
<td>60 000</td>
</tr>
<tr>
<td>ν</td>
<td>—</td>
<td>0.3</td>
</tr>
<tr>
<td>n</td>
<td>—</td>
<td>5</td>
</tr>
<tr>
<td>μ</td>
<td>MPa s</td>
<td>$10^5$</td>
</tr>
<tr>
<td>$H_{kb}$ for $b = 1$ to $6$</td>
<td>MPa</td>
<td>$400 \times 10^3$</td>
</tr>
<tr>
<td>$\beta_{kb}$ for $b = 1$ to $6$</td>
<td>—</td>
<td>1</td>
</tr>
</tbody>
</table>

### Table 2. SMA model temperature-dependent material parameters.

<table>
<thead>
<tr>
<th>Temperature (°C)</th>
<th>Material parameters (MPa)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$T_1 = 20 ^\circ C$</td>
<td>$\kappa$, $\kappa_{kb}$, $b = 1$, 2, $\kappa_{kb}$, $b = 3$, $\kappa_{kb}$, $b = 4$, $\kappa_{kb}$, $b = 5$, $\kappa_{kb}$, $b = 6$</td>
</tr>
<tr>
<td>$T_2 = 65 ^\circ C$ (50 °C for $b = 4$)</td>
<td>20 0.2 130 0.001 21</td>
</tr>
<tr>
<td>$T_3 = 115 ^\circ C$ (120 °C for $b = 4$)</td>
<td>20 62.2 1.00E + 21 0.001 21 52</td>
</tr>
<tr>
<td>$T_4 = 200 ^\circ C$</td>
<td>20 53.7 400 21</td>
</tr>
</tbody>
</table>

#### Note: The intermediate values in the above table are interpolated linearly between the tabulated values.

### Table 3. SMA model stress-dependent material parameters.

<table>
<thead>
<tr>
<th>Stress levels (MPa)</th>
<th>Scale factors</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\eta_{kb}$, $b = 1$, 2</td>
<td>$\eta_{kb}$, $b = 3$, $\eta_{kb}$, $b = 4$, $\eta_{kb}$, $b = 5$, $\eta_{kb}$, $b = 6$</td>
</tr>
<tr>
<td>10</td>
<td>0.070, 0.125 1.000 1.000 0.900 0.100</td>
</tr>
<tr>
<td>50</td>
<td>0.150, 0.300 1.200 0.304</td>
</tr>
<tr>
<td>80</td>
<td>0.600, 0.800 1.200 0.700</td>
</tr>
<tr>
<td>100</td>
<td>1.000 1.000 1.000 1.000</td>
</tr>
<tr>
<td>150</td>
<td>1.300 1.500 1.667 1.500</td>
</tr>
<tr>
<td>200</td>
<td>2.350 1.950 1.400 2.500</td>
</tr>
<tr>
<td>300</td>
<td>4.000 2.700 6.667 3.000</td>
</tr>
<tr>
<td>375</td>
<td>10.000 3.450 22.833 5.872</td>
</tr>
</tbody>
</table>

#### Note: The intermediate values in the above table are interpolated linearly between the tabulated values.
loading, followed by first heat and first cool) response (see broken line in figures 1(a) and (b)); i.e., small strain (∼0.23%) produced during the loading to 80 MPa at room temperature (24°C); followed by no significant strain (increase of ∼0.11%) during heating to the UCT and a strain of ∼2.89%, produced during subsequent cooling to the LCT; and (2) the evolutionary response of martensite strain (ε_M) from ∼2.8% to ∼5% in the SMA model and from ∼2.6% to ∼5% in the experiment; and austenite strain (ε_A) from ∼0.16% to ∼1.5% in the SMA model and from ∼0.48% to ∼1.5% in the experiment; producing an actuation strain (ε_ACT = |ε_M − ε_A|) from ∼2.7% to ∼3.3% in the SMA model and from ∼2.2% to ∼3.5% in the experiment; during ten thermal cycles (see figures 1(c) and (d)).

3. Numerical simulation

The following simulation was designed to predict the deformation response of a 55NiTi helical spring actuator during initial loading, followed by ten thermal cycles under applied constant axial load. All simulations utilize the as-calibrated model described in the previous section. All simulations were performed prior to their respective validating experiments.

3.1. Geometric and modeling details

The helical spring used in the simulation had an initial pitch angle (α) of 4.7°, pitch length (P) of 6 mm, 3 active coils (n), mean coil diameter (D_m) of 23.1 mm, wire diameter (d) of
2.16 mm and a spring index \( \left( \frac{D_m}{d} \right) \) of 10.7, as shown in figure 2.

In addition to the aforementioned geometric details, there are several important modeling considerations. First, the computational model of the helical-coil was created based on the three-dimensional theory of continuum mechanics; i.e., with only three translational displacements fields being relevant. This proved convenient in handling the multi-axial stress effects anticipated in the use of the helical structure during the thermal actuation cycles under the bias axial load. Second, an imaginary axis was created, which remained fixed (no rotation/no translation) during the deformation. This was achieved by fixing all the nodes of a half turn on the upper part of the modeled four-coil spring to all active translational degrees of freedom and connecting the nodes of a half turn on the lower part of the spring to an imaginary reference point located on the axis of the helix by a rigid body constraint (kinematic constraints in ABAQUS 2012), thus, leaving three coils to remain active to contribute to actuation. The reference point facilitates the application of an axial load and recording the axial deformation, as well as to measure the possible secondary response developed from restraining and allowing the rotations of the coil ends. Third, as elaborated below, the present study focuses on the experimental validation for aspects of the so-called primary response (i.e., axial deformation and associated actuation stroke) of the coil under two different boundary conditions. In addition, comparisons of the simulation-versus-experimental results for the so-called secondary response (i.e., twist and reaction torque) will also be presented.

3.2. Boundary conditions, loading methodology and mesh convergence
An axial load \( (F) \) of 8.8 N was estimated analytically (using methodology presented in Ancker and Goodier 1958) to be applied at the reference point at 30 °C to produce a maximum effective stress magnitude of approximately 100 MPa in the coils. The applied axial load was kept constant while performing ten thermal cycles (henceforth referred to isoforce) in the temperature range of 30 °C and 165 °C (spanning the whole transformation temperature range of the present 55NiTi material). Also, here, the axial displacement produced during the isoforce thermal cycling test was measured as the primary deformation response. Two cases of the end boundary conditions were implemented at the reference point in the loading end with respect to the experiments: Case (1) the loading end is restrained against twist, and Case (2) the loading end is free to twist. The reaction torque/moment and angle of twist constitute secondary responses in Case (1) and Case (2), respectively, both of which were measured at the reference point.

A mesh convergence study was conducted for Case (1) to determine the suitable mesh density required in the numerical simulation. On the basis of the results obtained for the axial displacement at the free end as well as the effective stress distribution over the wire cross section, a mesh containing a total of 1920 elements of the type C3D20R (quadratic, 20-noded, three-dimensional, brick) was determined to be sufficient for convergence of the deformation and stress responses. These 1920 elements were distributed as follows: 4 (number of coils) \( \times \) 12 (in SMA wire cross section) \( \times \) 40 (along the length of one SMA coil) = 4 (number of coils) \( \times \) 480 (total number of elements per single SMA coil).

4. Experimental procedure
4.1. Spring fabrication
Wire was electro discharge machined (to a wire diameter of 2.16 mm) from the same 55NiTi extrusion used to calibrate
the SMA model. The 55NiTi wire was wound onto a mandrel with a helical groove (with the geometric parameters detailed in section 3.1 and figure 2) at room temperature. A sleeve was slid over the wire and mandrel to ensure that the wire maintained its desired spring shape throughout the shape-setting process. The spring was shape-set at 450 °C for 30 min and then ice water-quenched. It has previously been shown that the shape-setting process does not have a significant effect on subsequent thermomechanical behavior in this particular material system (Benafan et al 2013b). The as-shape-set spring consisted of three active coils (four in total), with a coil diameter of 23.1 mm and a free length of 19 mm. This process was repeated to produce a total of two identical springs, one for each of the two aforementioned cases, Case (1) and Case (2).

4.2. Experimental setup

A modular test setup (figure 3) for the purpose of testing SMA springs (with various end constraints) was designed and assembled. A detailed description of the experimental setup can be found in Nicholson et al (2014). The following two configurations were used to experimentally validate Case (1) and Case (2). For Case (1), one spring mount was coupled to the torque cell (having a range of 0–176.5 N mm), to measure the torque produced during actuation when the spring was constrained from rotating. For Case (2), one spring mount was coupled to a rotary encoder, to measure the degree of twist produced during actuation when the spring was free to rotate. For both cases care was taken to ensure the boundary conditions in the experiment mirrored (as close as possible) the boundary conditions applied in the simulation (i.e., rigid with no conductive heat loss at the point of contact with each spring mount).

4.3. Thermomechanical testing

Using the setup described in section 4.2, the following tests were performed. Following the installation of the spring into the setup, two stress-free thermal cycles (between room temperature and 165 °C) were performed (to relieve any residual stresses produced by processing, machining or installation of the spring) prior to performing the experiment. Following the two no-load thermal cycles, an axial tensile load of 8.8 N was applied to the spring at room temperature. Under this constant load, 10 thermal cycles were performed between a LCT of 35 °C and an UCT of 165 °C. This thermomechanical loading procedure was performed for Case (1) and Case (2) using a virgin, as-shape-set spring for each case, respectively.

5. Results and discussion

5.1. Effective stresses and state of stress components

The simulations for both Cases (1) and (2) showed that the axially applied force of 8.8 N produced an estimated effective stress of around 100 MPa at 30 °C. The effective stress and six individual stress components (three normal and three shear stresses) after loading for Case (1) are shown in figure 4. Note that the two different cases both led to the same variation (within error) in magnitude of the effective stress after isothermal loading, thus only Case (1) was shown. In particular, there was significant spatial variation of the effective stress distribution (i.e., stress intensity) indicated by a range of values from 3 to 116 MPa (with an average value of nearly 60 MPa) across the wire cross section, and ranging from 70 to 116 MPa along the length of the coils.
Figure 4. Distribution (in MPa) of effective stress and six individual components of the stress tensor (in cylindrical coordinate system) of an SMA coil under an applied axial force of 8.8 N for Case (1), i.e., loading end restrained against twist, prior to thermal cycling.
Results from the numerical simulation were compared to stress values calculated analytically using the equations from Ancker and Goodier (1958) at point A shown in figure 4 (details of these calculations are provided in appendix A.3). Both theory and numerical analysis (as shown in figure 4) show the stress components $\sigma_{RR}$, $\tau_{R\theta}$ and $\tau_{RZ}$ to be negligible at the inner diameter surface (i.e., at point A shown in figure 4) of the spring coil. Stress components $\sigma_{\theta\theta}$ and $\sigma_{ZZ}$ found to be 4.55 and 15.87 MPa, respectively, from the model which was comparable to $\sigma_{\theta\theta}$ and $\sigma_{ZZ}$ found to be 0.098 and 9.55 MPa, respectively, from theory. The difference observed between model and theory for $\sigma_{\theta\theta}$ and $\sigma_{ZZ}$ was small when compared relative to the maximum effective stress. Shear stress component $\tau_{\theta Z}$ was found to be 61.15 MPa in comparison to 57.735 MPa calculated from theory.

As described by theory from Timoshenko and Goodier (1970), when the axial load $P$ is applied to the helical spring, a torsional couple $PD_m/2$ is generated which produces significant shear stress components $\tau_{R\theta}$ and $\tau_{RZ}$, which are independent of the pitch angle $\alpha$. As evident from figure 4, the variations of $\tau_{R\theta}$ and $\tau_{RZ}$ stress components are more significant over the entire surface and thickness of the coils when compared to the other stress components.

The above results demonstrate the complex three-dimensional nature of the state of stress in the SMA coils when a simple axial force is applied. Thermal cycling was performed under this complex, multi-axial, state of stress while the applied force was kept constant. Furthermore, the heterogeneity in the stress distribution was observed to be varying throughout the thermal cycles at the end of loading, cooling and heating branches. The distribution of the effective stress in the helical coil at the key stages of the isoforce thermal cycling test are shown in figure 4 of the spring coil. Stress components $\sigma_{\theta\theta}$ and $\sigma_{ZZ}$ vary throughout the thermal cycles at the end of loading, cooling and heating branches. The effective stress in the inner surface of the coils decreases to the corresponding value of the end of cooling branch (refer to States (3), (5) and (7) in figure 5) due to the changes in the geometric configuration of the spring during actuation. The helical spring coil experiences a wide range of varying stress magnitudes at various material points for the martensitic transformation, thus introducing a complex stress state for the actuation of the 55NiTi coil during heating and cooling.

### 5.2. Deformation states

The numerical simulation and experimental counterpart for the deformation response of Case (1) are depicted in figure 6 for the first three thermal cycles. This includes screen captures from the animation of the numerical simulation and from a video of the experiment. Eight distinct deformation states (labeled 1–8) were identified as follows. The helical-coil was initially loaded axially to 8.8 N at 30°C and that constituted the first loading stage from the initial undeformed state 1 to the mechanically deformed state 2 after the isothermal loading phase of the experiment. Keeping the applied axial load constant (thus, isoforce condition), the spring was heated to 165°C (i.e., from state 2 to state 3) and then cooled to 30°C (i.e., from state 3 to state 4), thus completing the first thermal cycle. Subsequently deformation states for two additional thermal cycles are presented. The states 5 and 6, 7 and 8 represent the deformation states at 165°C and 30°C for the second and third thermal cycle, respectively. These deformation states (1, 2, 3, 4, 5, 6, 7 and 8) are marked in the axial displacement-versus-temperature response for the first three thermal cycles for experiment and simulation in figures 7(a) and (b), respectively.

Recall that the initial undeformed helical-coil had an initial helix angle of 4.7° (state 1). Following the first thermal cycle (state 4), the helix angle increased to 45° in the numerical simulation and 46.5° in the experiment. The numerical model exhibited very similar deformation states with respect to the experimental counterpart, showing identical patterns of...
the significant bending and twisting of SMA wires (see state 4, 6, and 8 in figure 6). After the initial isothermal loading, a small increase in the helix angle can be seen in both the numerical simulation and the experiment (comparing the screen captures at state 2 with respect to 1 in figure 6). Similarly, the numerical simulation correctly predicted the experimental behavior showing a negligible change in the helix angle between states 2 and 3. Note, that in both the numerical simulation as well as the experimental measurements, the magnitude of the axial displacement produced during the initial loading and the first heating step were much smaller in comparison to the significantly larger magnitude observed after the subsequent cooling step. This signifies the importance of being able to capture the initial transient behavior of the actuator using the SMA model. In addition, the numerical simulation shows similar deformation states after heating and cooling steps for the first three thermal cycles.

5.3. Axial displacement versus temperature response and the effect of boundary conditions

Figures 7 and 8 show for Case (1) and Case (2), respectively, the comparison of the axial displacement versus
temperature (°C) response from numerical simulation (SMA model) and experiments under isoforce conditions for 10 thermal cycles (with the transient response for loading and the 1st thermal cycle shown as dashed lines in part a and b). With the boundary condition of Case (1), where the loading end is restrained against twist rotation, an axial displacement of 9.46 mm was predicted from the numerical simulation (figure 7(a)) after the isothermal loading step. In contrast, a value of 7.22 mm was measured during the experiment (figure 7(b)). After the heating step, the axial displacement reduced to 6.34 mm in the numerical simulation whereas it increased to 11.50 mm in the experiment. Finally, after the subsequent cooling step (state 4 in figure 6), the axial displacement increased to nearly 92 mm in the numerical simulation whereas it increased to approximately 103 mm in the experiment, thus producing a nearly comparable actuation stroke of 86 mm in the numerical simulation versus 91.7 mm in the experiment (figure 7(b)). Similarly, in Case (2), where the loading end is free to twist, during the first cooling step of the thermal cycle, the axial displacement was measured to be 94 mm in the large-scale simulation result in comparison to 104 mm from the experiment. Again, these results lead to nearly comparable actuation strokes, yielding 88 mm in the SMA model simulation versus 92.6 mm in the experiment (figures 8(a) and (b)). Both Cases (1) and (2) exhibited the same (within error) axial displacement response during both the initial isothermal loading at 30 °C, and also during the heating step to 165 °C. A significant amount of error was observed in the temperature response between model and experiment. Possible reasons for this error are discussed in detail in section 5.4.
Figure 8. Isoforce test for the helical-coil actuator under Case (2), i.e., the loading end free to rotate, for ten thermal cycles between 30 °C and 165 °C: axial displacement-versus-temperature response: (a) SMA model and (b) experiment; and axial displacement in the martensite state ($\delta_M$), austenite state ($\delta_A$) and corresponding actuation strain ($\delta_{ACT} = |\delta_M - \delta_A|$): (c) SMA model and (d) experiment.

Table 4. Summary and comparison of experimental and model results for both Case 1, i.e., loading end restrained against twist and Case 2, i.e., loading end free to twist.

<table>
<thead>
<tr>
<th>Displacement 'δ' (mm) at different stages in thermal cycles</th>
<th>Case 1: Loading end restrained against twist</th>
<th>Case 2: Loading end free to twist</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>SMA model</td>
<td>Experiment</td>
</tr>
<tr>
<td>Initial at 30 °C</td>
<td>0.00</td>
<td>0.00</td>
</tr>
<tr>
<td>After applying axial load of 8.8 N at 30 °C</td>
<td>9.46</td>
<td>7.22</td>
</tr>
<tr>
<td>After the 1st heating step at 165 °C, $\delta_A$</td>
<td>6.34</td>
<td>11.50</td>
</tr>
<tr>
<td>After the 1st cooling step at 30 °C, $\delta_M$</td>
<td>91.98</td>
<td>103.27</td>
</tr>
<tr>
<td>Actuation stroke at 1st Cycle ($\delta_{ACT} = \delta_M - \delta_A$)</td>
<td>85.64</td>
<td>91.77</td>
</tr>
<tr>
<td>After the 10th heating step at 165 °C, $\delta_A$</td>
<td>30.40</td>
<td>34.15</td>
</tr>
<tr>
<td>After the 10th cooling step at 30 °C, $\delta_M$</td>
<td>125.81</td>
<td>137.22</td>
</tr>
<tr>
<td>Actuation stroke at 10th Cycle ($\delta_{ACT} = \delta_M - \delta_A$)</td>
<td>95.41</td>
<td>103.07</td>
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</tbody>
</table>
Figure 9. Comparison of secondary responses from experiment (on left y-axis) and SMA model (on right y-axis) in the martensite state (30 °C) and austenite state (165 °C) for 10 thermal cycles: (a) reaction moment in Case (1), i.e., loading end restrained against twist and (b) angle of twist in Case (2), i.e., loading end free to twist.

For convenience in studying the evolution in the axial displacement response, the magnitude of the actuation strokes under the effect of the changes in the boundary conditions, a summary of the results of Cases (1) and (2) are presented in table 4. The numerical simulation and the experimental results indicated that the axial displacement attained after the cooling step was greater by ~2 mm when comparing Case (1) to Case (2). The percentage difference is calculated with respect to the respective experimental values. During the 10 thermal cycles, the numerical simulation showed less evolution of the axial displacement with respect to the experimental results (see figures 7(a), (c) and (b), (d) for Case (1) and figures 8(a), (c) and (b), (d) for Case (2) from SMA model and experimental results, respectively). Figures 7(c), (d) and 8(c), (d) give the details of the individual accumulated axial displacements, with cycles, at the martensite (at the LCT state (δM) and austenite (at the UCT) state (δA) and the actuation displacement/stroke (δACT) for experimental results of Cases (1) and (2), respectively. In summary, the numerical simulation is in good qualitative as well as quantitative agreement with the experimental results for the three active coil helical spring actuation considered here.

It is also important to note here that the above results from both numerical simulation and the validation experiment of the helical-coil actuator, under the complex stress state described in section 5.1, capture the overall deformation associated with the transient response observed under the simple, homogeneous, uniaxial, tensile isobaric test conditions that were used in characterizing the 55NiTi material. More specifically, this can be further appreciated by contrasting figures 7(a) and 8(a), versus figure 1(a) from the model, and figures 7(b) and 8(b) versus figure 1(b) from the experiments.

In addition to the axial displacement obtained as a primary response, the reaction torque/moment and angle of twist was measured as a secondary response in Case (1) and Case (2), respectively. Figures 9(a) and (b) shows the comparison of the reaction moment and angle of twist obtained after the heating step (at the UCT) and the cooling step (at the LCT) in Case (1) and Case (2) from experiment (in left y-axis) as well as numerical simulation (in right y-axis), respectively. Despite the differences in magnitudes of these secondary responses, the results from the model show a similar pattern of evolution with respect to their experimental counterpart for the ten thermal cycles investigated.

5.4. Deviations

Despite the overall good predictions of the experiment by the numerical simulation as described in sections 5.2 and 5.3, error was observed (see table 4). While, there are several possible reasons for this error the following were deemed to be most significant:

1. The model was parameterized based upon the data from the uniaxial, tensile, isobaric tests only, whereas the axially loaded helical-coil was shown to experience a significant multi-axial state of stress (see figure 4). Thus, the model may benefit by being more appropriately calibrated from additional tests involving compression and shear data. These types of additional tests are currently not available.

2. The numerical model was unable to account for specific details of the clamping procedures utilized in the experiments to implement the different boundary conditions (as described in sections 3.2 and 4.2). In this regard, note that the experimental setup utilizes the contact force-based clamping support arrangements (see figure 6 in experiments) as opposed to the simple kinematical boundary conditions utilized in the numerical simulation. Significant differences observed in the magnitudes of the secondary responses (see figure 9) provide evidence of the source of this error.

3. Varying contact between the mechanically fastened thermocouple and the spring can result in error
observed in the temperature measurements (relative to observed in the temperature measurements observed during uniaxial loading which had welded thermocouples). This was also the source of the 'lack of smoothness' observed in the axial displacement versus temperature response for the spring experiments (see figures 7(b) and 8(b)). Furthermore, induction heating was used during the isobaric tests used to calibrate the SMA model, whereas Joule heating was implemented during the validation experiment for the helical coil actuator. The source of error mentioned in item (1) can also contribute to the discrepancies observed in temperature measurements.

(4) In connection with item (3) above, it is anticipated that the error observed in the temperature measurement will manifest in error in the LCT and UCT employed in the experiment which have been shown to play an important role in the strain development (Padula et al. 2012). In contrast, the numerical simulation maintained fixed values for these two temperature limits.

A greater actuation stroke (up to 2 mm, see table 4) was obtained for Case (2) than for Case (1) in both experiment and model. This difference is expected to increase with further cycling (Nicholson et al. 2014). However, for the number of cycles investigated here (up to 10) this result was deemed inconclusive, as it was within error.

6. Conclusions

In this work, a framework developed for a multi-axial, multi-mechanism based constitutive model for the comprehensive representation of the evolutionary response of SMAs under general thermomechanical loading conditions was experimentally validated using SMA helical-coil actuators. During this investigation two different conditions of end boundary constraint were investigated for the transient and evolutionary responses of the SMA helical-coil actuators. The simulation proved to accurately predict the experiments and the followings conclusions were made.

(1) Both the simulation and experimental validation of the SMA helical-coil actuator demonstrated similar transient and evolutionary deformation responses under inherent complex stress/deformation states. These responses showed similar behavior to the strain-versus-temperature response obtained under the simple stress/deformation state for the uniaxial, isobaric tensile test condition.

(2) No significant differences (within error, as described in section 5.4) were observed in the transient and evolutionary (for up to 10 thermal cycles investigated here) axial displacement (primary) responses between Cases (1) and (2) and between simulation and experiment of each case, respectively.

(3) A significant difference in magnitude of the secondary responses of Cases (1) and (2) was observed between simulation and experiment. However, a similar pattern of evolution was observed between simulations and experiment over the ten thermal cycles investigated here. This successful experimental validation of the simulated response of SMA helical-coil actuators proves the feasibility of modeling and designing SMA actuators, in complex forms, to operate over an extended number cycles. The effect of boundary conditions on SMA actuators should be considered during the design process and the practicality of modeling such effects was conveyed here.

Acknowledgments

This work was supported by the Fundamental Aeronautics Program, Fixed-Wing, Project No. NNH10ZEA001N-SFW1, Grant No. NNX11AI57A to the University of Akron with the University of Central Florida as Sub Contractor. The authors would like to acknowledge Dr O Benafan for his technical guidance and programmatic support during the different phases of the project. The authors thank Dr O Benafan for helpful technical discussions regarding the experiments.

Appendix

A.1. Summary of model formulation

In the model formulation, the total strain tensor \(\varepsilon_{ij}\) (and its rate \(\dot{\varepsilon}_{ij}\)) is decomposed into reversible/elastic, \(\varepsilon_{ij}^e\) and irreversible/inelastic, \(\varepsilon_{ij}^i\), components in a generalized 3D space. In particular, the tensor \(\varepsilon_{ij}^i\) is utilized here to implicitly account for all transformation-induced deformations in the SMA

\[
\varepsilon_{ij} = \varepsilon_{ij}^e + \varepsilon_{ij}^i.
\] (A.1)

The stress tensor, \(\sigma_{ij}\), is decomposed into an effective stress, \((\sigma_{ij} - \alpha_{ij})\), and internal state tensorial variable, \(\alpha_{ij} = \sum_{b=1}^{6} \alpha_{ij}^b\), where \(b = 1\) to \(6\) indicates the number of inelastic mechanisms whose internal stress-like, and conjugate strain-like, internal variables are denoted as \(\alpha_{ij}^b\) and \(\gamma_{ij}^b\), respectively. These are utilized to regulate the energy storage \((b = 1, 2, 3)\) and energy dissipations \((b = 4, 5, 6)\) during the evolution of the thermomechanical response of the material.

There are two fundamental energy potentials; i.e., a Gibb’s complementary function, \(\Phi\), and a dissipation function, \(\Omega((\sigma_{ij} - \alpha_{ij}), \alpha_{ij}^b)\), where the subscripts R and IR indicate reversible (elastic) and irreversible (inelastic) components

\[
\Phi = \Phi_R(\sigma_{ij}) + \Phi_{IR}(\sigma_{ij}, \alpha_{ij}^b),
\] (A.2)

\[
\Phi_R(\sigma_{ij}) = \frac{1}{2} \sigma_{ij} E_{ijkl} \varepsilon_{kl}
\] (A.3)

\[
\Phi_{IR}(\sigma_{ij}, \alpha_{ij}^b) = \sigma_{ij}^e \varepsilon_{ij}^e + \sum_{b=1}^{N} \mathcal{H}_{bb}^b,
\] (A.4)
\begin{align}
\Omega((\sigma_i - \alpha_i), \alpha_i^{(b)}) &= \int \frac{\kappa^2 F^n}{2\mu} dF. \quad \text{(A.5)}
\end{align}

The evolutionary equations for the inelastic transformation strain, as well as the internal state variables, are obtained as in equations (A.6) and (A.7); where, \( E_{ijkl} \) is the isotropic fourth-order tensor of the elastic moduli (Young’s modulus, \( E \) and Poisson’s ratio \( \nu \))

\begin{align}
\dot{\varepsilon}_{ij} - \varepsilon_i^{(b)} &= \frac{d}{dr} \left( \frac{\partial \Phi_{\varepsilon}}{\partial \sigma_{ij}} \right) = \varepsilon_{ij}^{(b)} \theta_D, \quad \text{(A.6)}
\end{align}

\begin{align}
\dot{\alpha}_{ij}^{(b)} &= \left[ \frac{\partial^2 \Phi_{\varepsilon}}{\partial \sigma_{ij} \partial \sigma_{il}} \right] \varepsilon_i^{(b)} \
\dot{\varepsilon}_i^{(b)} &= \frac{\partial \Omega}{\partial \sigma_{ij}} \quad \text{and} \quad \varepsilon_i^{(b)} = - \frac{\partial \Omega}{\partial \alpha_i^{(b)}}. \quad \text{(A.7)}
\end{align}

The functions that provide the driving force for the energy storage and dissipations during the evolution of the thermomechanical response are given below:

\begin{align}
F(\sigma_i - \alpha_i) &= \frac{1}{\kappa^2} \left[ \frac{1}{2\rho} (\sigma_i - \alpha_i) \mathcal{M}_{ijkl}(\sigma_i - \alpha_i) \right], \quad \text{(A.9)}
\end{align}

\begin{align}
H_{ib}^{(b)} &= \begin{cases} 
\frac{\kappa_{ib}(\beta_{ib}^b)}{\rho_{ib}} \int h(\sigma_i - \alpha_i) \mathcal{M}_{ijkl} \, d\sigma_i, & \text{for } b = 1, 2, 3, \\
\frac{\kappa_{ib}(\beta_{ib}^b)}{\rho_{ib}} \int h(\sigma_i - \alpha_i) \mathcal{M}_{ijkl} \, d\sigma_i, & \text{for } b \geq 4,
\end{cases} \quad \text{(A.10)}
\end{align}

\begin{align}
G_{ib}^{(b)}(\alpha_{ij}^{(b)}) &= \begin{cases} 
\rho_{ib}(\kappa_{ib} H_{ib} (\sqrt{\dot{g}} - \gamma_{ij}^{(b)} - 1), & \text{for } b = 1, 2, \\
\rho_{ib}(\kappa_{ib} H_{ib} [1 - (\sqrt{\dot{g}} - \gamma_{ij}^{(b)})^2], & \text{for } b = 3, \\
h(\sigma_i - \alpha_i) \mathcal{M}_{ijkl} \gamma_{ij}^{(b)} \theta_D, & \text{for } b \geq 4;
\end{cases} \quad \text{(A.11)}
\end{align}

\begin{align}
G_{ib}^{(b)}(\alpha_{ij}^{(b)}) &= \frac{1}{2\kappa_{ib}^2} \alpha_{ij}^{(b)} \mathcal{M}_{ijkl}(\alpha_{ij}^{(b)}), \quad \text{(A.13)}
\end{align}

\begin{align}
g_{ib}^{(b)}(\gamma_{ij}^{(b)}) &= \gamma_{ij}^{(b)}, \quad \text{(A.14)}
\end{align}

\begin{align}
\rho &= \frac{1 + c \sqrt{d}}{1 + c \sqrt{d} + k_3}, \quad \text{(A.15)}
\end{align}

In the above equations, \( h(L) \) is a Heaviside function with argument being the loading index; \( L = \alpha_{ij}^{(b)} \Gamma_{ij} \), where

\begin{align}
\Gamma_{ij} &= \partial F / \partial (\sigma_i - \alpha_i); \quad \mathcal{M}_{ijkl} = \frac{1}{3} (\delta_{ik} \delta_{lj} + \delta_{ij} \delta_{lk} - \frac{1}{3} \delta_{ij} \delta_{kl}); \quad \text{with } \delta_{ij} = \text{Kronecker delta}; \quad \text{and } H_{ib} \text{, } \beta_{ib} \text{, and } \kappa_{ib} \text{ are material parameters for the individual hardening mechanism.}
\end{align}

\[ k_3 = \cos \theta \beta, \] where \( \beta \) is Lode’s angle calculated from the invariants of the effective stress \( (\sigma_i - \alpha_i) \), see Chen and Saleeb (1994). For further elaboration on the details of the model, see Saleeb et al (2011, 2013a, 2013b).

A.2. Parameterization procedure

With regard to the calibration of the SMA model for any thermomechanical response, there are a total of 25 material parameters to be evaluated. Among these are the elastic modulus, ‘\( E \)’ and Poisson’s ratio, ‘\( \nu \)’ to account for the elastic/reversible part, while the remaining 23 parameters are devoted for the inelastic, non-linear part of the model. These are categorized into two sets: (1) parameters accounting for the inelastic/transformation strain (rate dependency factors, ‘\( n \)’ and ‘\( \mu \)’; threshold ‘\( \kappa \)’ in the transformation function; and distortion material parameters \( c, d \)), and (2) threshold values \( \kappa_{ib} \text{ and exponent } \beta_{ib} \text{ for mechanisms } b = 1 \text{ to } 6 \) in the evolution equations of the internal state variables. Note that the two material parameters \( (c, d) \) account for differences in the material deformation response under various loading modes, such as tension, compression, or shear, also referred to here as intrinsic asymmetry in tension and compression (ATC) parameters.

The above mentioned 25 material parameters can be further grouped into two groups: (1) 17 material parameters (\( E, \nu, n, \mu, d, H_{ib} \) and \( \beta_{ib} \) for \( b = 1 \) to 6) which remains fixed, and (2) another set of eight material parameters \( (\kappa, c, \text{ and } \kappa_{ib} \text{ for } b = 1 \text{ to } 6) \) that account for the possible temperature and/or stress-state dependencies of the thermomechanical SMA responses such as strain evolution during thermal cycling, asymmetry in responses for tension, compression, and shear loading modes, and any other unique SMA behaviors. The functional temperature/stress dependency of the 7 key reference thresholds \( (\kappa, \kappa_{ib} \text{ for } b = 1 \text{ to } 6) \) are expressed as \( k^{(c)} = \kappa (T) \cdot \eta (\sigma_i) \) and \( \kappa^{(c)}(T) = \kappa(T) \cdot \eta_{ib}(\sigma_i) \); where \( T \) is the temperature, \( \eta (or \eta_{ib}) \) is a non-dimensional factor that is dependent on the stress intensity only and the multi-axial stress intensity is defined as \( \sigma_{ps} = \sqrt{3} (\sigma_i \mathcal{M}_{ijkl} \sigma_{kl})^{2} \) (Saleeb et al 2013a, 2013b, 2013c).

A.3. Calculation of effective stress and stress components

The states of effective and individual stress components obtained from the numerical simulation (see figure 4) at the end of applied axial load of 8.8 N at 30°C were compared to the values obtained from analytical helical coil theory from Ancker and Goodier (1958). The stress formulas provide the approximate results for the stress component values in cylindrical coordinate system at the inner point in the horizontal diameter of the wire cross-section (i.e., point A as shown in figure 4).

From Ancker and Goodier (1958):

\begin{align}
\sigma_{RR} &= \tau_{R\theta} = \tau_{RZ} = 0, \quad \text{(A.16)}
\end{align}

\begin{align}
\sigma_{R\theta} &= \frac{8PD}{\pi d^3} \left[ \frac{(-2 + v + 4v^2)}{4(1 + v)C} \tan \alpha + \cdots \right], \quad \text{(A.17)}
\end{align}
\[ \sigma_{zz} = \frac{8PD}{\pi d^3} \left[ 2 \tan \alpha + \frac{(11 + 12 \nu)}{4(1 + \nu)} \tan \alpha + \cdots \right], \quad (A18) \]

\[ \tau_{zz} = \frac{8PD}{\pi d^3} \left[ 1 + \frac{5}{4C} + \frac{7}{8C^2} + \cdots \right], \quad (A19) \]

\[ \tau_{\text{effective}} = \sqrt{\sigma_{\theta \theta}^2 + \sigma_{zz}^2 - \sigma_{\theta \theta} \sigma_{zz} + 3 \tau_{\theta z}^2}. \quad (A20) \]

The helical spring used in the simulation has an initial pitch angle (\( \alpha \)) of 4.7°, mean coil diameter (\( D = D_{\text{m}} \)) of 23.1 mm, wire diameter (d) of 2.16 mm and a spring index (\( C = D_{\text{m}}/d \)) of 10.7 (as described in section 3.1). Also the Poisson’s ration utilized here is \( \nu = 0.3 \) and the applied axial load is \( P = 8.8 \) N. From equations (A17)–(A20) values were obtained for the following stress components: \( \sigma_{\theta \theta} \approx 0.098 \) MPa, \( \sigma_{zz} \approx 9.55 \) MPa, \( \tau_{\theta z} \approx 57.74 \) MPa and \( \tau_{\text{effective}} \approx 100.45 \) MPa. The stress values obtained from the numerical model from the applied axial load of 8.8 N were as follows: \( \sigma_{\theta \theta} \approx 0.24 \) MPa, \( \sigma_{zz} \approx 4.55 \) MPa, \( \sigma_{zz} \approx 15.87 \) MPa, \( \tau_{\theta z} \approx 0.02 \) MPa, \( \tau_{\theta z} \approx 61.15 \) MPa, \( \tau_{\theta z} \approx 0.85 \) MPa and \( \tau_{\text{effective}} \approx 107.68 \) MPa. Considering the magnitudes, all the stress components are in excellent agreement between the theoretical and numerical values. This result was also evident from the magnitudes corresponding to the color of the stress contours on the inner diameter region of the helical coil as shown in Figure 4. The good agreement between between theory and model observed here can be attributed to the notion that deformation from the applied 8.8 N load was mostly elastic with limited detwining and reorientation of martensite variants.

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