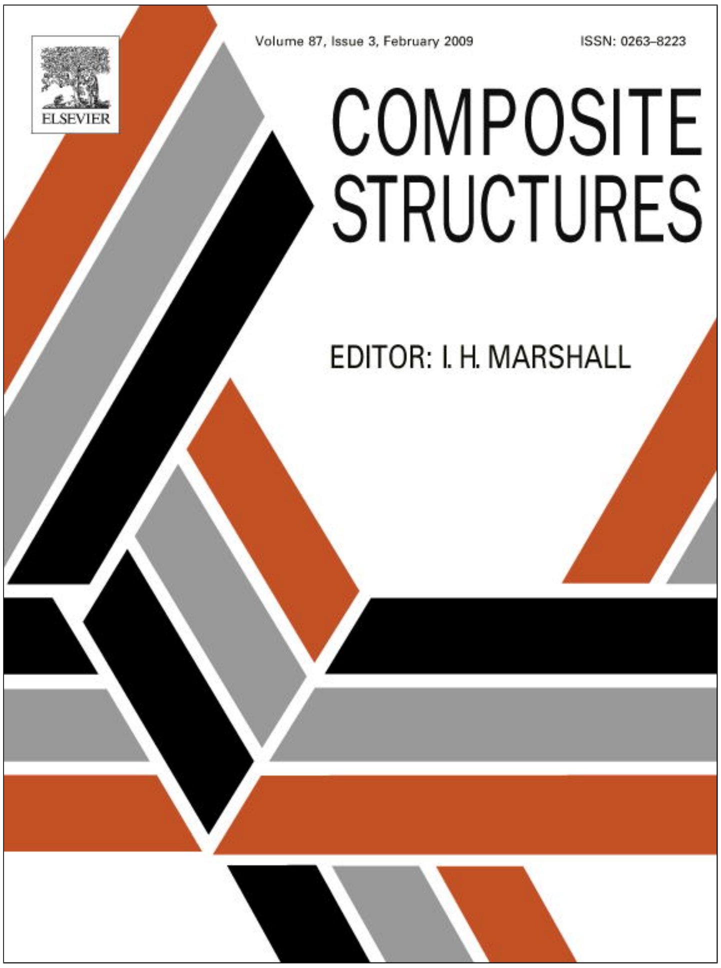
ELSEVIER
Volume 87, Issue 3, February 2009
ISSN: 0263-8223
COMPOSITE STRUCTURES
EDITOR: I. H. MARSHALL

Available online at www.sciencedirect.com
ScienceDirect
Composite Structures 87 (2009) 206–214
COMPOSITE STRUCTURES
www.elsevier.com/locate/compstruct

Effective properties of multilayered functionally graded multiferroic composites

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Available online 26 January 2008

Abstract

A micromechanics approach is employed to derive the effective properties (including the thermal properties) of a multilayered functionally graded multiferroic composite with 2–2 connectivity among the phases. Concise matrix expressions of the effective properties of the layered composite are presented. The derived formulas are then applied to find the explicit expressions of the effective properties for three practical cases: (a) a multiferroic composite composed of an orthotropic piezoelectric phase and an orthotropic magnetostrictive phase; (b) a multiferroic composite composed of an orthotropic piezoelectric phase, an orthotropic magnetostrictive phase and an orthotropic elastic substrate; (c) a multiferroic composite composed of a functionally graded orthotropic piezoelectric phase and a functionally graded orthotropic magnetostrictive phase. Our results clearly show that: (i) the magnetoelectric coupling effect for case (b) dramatically drops as the volume of the elastic substrate increases; and (ii) the magnetoelectric coupling effect for case (c) can be significantly enhanced or reduced depending on the material gradient manner for the functionally graded piezoelectric and magnetostrictive phases.

Keywords: Multiferroics; Functionally graded material; 2–2 connectivity; Effective properties

1. Introduction

Functionally graded materials (FGMs) and composites possess various attractive properties and have been the fields of intensive investigation. Recently, a first-order shear deformation model was proposed for isotropic elastic FGM plate [1]. The structural stability of FGM panels under aero-thermal loads [2] and dynamic stability of FGM cylindrical shells under a period axial loading [3] were also studied numerically. While Chen et al. [4] calculated the dispersion curves for an elastic FGM plate, Yang and Chen [5] analyzed the free vibration and buckling of elastic FGM beams weakened by edge cracks. Besides the elastic FGM structures, piezoelectric FGM structures were also investigated, as in [6] where the transient piezothermoelastic behavior of an FGM thermopiezoelectric hollow sphere was studied to demonstrate the influence of the FGMs on the field quantities.

Recently, the coupling between the magnetic and electric fields has attracted wide attention in composites as this is an intrinsic fascinating property in multiferroics [7–9]. This coupling, also called magnetoelectric (ME) effect, can be described as an induced electric polarization under an external magnetic field or an induced magnetization under an external electric field. It has been found that the ME effect in artificial multiferroic composites consisting of ferromagnetic and ferroelectric phases, which is achieved through the product property, can be several orders larger than that observed in natural single-phase materials, such as the antiferromagnetic Cr_2O_3 crystal [8–12]. Up to now several approaches have been developed to predict the ME effect in multiferroic particulate or laminate composites: (i) the micromechanics approach such as the Mori–Tanaka mean

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doi:10.1016/j.compstruct.2008.01.006

field method, the dilute concentration method and the self-consistent method [13,14]; (ii) the Green's function method [15]; (iii) the method by Milgrom and Shtrikman for two-phase fibrous composites, which establishes a correspondence between the uncoupled and coupled problems [16,17]; (iv) the method by Harshé for layered composites with 2–2 connectivity [18–21]; (v) the equivalent circuit method (particularly for the analysis of the resonance ME effect) [22,23].

In this research we extend the micromechanics approach proposed by Qu and Cherkaoui [24] to the study of the effective properties including the ME effect and the thermal properties of a magnetoelectric multiferroic multilayer composite with 2–2 connectivity of the phases. We find that this micromechanics approach is rather efficient in the sense that: (i) explicit expressions of the effective properties of the layered multiferroic composites can be obtained; (ii) the ME effect of the 2–2 type piezoelectric-magnetostrictive films on an elastic substrate can be expediently investigated and the influence of the substrate on the ME response is found to be in qualitative agreement with recent observations and calculations [7,15,25]; (iii) the ME effect of the multilayered FGM multiferroic composites, which have recently been successfully fabricated [26], can also be easily obtained.

2. Effective properties of multilayered multiferroic composites

The linear constitutive equations of a homogeneous multiferroic material can be written as

$$\begin{aligned} \sigma_{ij} &= C_{ijkl}S_{kl} - e_{kij}E_k - q_{kij}H_k - \beta_{ij}\theta, \\ D_k &= e_{kij}S_{ij} + \varepsilon_{kl}E_l + \alpha_{kl}E_l + p_k\theta, \\ B_k &= q_{kij}S_{ij} + \alpha_{kl}E_l + \mu_{kl}E_l + m_k\theta, \end{aligned} \tag{1}$$

where σ_{ij} and S_{ij} are the stress and strain components; D_i and E_i are the electric displacement and electric fields; B_i and H_i are the magnetic flux and magnetic fields; θ is the temperature change; C_{ijkl}, ε_{kl} and μ_{kl} are the elastic, the dielectric permittivity, and magnetic permeability coefficients, respectively; e_{kij}, q_{kij} and α_{kl} are the piezoelectric, piezomagnetic and magnetoelectric coefficients, respectively; β_{ij}, p_k and m_k are the thermal stress, pyroelectric and pyromagnetic coefficients, respectively.

If we define the following vector notations:

$$\boldsymbol{\sigma}_n = \begin{bmatrix} \sigma_{33} \\ \sigma_{23} \\ \sigma_{13} \\ D_3 \\ B_3 \end{bmatrix}, \quad \boldsymbol{\sigma}_t = \begin{bmatrix} \sigma_{11} \\ \sigma_{22} \\ \sigma_{12} \\ D_1 \\ D_2 \\ B_1 \\ B_2 \end{bmatrix}, \quad \mathbf{S}_n = \begin{bmatrix} S_{33} \\ 2S_{23} \\ 2S_{13} \\ -E_3 \\ -H_3 \end{bmatrix}, \quad \mathbf{S}_t = \begin{bmatrix} S_{11} \\ S_{22} \\ 2S_{12} \\ -E_1 \\ -E_2 \\ -H_1 \\ -H_2 \end{bmatrix}, \tag{2}$$

then the constitutive Eq. (1) can be equivalently written the following concise matrix form

$$\boldsymbol{\sigma}_n = \mathbf{C}_{nn}\mathbf{S}_n + \mathbf{C}_{nt}\mathbf{S}_t - \boldsymbol{\beta}_n\theta, \boldsymbol{\sigma}_t = \mathbf{C}_{tn}\mathbf{S}_n + \mathbf{C}_{tt}\mathbf{S}_t - \boldsymbol{\beta}_t\theta, \tag{3}$$

where $\mathbf{C}_{nn}$, $\mathbf{C}_{nt}$, $\mathbf{C}_{tn}$, and $\mathbf{C}_{tt}$ are the generalized Voigt matrices given by

$$\mathbf{C}_{nn} = \mathbf{C}_{nn}^{\mathrm{T}} = \begin{bmatrix} C_{33} & C_{34} & C_{35} & e_{33} & q_{33} \\ C_{34} & C_{44} & C_{45} & e_{34} & q_{34} \\ C_{35} & C_{45} & C_{55} & e_{35} & q_{35} \\ e_{33} & e_{34} & e_{35} & -\varepsilon_{33} & -\alpha_{33} \\ q_{33} & q_{34} & q_{35} & -\alpha_{33} & -\mu_{33} \end{bmatrix}, \tag{4}$$

$$\mathbf{C}_{nt} = \begin{bmatrix} C_{13} & C_{23} & C_{36} & e_{13} & e_{23} & q_{13} & q_{23} \\ C_{14} & C_{24} & C_{46} & e_{14} & e_{24} & q_{14} & q_{24} \\ C_{15} & C_{25} & C_{56} & e_{15} & e_{25} & q_{15} & q_{25} \\ e_{31} & e_{32} & e_{36} & -\varepsilon_{13} & -\varepsilon_{23} & -\alpha_{31} & -\alpha_{32} \\ q_{31} & q_{32} & q_{36} & -\alpha_{13} & -\alpha_{23} & -\mu_{13} & -\mu_{23} \end{bmatrix}, \mathbf{C}_{tn} = \mathbf{C}_{nt}^{\mathrm{T}}, \tag{5}$$

$$\mathbf{C}_{tt} = \mathbf{C}_{tt}^{T} = \begin{bmatrix} C_{11} & C_{12} & C_{16} & e_{11} & e_{21} & q_{11} & q_{21} \\ C_{12} & C_{22} & C_{26} & e_{12} & e_{22} & q_{12} & q_{22} \\ C_{16} & C_{26} & C_{66} & e_{16} & e_{26} & q_{16} & q_{26} \\ e_{11} & e_{12} & e_{16} & -\varepsilon_{11} & -\varepsilon_{12} & -\alpha_{11} & -\alpha_{12} \\ e_{21} & e_{22} & e_{26} & -\varepsilon_{12} & -\varepsilon_{22} & -\alpha_{21} & -\alpha_{22} \\ q_{11} & q_{12} & q_{16} & -\alpha_{11} & -\alpha_{21} & -\mu_{11} & -\mu_{12} \\ q_{21} & q_{22} & q_{26} & -\alpha_{12} & -\alpha_{22} & -\mu_{12} & -\mu_{22} \end{bmatrix}, \tag{6}$$

$$\begin{aligned} \boldsymbol{\beta}_n &= [\beta_{33} \quad \beta_{23} \quad \beta_{13} \quad -p_3 \quad -m_3]^{\mathrm{T}}, \\ \boldsymbol{\beta}_t &= [\beta_{11} \quad \beta_{22} \quad \beta_{12} \quad -p_1 \quad -p_2 \quad -m_1 \quad -m_2]^{\mathrm{T}}, \end{aligned} \tag{7}$$

with T denoting matrix transpose. It is observed that $\mathbf{C}_{nn}$ and $\mathbf{C}_{tt}$ are symmetric but not positive definite. Now we consider a multiferroic multilayered composite consisting of N layers of homogeneous multiferroic materials as shown in Fig. 1. In the following we will attach a superscript (k) to the quantities associated with layer k $(k = 1, 2, \ldots, N)$. Therefore, for the kth multiferroic layer, it follows from Eq. (1) that

$$\begin{aligned} \boldsymbol{\sigma}_n^{(k)} &= \mathbf{C}_{nn}^{(k)}\mathbf{S}_n^{(k)} + \mathbf{C}_{nt}^{(k)}\mathbf{S}_t^{(k)} - \boldsymbol{\beta}_n^{(k)}\theta^{(k)}, \\ \boldsymbol{\sigma}_t^{(k)} &= \mathbf{C}_{tn}^{(k)}\mathbf{S}_n^{(k)} + \mathbf{C}_{tt}^{(k)}\mathbf{S}_t^{(k)} - \boldsymbol{\beta}_t^{(k)}\theta^{(k)}, \end{aligned} \tag{8}$$

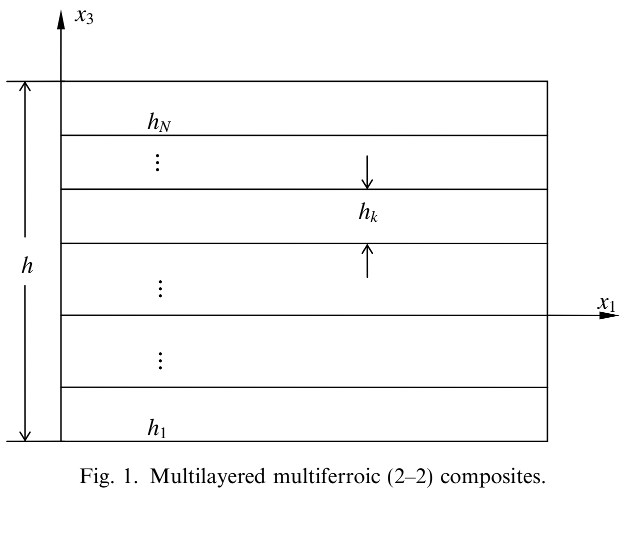

Fig. 1. Multilayered multiferroic (2–2) composites.

It is further assumed that: (1) each homogeneous layer is under a uniform state of deformation; (2) the temperature is uniform over the whole representative volume element (RVE); (3) the layers are perfectly bonded together, i.e., tractions, normal electric displacement, normal magnetic flux, displacements, electric potential and magnetic potential are all continuous across the layer interface. Based on these assumptions, we then have

$$\boldsymbol{\sigma}_n^{(k)} = \boldsymbol{\sigma}_n, \mathbf{S}_t^{(k)} = \mathbf{S}_t, \theta^{(k)} = \theta, \text{for } k = 1, 2, \ldots, N \tag{9}$$

In view of Eq. (9), Eq. $(8)_1$ can then be further cast into

$$\mathbf{S}_n^{(k)} = (\mathbf{C}_{nn}^{(k)})^{-1}\boldsymbol{\sigma}_n - (\mathbf{C}_{nn}^{(k)})^{-1}\mathbf{C}_{nt}^{(k)}\mathbf{S}_t + (\mathbf{C}_{nn}^{(k)})^{-1}\boldsymbol{\beta}_n^{(k)}\theta. \tag{10}$$

Taking the average of $\mathbf{S}_n^{(k)}$ over the RVE, we obtain the following

$$\mathbf{S}_n = \sum_{k=1}^{N} v_k \mathbf{S}_n^{(k)} = \sum_{k=1}^{N} v_k (\mathbf{C}_{nn}^{(k)})^{-1}\boldsymbol{\sigma}_n - \sum_{k=1}^{N} v_k (\mathbf{C}_{nn}^{(k)})^{-1}\mathbf{C}_{nt}^{(k)}\mathbf{S}_t + \sum_{k=1}^{N} v_k (\mathbf{C}_{nn}^{(k)})^{-1}\boldsymbol{\beta}_n^{(k)}\theta, \tag{11}$$

where $v_k = h_k/h$ is the volume fraction of the kth layer. By rearranging the terms in Eq. (11), we arrive at

$$\boldsymbol{\sigma}_n = \overline{\mathbf{C}}_{nn}\mathbf{S}_n + \overline{\mathbf{C}}_{nt}\mathbf{S}_t - \bar{\boldsymbol{\beta}}_n\theta, \tag{12}$$

where

$$\overline{\mathbf{C}}_{nn} = \left[\sum_{k=1}^{N} v_k (\mathbf{C}_{nn}^{(k)})^{-1}\right]^{-1}, \tag{13}$$

$$\overline{\mathbf{C}}_{nt} = \overline{\mathbf{C}}_{nn}\left[\sum_{k=1}^{N} v_k (\mathbf{C}_{nn}^{(k)})^{-1}\mathbf{C}_{nt}^{(k)}\right], \tag{14}$$

$$\bar{\boldsymbol{\beta}}_n = \overline{\mathbf{C}}_{nn}\sum_{k=1}^{N} v_k (\mathbf{C}_{nn}^{(k)})^{-1}\boldsymbol{\beta}_n^{(k)}. \tag{15}$$

Substitution of Eq. (10) into Eq. $(8)_2$ yields

$$\boldsymbol{\sigma}_t^{(k)} = \mathbf{C}_{tn}^{(k)}(\mathbf{C}_{nn}^{(k)})^{-1}\boldsymbol{\sigma}_n + [\mathbf{C}_{tt}^{(k)} - \mathbf{C}_{tn}^{(k)}(\mathbf{C}_{nn}^{(k)})^{-1}\mathbf{C}_{nt}^{(k)}]\mathbf{S}_t + [\mathbf{C}_{tn}^{(k)}(\mathbf{C}_{nn}^{(k)})^{-1}\boldsymbol{\beta}_n^{(k)} - \boldsymbol{\beta}_t^{(k)}]\theta. \tag{16}$$

Taking the average of $\boldsymbol{\sigma}_t^{(k)}$ over the RVE, we then obtain

$$\boldsymbol{\sigma}_t = \sum_{k=1}^{N} v_k \boldsymbol{\sigma}_t^{(k)} = \sum_{k=1}^{N} v_k \mathbf{C}_{tn}^{(k)}(\mathbf{C}_{nn}^{(k)})^{-1}\boldsymbol{\sigma}_n + \sum_{k=1}^{N} v_k [\mathbf{C}_{tt}^{(k)} - \mathbf{C}_{tn}^{(k)}(\mathbf{C}_{nn}^{(k)})^{-1}\mathbf{C}_{nt}^{(k)}]\mathbf{S}_t + \sum_{k=1}^{N} v_k [\mathbf{C}_{tn}^{(k)}(\mathbf{C}_{nn}^{(k)})^{-1}\boldsymbol{\beta}_n^{(k)} - \boldsymbol{\beta}_t^{(k)}]\theta. \tag{17}$$

Substitution of Eq. (12) into Eq. (17) results in

$$\boldsymbol{\sigma}_t = \overline{\mathbf{C}}_{tn}\mathbf{S}_n + \overline{\mathbf{C}}_{tt}\mathbf{S}_t - \bar{\boldsymbol{\beta}}_t\theta, \tag{18}$$

where

$$\overline{\mathbf{C}}_{tn} = \left[\sum_{k=1}^{N} v_k \mathbf{C}_{tn}^{(k)}(\mathbf{C}_{nn}^{(k)})^{-1}\right]\overline{\mathbf{C}}_{nn}, \tag{19}$$

$$\overline{\mathbf{C}}_{tt} = \sum_{k=1}^{N} v_k \mathbf{C}_{tt}^{(k)} + \sum_{k=1}^{N} v_k \mathbf{C}_{tn}^{(k)}(\mathbf{C}_{nn}^{(k)})^{-1}(\overline{\mathbf{C}}_{nt} - \mathbf{C}_{nt}^{(k)}), \tag{20}$$

$$\bar{\boldsymbol{\beta}}_t = \sum_{k=1}^{N} v_k \boldsymbol{\beta}_t^{(k)} + \sum_{k=1}^{N} v_k \mathbf{C}_{tn}^{(k)}(\mathbf{C}_{nn}^{(k)})^{-1}(\bar{\boldsymbol{\beta}}_n - \boldsymbol{\beta}_n^{(k)}). \tag{21}$$

It can be easily proved that the following symmetry properties exist

$$\overline{\mathbf{C}}_{nn} = \overline{\mathbf{C}}_{nn}^{\mathrm{T}}, \quad \overline{\mathbf{C}}_{tt} = \overline{\mathbf{C}}_{tt}^{\mathrm{T}}, \quad \overline{\mathbf{C}}_{tn} = \overline{\mathbf{C}}_{nt}^{\mathrm{T}}. \tag{22}$$

Now the effective properties of the multilayered multiferroic composite have been completely determined. It is observed that the above derivations are basically an extension of the approach proposed by Qu and Cherkaoui [24] to multiferroic composites including also the thermal effect. Next we present three practical cases as applications of the above formulas, which further demonstrate the efficiency and versatility of the proposed approach.

3. Applications

3.1. Effective properties of a multiferroic composite composed of an orthotropic piezoelectric phase and an orthotropic magnetostrictive phase

By using Eqs. (13)–(15) and (19)–(21) the effective properties of the orthotropic piezoelectric-magnetostrictive bilayer composite as shown in Fig. 2 can be explicitly determined as follows

The nonzero components of $\overline{\mathbf{C}}_{nn}$:

$$\begin{aligned}
\overline{C}_{33} &= \frac{1}{\Delta}\left(\frac{v_{\mathrm{p}}C_{33}^{(p)}}{\varepsilon_{33}^{(p)}\widetilde{C}_{33}^{(p)}} + \frac{v_{\mathrm{m}}}{\varepsilon_{33}^{(m)}}\right)\left(\frac{v_{\mathrm{m}}C_{33}^{(m)}}{\mu_{33}^{(m)}\widetilde{C}_{33}^{(m)}} + \frac{v_p}{\mu_{33}^{(p)}}\right),\\
\overline{C}_{44} &= \frac{C_{44}^{(p)}C_{44}^{(m)}}{v_{\mathrm{p}}C_{44}^{(m)} + v_{\mathrm{m}}C_{44}^{(p)}}, \quad \overline{C}_{55} = \frac{C_{55}^{(p)}C_{55}^{(m)}}{v_{\mathrm{p}}C_{55}^{(m)} + v_{\mathrm{m}}C_{55}^{(p)}},\\
\bar{e}_{33} &= \frac{v_{\mathrm{p}}e_{33}}{\Delta\varepsilon_{33}^{(p)}\widetilde{C}_{33}^{(p)}}\left(\frac{v_{\mathrm{m}}C_{33}^{(m)}}{\mu_{33}^{(m)}\widetilde{C}_{33}^{(m)}} + \frac{v_{\mathrm{p}}}{\mu_{33}^{(p)}}\right), \quad \bar{q}_{33} = \frac{v_{\mathrm{m}}q_{33}}{\Delta\mu_{33}^{(m)}\widetilde{C}_{33}^{(m)}}\left(\frac{v_{\mathrm{p}}C_{33}^{(p)}}{\varepsilon_{33}^{(p)}\widetilde{C}_{33}^{(p)}} + \frac{v_{\mathrm{m}}}{\varepsilon_{33}^{(m)}}\right),\\
\bar{\varepsilon}_{33} &= \frac{1}{\Delta}[\left(\frac{v_{\mathrm{p}}}{\widetilde{C}_{33}^{(p)}} + \frac{v_{\mathrm{m}}}{\widetilde{C}_{33}^{(m)}}\right)\left(\frac{v_{\mathrm{m}}C_{33}^{(m)}}{\mu_{33}^{(m)}\widetilde{C}_{33}^{(m)}} + \frac{v_{\mathrm{p}}}{\mu_{33}^{(p)}}\right) + \frac{v_{\mathrm{m}}^2 q_{33}^2}{\mu_{33}^{(m)2}\widetilde{C}_{33}^{(m)2}}],\\
\bar{\mu}_{33} &= \frac{1}{\Delta}[\left(\frac{v_{\mathrm{p}}}{\widetilde{C}_{33}^{(p)}} + \frac{v_{\mathrm{m}}}{\widetilde{C}_{33}^{(m)}}\right)\left(\frac{v_{\mathrm{p}}C_{33}^{(p)}}{\varepsilon_{33}^{(p)}\widetilde{C}_{33}^{(p)}} + \frac{v_{\mathrm{m}}}{\varepsilon_{33}^{(m)}}\right) + \frac{v_{\mathrm{p}}^2 e_{33}^2}{\varepsilon_{33}^{(p)2}\widetilde{C}_{33}^{(p)2}}],\\
\bar{\alpha}_{33} &= -\frac{v_{\mathrm{p}}v_{\mathrm{m}}e_{33}q_{33}}{\varepsilon_{33}^{(p)}\mu_{33}^{(m)}\widetilde{C}_{33}^{(p)}\widetilde{C}_{33}^{(m)}\Delta},
\end{aligned} \tag{23}$$

where $\widetilde{C}_{33}^{(p)} = C_{33}^{(p)} + e_{33}^2/\varepsilon_{33}^{(p)}$ and $\widetilde{C}_{33}^{(m)} = C_{33}^{(m)} + q_{33}^2/\mu_{33}^{(m)}$ are, respectively the piezoelectrically and piezomagnetically stiffened elastic constants, and

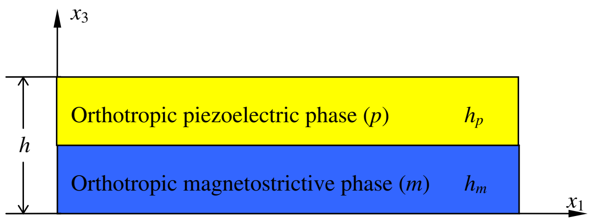

Fig. 2. A multiferroic composite composed of an orthotropic piezoelectric phase and an orthotropic magnetostrictive phase.

$$\varDelta = \left(\frac{v_{\rm p}}{\widetilde{C}_{33}^{(p)}} + \frac{v_{\rm m}}{\widetilde{C}_{33}^{(m)}}\right)\left(\frac{v_{\rm p}C_{33}^{(p)}}{\varepsilon_{33}^{(p)}\widetilde{C}_{33}^{(p)}} + \frac{v_{\rm m}}{\varepsilon_{33}^{(m)}}\right)\left(\frac{v_{\rm m}C_{33}^{(m)}}{\mu_{33}^{(m)}\widetilde{C}_{33}^{(m)}} + \frac{v_{\rm p}}{\mu_{33}^{(p)}}\right) + \frac{v_{\rm m}^2 q_{33}^2}{\mu_{33}^{(m)2}\widetilde{C}_{33}^{(m)2}}\left(\frac{v_{\rm p}C_{33}^{(p)}}{\varepsilon_{33}^{(p)}\widetilde{C}_{33}^{(p)}} + \frac{v_{\rm m}}{\varepsilon_{33}^{(m)}}\right) + \frac{v_{\rm p}^2 e_{33}^2}{\varepsilon_{33}^{(p)2}\widetilde{C}_{33}^{(p)2}} \times \left(\frac{v_{\rm m}C_{33}^{(m)}}{\mu_{33}^{(m)}\widetilde{C}_{33}^{(m)}} + \frac{v_{\rm p}}{\mu_{33}^{(p)}}\right). \tag{24}$$

The nonzero components of $\overline{\mathbf{C}}_{nt}(=\overline{\mathbf{C}}_{tn}^T)$:

$$\overline{C}_{13} = \frac{v_{\rm p}[\overline{C}_{33}(\varepsilon_{33}^{(p)}C_{13}^{(p)} + e_{33}e_{31}) + \bar{e}_{33}(e_{33}C_{13}^{(p)} - e_{31}C_{33}^{(p)})]}{\varepsilon_{33}^{(p)}\widetilde{C}_{33}^{(p)}} + \frac{v_{\rm m}[\overline{C}_{33}(\mu_{33}^{(m)}C_{13}^{(m)} + q_{33}q_{31}) + \bar{q}_{33}(q_{33}C_{13}^{(m)} - q_{31}C_{33}^{(m)})]}{\mu_{33}^{(m)}\widetilde{C}_{33}^{(m)}},$$

$$\overline{C}_{23} = \frac{v_{\rm p}[\overline{C}_{33}(\varepsilon_{33}^{(p)}C_{23}^{(p)} + e_{33}e_{32}) + \bar{e}_{33}(e_{33}C_{23}^{(p)} - e_{32}C_{33}^{(p)})]}{\varepsilon_{33}^{(p)}\widetilde{C}_{33}^{(p)}} + \frac{v_{\rm m}[\overline{C}_{33}(\mu_{33}^{(m)}C_{23}^{(m)} + q_{33}q_{32}) + \bar{q}_{33}(q_{32}C_{33}^{(m)} - q_{33}C_{23}^{(m)})]}{\mu_{33}^{(m)}\widetilde{C}_{33}^{(m)}}, \tag{25a}$$

$$\bar{e}_{24} = \frac{v_{\rm p}e_{24}\overline{C}_{44}}{C_{44}^{(p)}}, \quad \bar{e}_{15} = \frac{v_{\rm p}e_{15}\overline{C}_{55}}{C_{55}^{(p)}}, \quad \bar{q}_{24} = \frac{v_{\rm m}q_{24}\overline{C}_{44}}{C_{44}^{(m)}}, \quad \bar{q}_{15} = \frac{v_{\rm m}q_{15}\overline{C}_{55}}{C_{55}^{(m)}}, \tag{25b}$$

$$\bar{e}_{31} = \frac{v_{\rm p}[\bar{e}_{33}(\varepsilon_{33}^{(p)}C_{13}^{(p)} + e_{33}e_{31}) + \bar{\varepsilon}_{33}(e_{31}C_{33}^{(p)} - e_{33}C_{13}^{(p)})]}{\varepsilon_{33}^{(p)}\widetilde{C}_{33}^{(p)}} + \frac{v_{\rm m}[\bar{e}_{33}(\mu_{33}^{(m)}C_{13}^{(m)} + q_{33}q_{31}) + \bar{\alpha}_{33}(q_{33}C_{13}^{(m)} - q_{31}C_{33}^{(m)})]}{\mu_{33}^{(m)}\widetilde{C}_{33}^{(m)}},$$

$$\bar{e}_{32} = \frac{v_{\rm p}[\bar{e}_{33}(\varepsilon_{33}^{(p)}C_{23}^{(p)} + e_{33}e_{32}) + \bar{\varepsilon}_{33}(e_{32}C_{33}^{(p)} - e_{33}C_{23}^{(p)})]}{\varepsilon_{33}^{(p)}\widetilde{C}_{33}^{(p)}} + \frac{v_{\rm m}[\bar{e}_{33}(\mu_{33}^{(m)}C_{23}^{(m)} + q_{33}q_{32}) + \bar{\alpha}_{33}(q_{32}C_{33}^{(m)} - q_{33}C_{23}^{(m)})]}{\mu_{33}^{(m)}\widetilde{C}_{33}^{(m)}},$$

$$\bar{q}_{31} = \frac{v_{\rm p}[\bar{q}_{33}(\varepsilon_{33}^{(p)}C_{13}^{(p)} + e_{33}e_{31}) + \bar{\alpha}_{33}(e_{31}C_{33}^{(p)} - e_{33}C_{13}^{(p)})]}{\varepsilon_{33}^{(p)}\widetilde{C}_{33}^{(p)}} + \frac{v_{\rm m}[\bar{q}_{33}(\mu_{33}^{(m)}C_{13}^{(m)} + q_{33}q_{31}) + \bar{\mu}_{33}(q_{31}C_{33}^{(m)} - q_{33}C_{13}^{(m)})]}{\mu_{33}^{(m)}\widetilde{C}_{33}^{(m)}},$$

$$\bar{q}_{32} = \frac{v_{\rm p}[\bar{q}_{33}(\varepsilon_{33}^{(p)}C_{23}^{(p)} + e_{33}e_{32}) + \bar{\alpha}_{33}(e_{32}C_{33}^{(p)} - e_{33}C_{23}^{(p)})]}{\varepsilon_{33}^{(p)}\widetilde{C}_{33}^{(p)}} + \frac{v_{\rm m}[\bar{q}_{33}(\mu_{33}^{(m)}C_{23}^{(m)} + q_{33}q_{32}) + \bar{\mu}_{33}(q_{33}C_{23}^{(m)} - q_{32}C_{33}^{(m)})]}{\mu_{33}^{(m)}\widetilde{C}_{33}^{(m)}}. \tag{25c}$$

The nonzero components of $\overline{\mathbf{C}}_{tt}$:

$$\overline{C}_{11} = v_{\rm p}C_{11}^{(p)} + v_{\rm m}C_{11}^{(m)} + \frac{v_{\rm p}[\varepsilon_{33}^{(p)}C_{13}^{(p)}(\overline{C}_{13} - C_{13}^{(p)}) + e_{33}C_{13}^{(p)}(\bar{e}_{31} - e_{31}) + e_{33}e_{31}(\overline{C}_{13} - C_{13}^{(p)}) - e_{31}(\bar{e}_{31} - e_{31})C_{33}^{(p)}]}{\varepsilon_{33}^{(p)}\widetilde{C}_{33}^{(p)}} + \frac{v_{\rm m}[\mu_{33}^{(m)}C_{13}^{(m)}(\overline{C}_{13} - C_{13}^{(m)}) + q_{33}C_{13}^{(m)}(\bar{q}_{31} - q_{31}) + q_{33}q_{31}(\overline{C}_{13} - C_{13}^{(m)}) - q_{31}C_{33}^{(m)}(\bar{q}_{31} - q_{31})]}{\mu_{33}^{(m)}\widetilde{C}_{33}^{(m)}},$$

$$\overline{C}_{12} = v_{\rm p}C_{12}^{(p)} + v_{\rm m}C_{12}^{(m)} + \frac{v_{\rm p}[\varepsilon_{33}^{(p)}C_{13}^{(p)}(\overline{C}_{23} - C_{23}^{(p)}) + e_{33}C_{13}^{(p)}(\bar{e}_{32} - e_{32}) + e_{33}e_{31}(\overline{C}_{23} - C_{23}^{(p)}) - e_{31}(\bar{e}_{32} - e_{32})C_{33}^{(p)}]}{\varepsilon_{33}^{(p)}\widetilde{C}_{33}^{(p)}} + \frac{v_{\rm m}[\mu_{33}^{(m)}C_{13}^{(m)}(\overline{C}_{23} - C_{23}^{(m)}) + q_{33}C_{13}^{(m)}(\bar{q}_{32} - q_{32}) + q_{33}q_{31}(\overline{C}_{23} - C_{23}^{(m)}) - q_{31}C_{33}^{(m)}(\bar{q}_{32} - q_{32})]}{\mu_{33}^{(m)}\widetilde{C}_{33}^{(m)}},$$

$$\overline{C}_{22} = v_{\rm p}C_{22}^{(p)} + v_{\rm m}C_{22}^{(m)} + \frac{v_{\rm p}[\varepsilon_{33}^{(p)}C_{23}^{(p)}(\overline{C}_{23} - C_{23}^{(p)}) + e_{33}C_{23}^{(p)}(\bar{e}_{32} - e_{32}) + e_{33}e_{32}(\overline{C}_{23} - C_{23}^{(p)}) - e_{32}(\bar{e}_{32} - e_{32})C_{33}^{(p)}]}{\varepsilon_{33}^{(p)}\widetilde{C}_{33}^{(p)}} + \frac{v_{\rm m}[\mu_{33}^{(m)}C_{23}^{(m)}(\overline{C}_{23} - C_{23}^{(m)}) + q_{33}C_{23}^{(m)}(\bar{q}_{32} - q_{32}) + q_{33}q_{32}(\overline{C}_{23} - C_{23}^{(m)}) - q_{32}C_{33}^{(m)}(\bar{q}_{32} - q_{32})]}{\mu_{33}^{(m)}\widetilde{C}_{33}^{(m)}}, \overline{C}_{66} = v_{\rm p}C_{66}^{(p)} + v_{\rm m}C_{66}^{(m)}, \tag{26a}$$

$$\bar{\varepsilon}_{11} = v_{\rm p}\varepsilon_{11}^{(p)} + v_{\rm m}\varepsilon_{11}^{(m)} + \frac{v_{\rm p}v_{\rm m}e_{15}^2}{v_{\rm p}C_{55}^{(m)} + v_{\rm m}C_{55}^{(p)}}, \bar{\varepsilon}_{22} = v_{\rm p}\varepsilon_{22}^{(p)} + v_{\rm m}\varepsilon_{22}^{(m)} + \frac{v_{\rm p}v_{\rm m}e_{24}^2}{v_{\rm p}C_{44}^{(m)} + v_{\rm m}C_{44}^{(p)}},$$
$$\bar{\mu}_{11} = v_{\rm p}\mu_{11}^{(p)} + v_{\rm m}\mu_{11}^{(m)} + \frac{v_{\rm p}v_{\rm m}q_{15}^2}{v_{\rm p}C_{55}^{(m)} + v_{\rm m}C_{55}^{(p)}}, \bar{\mu}_{22} = v_{\rm p}\mu_{22}^{(p)} + v_{\rm m}\mu_{22}^{(m)} + \frac{v_{\rm p}v_{\rm m}q_{24}^2}{v_{\rm p}C_{44}^{(m)} + v_{\rm m}C_{44}^{(p)}},$$
$$\bar{\alpha}_{11} = -\frac{v_{\rm p}v_{\rm m}e_{15}q_{15}}{v_{\rm p}C_{55}^{(m)} + v_{\rm m}C_{55}^{(p)}}, \bar{\alpha}_{22} = -\frac{v_{\rm p}v_{\rm m}e_{24}q_{24}}{v_{\rm p}C_{44}^{(m)} + v_{\rm m}C_{44}^{(p)}}. \tag{26b}$$

The nonzero components of $\bar{\boldsymbol{\beta}}_n$:

$$\bar{\beta}_{33} = \frac{v_{\rm p}[(\varepsilon_{33}^{(p)}\overline{C}_{33} + e_{33}\bar{e}_{33})\beta_{33}^{(p)} + (\bar{e}_{33}C_{33}^{(p)} - e_{33}\overline{C}_{33})p_3^{(p)}]}{\varepsilon_{33}^{(p)}\widetilde{C}_{33}^{(p)}} + \frac{v_{\rm m}[(\mu_{33}^{(m)}\overline{C}_{33} + q_{33}\bar{q}_{33})\beta_{33}^{(m)} + (\bar{q}_{33}C_{33}^{(m)} - q_{33}\overline{C}_{33})m_3^{(m)}]}{\mu_{33}^{(m)}\widetilde{C}_{33}^{(m)}} + \frac{v_{\rm p}\bar{q}_{33}m_3^{(p)}}{\mu_{33}^{(p)}} + \frac{v_{\rm m}\bar{e}_{33}p_3^{(m)}}{\varepsilon_{33}^{(m)}},$$

$$\bar{p}_3 = \frac{v_{\rm p}[(\bar{\varepsilon}_{33}C_{33}^{(p)} + e_{33}\bar{e}_{33})p_3^{(p)} + (\bar{\varepsilon}_{33}e_{33} - \varepsilon_{33}^{(p)}\bar{e}_{33})\beta_{33}^{(p)}]}{\varepsilon_{33}^{(p)}\widetilde{C}_{33}^{(p)}} + \frac{v_{\rm m}[(\bar{\alpha}_{33}C_{33}^{(m)} + q_{33}\bar{e}_{33})m_3^{(m)} + (\bar{\alpha}_{33}q_{33} - \mu_{33}^{(m)}\bar{e}_{33})\beta_{33}^{(m)}]}{\mu_{33}^{(m)}\widetilde{C}_{33}^{(m)}} + \frac{v_{\rm p}\bar{\alpha}_{33}m_3^{(p)}}{\mu_{33}^{(p)}} + \frac{v_{\rm m}\bar{\varepsilon}_{33}p_3^{(m)}}{\varepsilon_{33}^{(m)}},$$

$$\bar{m}_3 = \frac{v_{\rm p}[(\bar{\alpha}_{33}C_{33}^{(p)} + e_{33}\bar{q}_{33})p_3^{(p)} + (\bar{\alpha}_{33}e_{33} - \varepsilon_{33}^{(p)}\bar{q}_{33})\beta_{33}^{(p)}]}{\varepsilon_{33}^{(p)}\widetilde{C}_{33}^{(p)}} + \frac{v_{\rm m}[(\bar{\mu}_{33}C_{33}^{(m)} + q_{33}\bar{q}_{33})m_3^{(m)} + (\bar{\mu}_{33}q_{33} - \mu_{33}^{(m)}\bar{q}_{33})\beta_{33}^{(m)}]}{\mu_{33}^{(m)}\widetilde{C}_{33}^{(m)}} + \frac{v_{\rm p}\bar{\mu}_{33}m_3^{(p)}}{\mu_{33}^{(p)}} + \frac{v_{\rm m}\bar{\alpha}_{33}p_3^{(m)}}{\varepsilon_{33}^{(m)}}. \quad (27)$$

The nonzero components of $\bar{\boldsymbol{\beta}}_t$:

$$\bar{\beta}_{11} = v_{\rm p}\beta_{11}^{(p)} + v_{\rm m}\beta_{11}^{(m)} + \frac{v_{\rm p}[(\varepsilon_{33}^{(p)}C_{13}^{(p)} + e_{33}e_{31})(\bar{\beta}_{33} - \beta_{33}^{(p)}) + (e_{31}C_{33}^{(p)} - e_{33}C_{13}^{(p)})(\bar{p}_3 - p_3^{(p)})]}{\varepsilon_{33}^{(p)}\widetilde{C}_{33}^{(p)}} + \frac{v_{\rm m}[(\mu_{33}^{(m)}C_{13}^{(m)} + q_{33}q_{31})(\bar{\beta}_{33} - \beta_{33}^{(m)}) + (q_{31}C_{33}^{(m)} - q_{33}C_{13}^{(m)})(\bar{m}_3 - m_3^{(m)})]}{\mu_{33}^{(m)}\widetilde{C}_{33}^{(m)}},$$

$$\bar{\beta}_{22} = v_{\rm p}\beta_{22}^{(p)} + v_{\rm m}\beta_{22}^{(m)} + \frac{v_{\rm p}[(\varepsilon_{33}^{(p)}C_{23}^{(p)} + e_{33}e_{32})(\bar{\beta}_{33} - \beta_{33}^{(p)}) + (e_{32}C_{33}^{(p)} - e_{33}C_{23}^{(p)})(\bar{p}_3 - p_3^{(p)})]}{\varepsilon_{33}^{(p)}\widetilde{C}_{33}^{(p)}} + \frac{v_{\rm m}[(\mu_{33}^{(m)}C_{23}^{(m)} + q_{33}q_{32})(\bar{\beta}_{33} - \beta_{33}^{(m)}) + (q_{32}C_{33}^{(m)} - q_{33}C_{23}^{(m)})(\bar{m}_3 - m_3^{(m)})]}{\mu_{33}^{(m)}\widetilde{C}_{33}^{(m)}}. \quad (28)$$

It is of interest to point out that our numerical results for the multiferroic $BaTiO_3$–$CoFe_2O_4$ layered composite (the material properties of $BaTiO_3$ and $CoFe_2O_4$ are taken from Ref. [27]) based on the above formulas are in agreement with Figs. 3, 4, and 5 in Ref. [14], while the values of ME coefficient $\bar{\alpha}_{11}$ are twice of those in Fig. 2 of Ref. [14].

3.2. Effective properties of a multiferroic composite composed of an orthotropic piezoelectric phase, an orthotropic magnetostrictive phase, and an orthotropic elastic substrate

We next consider the orthotropic piezoelectric–magnetostrictive bilayer film on an orthotropic elastic substrate, as shown in Fig. 3. Here, we are particularly interested in the influence of the elastic substrate on the ME effect. Using Eqs. (13)–(15) and (19)–(21), we can arrive at the following concise expressions for the three effective ME coefficients $\bar{\alpha}_{11}, \bar{\alpha}_{22}$ and $\bar{\alpha}_{33}$

$$\bar{\alpha}_{11} = -\frac{v_{\rm p}v_{\rm m}e_{15}q_{15}}{v_{\rm p}C_{55}^{(m)} + v_{\rm m}C_{55}^{(p)} + v_{\rm s}\frac{C_{55}^{(p)}C_{55}^{(m)}}{C_{55}^{(s)}}}, \quad (29)$$

$$\bar{\alpha}_{22} = -\frac{v_{\rm p}v_{\rm m}e_{24}q_{24}}{v_{\rm p}C_{44}^{(m)} + v_{\rm m}C_{44}^{(p)} + v_{\rm s}\frac{C_{44}^{(p)}C_{44}^{(m)}}{C_{44}^{(s)}}}, \quad (30)$$

$$\bar{\alpha}_{33} = -\frac{v_{\rm p}v_{\rm m}e_{33}q_{33}}{\varepsilon_{33}^{(p)}\mu_{33}^{(m)}\widetilde{C}_{33}^{(p)}\widetilde{C}_{33}^{(m)}\Delta_1}, \quad (31)$$

where

$$\Delta_1 = \left(\frac{v_{\rm p}}{\widetilde{C}_{33}^{(p)}} + \frac{v_{\rm m}}{\widetilde{C}_{33}^{(m)}} + \frac{v_{\rm s}}{C_{33}^{(s)}}\right)\left(\frac{v_{\rm p}C_{33}^{(p)}}{\varepsilon_{33}^{(p)}\widetilde{C}_{33}^{(p)}} + \frac{v_{\rm m}}{\varepsilon_{33}^{(m)}} + \frac{v_{\rm s}}{\varepsilon_{33}^{(s)}}\right) \times \left(\frac{v_{\rm m}C_{33}^{(m)}}{\mu_{33}^{(m)}\widetilde{C}_{33}^{(m)}} + \frac{v_{\rm p}}{\mu_{33}^{(p)}} + \frac{v_{\rm s}}{\mu_{33}^{(s)}}\right) + \frac{v_{\rm p}^2e_{33}^2}{\varepsilon_{33}^{(p)2}\widetilde{C}_{33}^{(p)2}}\left(\frac{v_{\rm m}C_{33}^{(m)}}{\mu_{33}^{(m)}\widetilde{C}_{33}^{(m)}} + \frac{v_{\rm p}}{\mu_{33}^{(p)}} + \frac{v_{\rm s}}{\mu_{33}^{(s)}}\right) + \frac{v_{\rm m}^2q_{33}^2}{\mu_{33}^{(m)2}\widetilde{C}_{33}^{(m)2}}\left(\frac{v_{\rm p}C_{33}^{(p)}}{\varepsilon_{33}^{(p)}\widetilde{C}_{33}^{(p)}} + \frac{v_{\rm m}}{\varepsilon_{33}^{(m)}} + \frac{v_{\rm s}}{\varepsilon_{33}^{(s)}}\right). \quad (32)$$

It is observed from Eqs. (29)–(32) that the existence of the elastic substrate will always cause a drop in the ME effect, which is in qualitative agreement with recent observations and calculations [7,15,25]. In order to demonstrate clearly the influence of the substrate on the ME effect, we show in Figs. 4 and 5 the dependence of the ME coefficients $\bar{\alpha}_{11}$ and $\bar{\alpha}_{33}$ on the $BaTiO_3$ volume fraction $v = v_{\rm p}/(v_{\rm p} + v_{\rm m})$ for a series of the substrate volume fraction $\bar{v}_{\rm s} = v_{\rm s}/(v_{\rm p} + v_{\rm m})$. During the calculation the magnetostrictive phase is $CoFe_2O_4$, and the pertinent material properties of the elastic substrate are: $C_{33}^{(s)} = 195 \times 10^9$ N/m^2, $C_{44}^{(s)} = C_{55}^{(s)} = 65 \times 10^9$ N/m^2, $\varepsilon_{33}^{(s)} = 9.6\varepsilon_0$, $\mu_{33}^{(s)} = 10\mu_0$. A dramatic, factor-of-2 decrease in $\bar{\alpha}_{11}$ (see Fig. 4) and a more dramatic, factor-of-3 decrease in $\bar{\alpha}_{33}$ (see Fig. 5) are observed when the $BaTiO_3$–$CoFe_2O_4$ film is deposited on the elastic substrate with a volume fraction of only 50% of the film. Further increase in the substrate volume $v_{\rm s}$

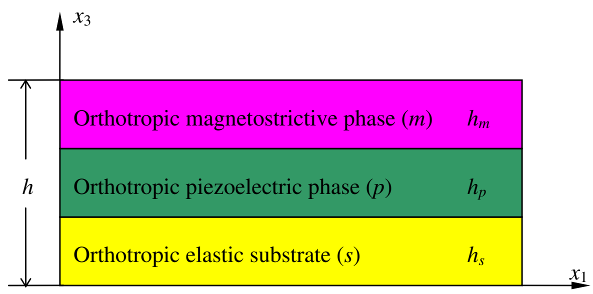

Fig. 3. A multiferroic composite composed of an orthotropic piezoelectric phase, an orthotropic magnetostrictive phase and an orthotropic elastic substrate.

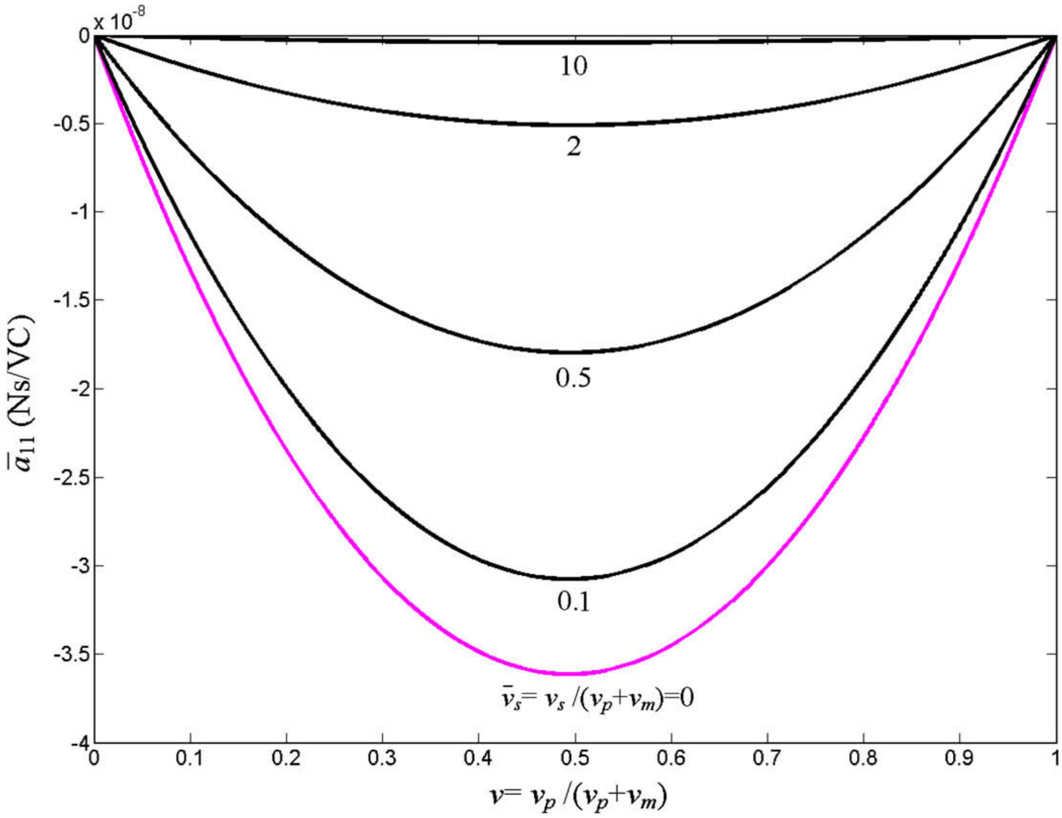

Fig. 4. Variation of the ME coefficient $\bar{\alpha}_{11}$ vs. the $BaTiO_3$ volume fraction for a series of the substrate volume fraction $\bar{v}_s$. The multiferroic composite is composed of a $CoFe_2O_4$–$BaTiO_3$ bilayer on an elastic substrate.

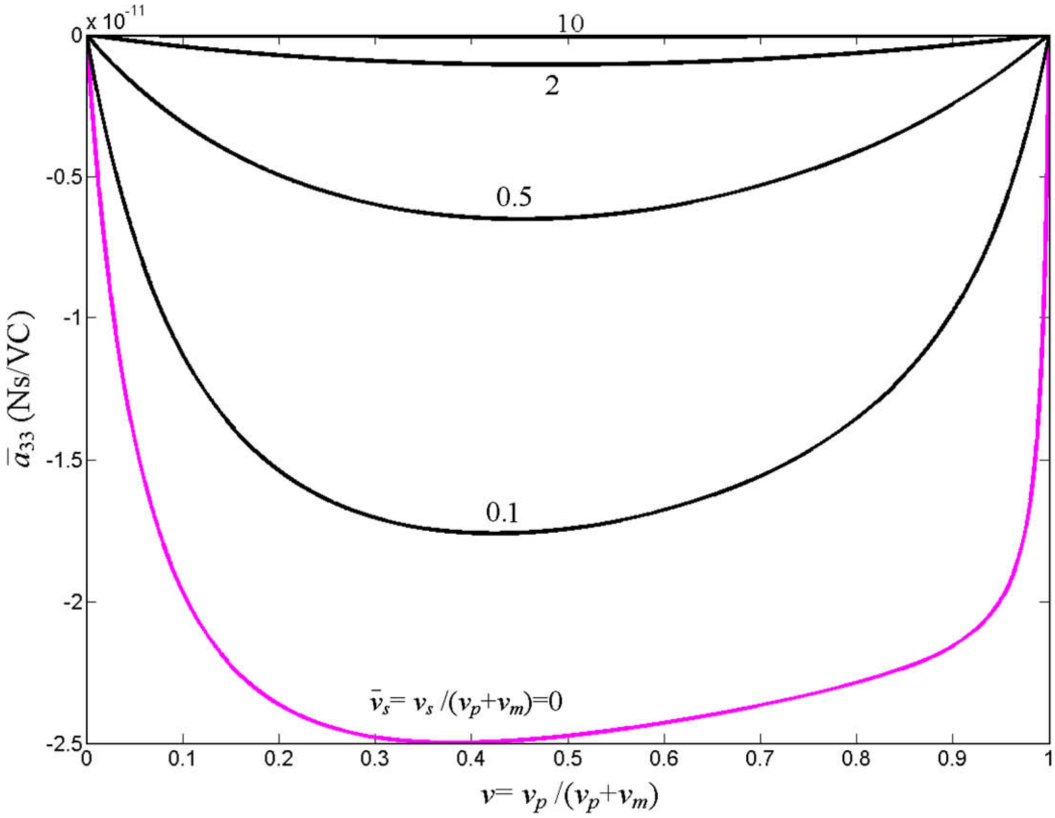

Fig. 5. Variation of the ME coefficient $\bar{\alpha}_{33}$ vs. the $BaTiO_3$ volume fraction for a series of the substrate volume fraction $\bar{v}_s$. The multiferroic composite is composed of a $CoFe_2O_4$–$BaTiO_3$ bilayer on an elastic substrate.

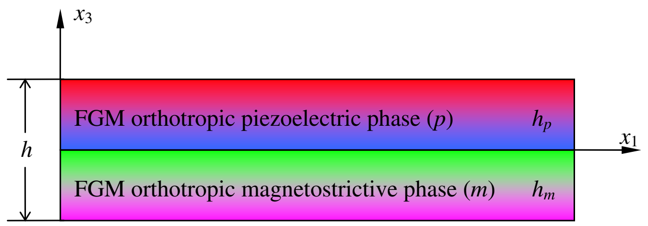

Fig. 6. A multiferroic composite composed of a functionally graded orthotropic piezoelectric phase and a functionally graded orthotropic magnetostrictive phase.

leads to continuous decreases in $\bar{\alpha}_{11}$ and $\bar{\alpha}_{33}$, and the ME coupling eventually vanishes when the volume of the elastic substrate of volume is roughly 10 times of the film volume.

3.3. Effective properties of a multiferroic composite composed of an orthotropic FGM piezoelectric phase and an orthotropic FGM magnetostrictive phase

Finally, we consider an orthotropic FGM piezoelectric phase located in the domain of $0 \leqslant x_3 \leqslant h_p$ which is perfectly bonded to an orthotropic FGM magnetostrictive phase located in the domain of $-h_m \leqslant x_3 \leqslant 0$, as shown in Fig. 6. Furthermore we assume that the material properties of the FGM piezoelectric and magnetostrictive phases vary along the thickness direction as

$$\begin{bmatrix} C_{ij}^{(p)}(x_3) & \varepsilon_{ij}^{(p)}(x_3) & e_{ij}(x_3) \end{bmatrix} = f(x_3)\begin{bmatrix} C_{ij}^{0(p)} & \varepsilon_{ij}^{0(p)} & e_{ij}^{0} \end{bmatrix}, \quad 0 \leqslant x_3 \leqslant h_p \tag{33}$$

$$\begin{bmatrix} C_{ij}^{(m)}(x_3) & \mu_{ij}^{(m)}(x_3) & q_{ij}(x_3) \end{bmatrix} = g(x_3)\begin{bmatrix} C_{ij}^{0(m)} & \mu_{ij}^{0(m)} & q_{ij}^{0} \end{bmatrix}, \quad -h_m \leqslant x_3 \leqslant 0 \tag{34}$$

where $C_{ij}^{0(p)}, \varepsilon_{ij}^{0(p)}, e_{ij}^{0}$ and $C_{ij}^{0(m)}, \mu_{ij}^{0(m)}, q_{ij}^{0}$ are the corresponding material constants at $x_3 = 0$. Apparently $f(0) = g(0) = 1$.

When calculating the effective properties, we can divide the two FGM phases into many thin homogeneous layers. By using Eqs. 13, 14, 15 and 19, 20, 21 and taking the limit (i.e., letting the thickness of the thin layer approach zero), the two in-plane effective ME coefficients $\bar{\alpha}_{11}$ and $\bar{\alpha}_{22}$ can be finally derived as

$$\begin{aligned} \bar{\alpha}_{11} &= -\frac{v_p v_m e_{15}^0 q_{15}^0}{C_{55}^{0(m)} h^{-1} \int_0^{h_p} \frac{dx}{f(x)} + C_{55}^{0(p)} h^{-1} \int_0^{h_m} \frac{dx}{g(-x)}}, \\ \bar{\alpha}_{22} &= -\frac{v_p v_m e_{24}^0 q_{24}^0}{C_{44}^{(m)} h^{-1} \int_0^{h_p} \frac{dx}{f(x)} + C_{44}^{0(p)} h^{-1} \int_0^{h_m} \frac{dx}{g(-x)}}. \end{aligned} \tag{35}$$

It is clear that one needs only to carry out two line integrals in order to arrive at the two ME coefficients.

In the following we consider two specific examples:

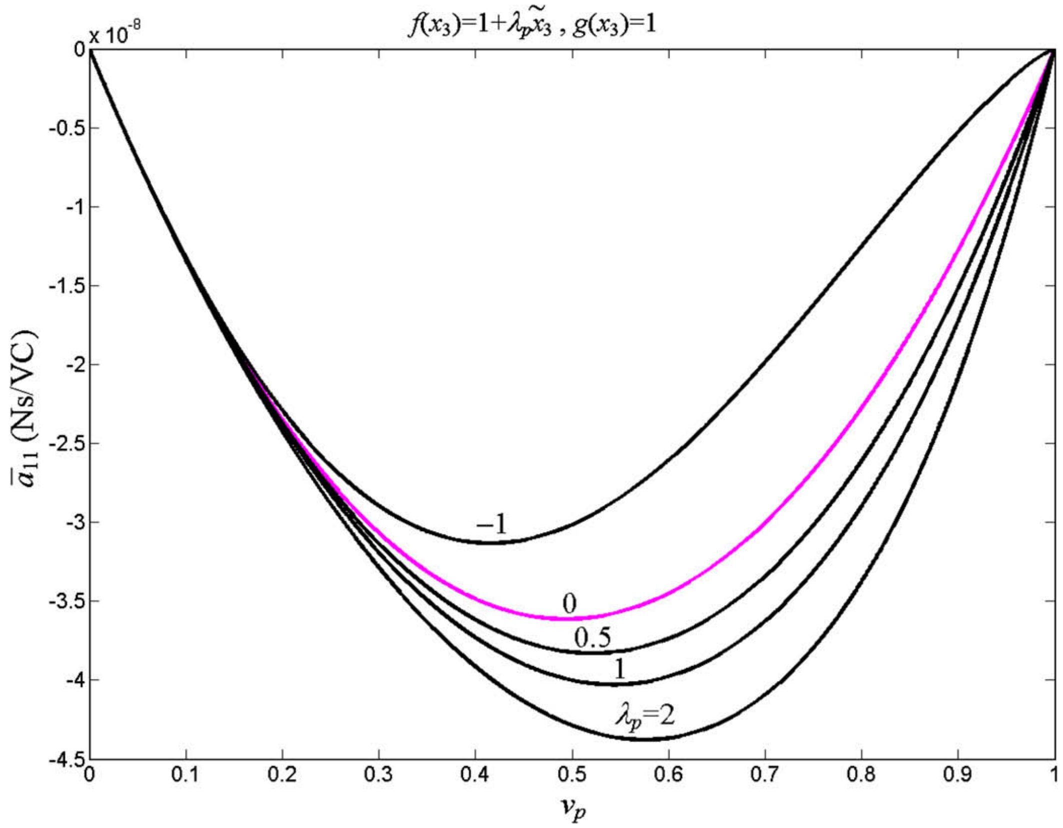

Fig. 7. Variation of the ME coefficient $\bar{\alpha}_{11}$ vs. the FGM $BaTiO_3$ volume fraction v_p for a series gradient parameter λ_p with a linearly varied FGM $BaTiO_3$ bonded to a homogeneous $CoFe_2O_4$ layer.

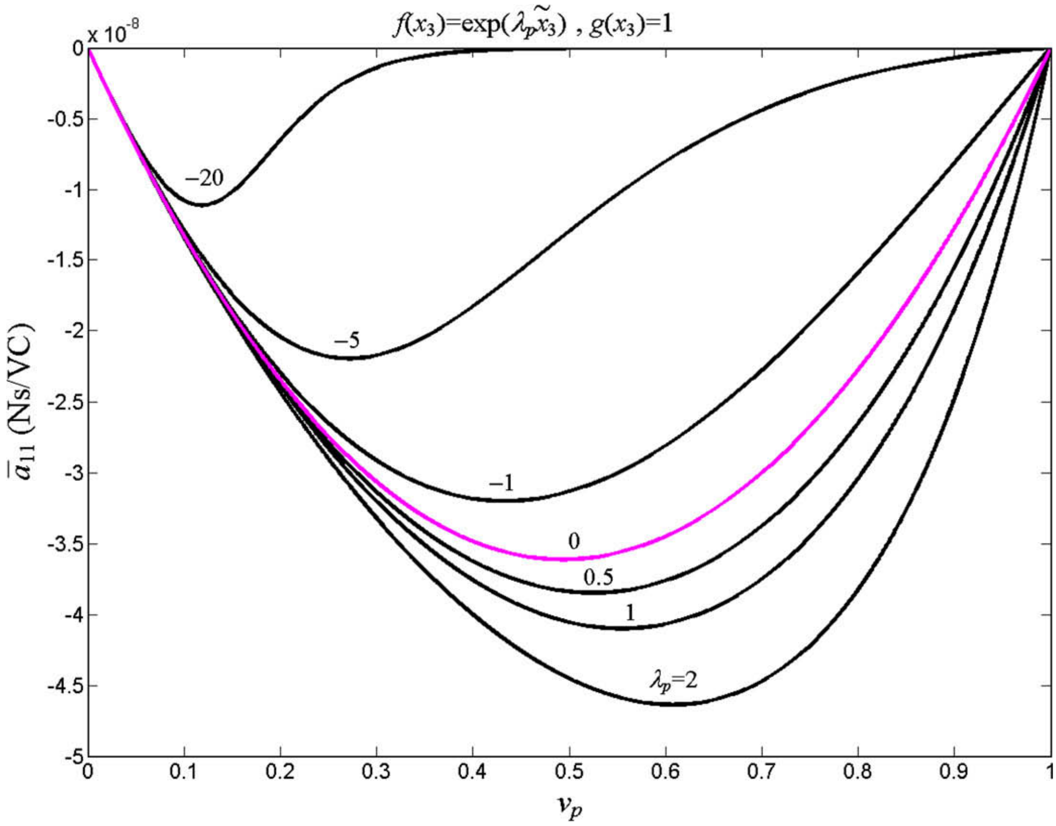

Fig. 8. Variation of the ME coefficient $\bar{\alpha}_{11}$ vs. the FGM $BaTiO_3$ volume fraction v_p for a series gradient parameter λ_p with an exponentially varied FGM $BaTiO_3$ bonded to a homogeneous $CoFe_2O_4$ layer.

(i) When both $f(x_3)$ and $g(x_3)$ are linear functions of x_3 described by

$$f(x_3) = 1 + \lambda_p \tilde{x}_3, \ \ g(x_3) = 1 - \lambda_m \tilde{x}_3, \quad (\lambda_p > -1/v_p, \ \ \lambda_m > -1/v_m), \tag{36}$$

where $\tilde{x}_3 = x_3/h$ and λ_p and λ_m are two dimensionless material constants. Then we have

$$\bar{\alpha}_{11} = -\frac{v_p v_m e_{15}^0 q_{15}^0}{\lambda_p^{-1}\ln(1+\lambda_p v_p)C_{55}^{0(m)} + \lambda_m^{-1}\ln(1+\lambda_m v_m)C_{55}^{0(p)}},$$
$$\bar{\alpha}_{22} = -\frac{v_p v_m e_{24}^0 q_{24}^0}{\lambda_p^{-1}\ln(1+\lambda_p v_p)C_{44}^{0(m)} + \lambda_m^{-1}\ln(1+\lambda_m v_m)C_{44}^{0(p)}}. \tag{37}$$

(ii) When both $f(x_3)$ and $g(x_3)$ are exponential functions of x_3 described by

$$f(x_3) = \exp(\lambda_p \tilde{x}_3), g(x_3) = \exp(-\lambda_m \tilde{x}_3), \tag{38}$$

Then we have

$$\bar{\alpha}_{11} = -\frac{v_p v_m e_{15}^0 q_{15}^0}{\lambda_p^{-1}[1-\exp(-\lambda_p v_p)]C_{55}^{0(m)} + \lambda_m^{-1}[1-\exp(-\lambda_m v_m)]C_{55}^{0(p)}},$$
$$\bar{\alpha}_{22} = -\frac{v_p v_m e_{24}^0 q_{24}^0}{\lambda_p^{-1}[1-\exp(-\lambda_p v_p)]C_{44}^{0(m)} + \lambda_m^{-1}[1-\exp(-\lambda_m v_m)]C_{44}^{0(p)}}. \tag{39}$$

We demonstrate in Figs. 7 and 8 the ME coefficient $\bar{\alpha}_{11}$ as a function of the $BaTiO_3$ volume fraction v_p and the gradient parameter λ_p for a linearly and exponentially varied FGM $BaTiO_3$ bonded to a homogeneous $CoFe_2O_4$ layer. During the calculation the material properties of the FGM $BaTiO_3$ at $x_3 = 0$ are again taken from Ref. [27]. It is observed that for both linearly and exponentially varied FGM $BaTiO_3$ the ME effect can be significantly enhanced when $\lambda_p > 0$ whilst it is reduced when $\lambda_p < 0$. It is also interesting to observe from Fig. 8 that the ME effect will reach a minimum when $\lambda_p \leqslant -20$ for an exponentially varied FGM $BaTiO_3$.

4. Conclusion

Theoretical modeling of the effective properties including the ME effect of multiferroic 2-2 connectivity composites has been rigorously developed based on the micromechanics scheme originally proposed by Qu and Cherkaoui for multilayered elastic composites.[19] As applications we first presented the explicit expressions for all the effective moduli of the multiferroic composite composed of an orthotropic piezoelectric phase and an orthotropic magnetostrictive phase. We then derived the three effective ME coefficients $\bar{\alpha}_{11}, \bar{\alpha}_{22}$ and $\bar{\alpha}_{33}$ for the multiferroic composite composed of an orthotropic piezoelectric phase, an orthotropic magnetostrictive phase and an orthotropic elastic substrate. Finally, we presented the two in-plane effective ME coefficients $\bar{\alpha}_{11}$ and $\bar{\alpha}_{22}$ for the multiferroic composite composed of an orthotropic FGM piezoelectric phase and an orthotropic FGM magnetostrictive phase. In addition, for the reduced simple cases, our results are in agreement with recent experimental observations and theoretical studies [7,14,15,25].

Acknowledgements

This work was supported in part by AFOSR FA9550-06-1-0317. The authors are also grateful for the constructive comments from the reviewers and the editor.

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