Time-decaying magnetoelectric effects in multiferroic fibrous composites with a viscous interface

E. Pan,[1,a)] X. Wang,[1,b)] and J. D. Albrecht[2]
[1]*Computer Modeling and Simulation Group, College of Engineering, University of Akron, Akron, Ohio 44325-3905, USA*
[2]*Air Force Research Laboratory, Wright-Patterson Air Force Base, Ohio 45433, USA*

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This paper addresses the time-dependent magnetoelectroelastic responses of multiferroic fibrous composites with a viscous interface. First, the problem of an isolated multiferroic fiber embedded in an infinite multiferroic matrix is rigorously solved. It is observed that the internal magnetoelectroelastic field such as stresses, electric displacements, and magnetic inductions inside an isolated multiferroic fiber is uniform but time dependent. The Mori–Tanaka mean-field method is then utilized to derive an extremely concise expression of the time-dependent effective moduli of the multiferroic fibrous composite. The numerical results demonstrate that the viscosity of the interface will cause a time-decaying magnetoelectric effect of the $BaTiO_3-CoFe_2O_4$ fibrous composite. As the time approaches infinity the magnetoelectric effect will approach zero due to the fact that a viscous interface will finally evolve into a free-sliding one which does not sustain shear stress. This interesting feature should be particularly important to the analysis and design of multiferroic composites where the interface is utilized to enhance the magnetoelectric effect.
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I. INTRODUCTION

Recently magnetoelectric (ME) multiferroic materials, which simultaneously possess both ferroelectric and ferromagnetic order in the same phase, have attracted significant attention from the scientific community due to their ME effect, dielectric polarization of a material under magnetic field, or induced magnetization under an electric field.[1–3] Particularly multiferroic composites made of ferromagnetic and ferroelectric phases can exhibit a strong extrinsic ME effect through the product property, which is absent in the constituents and which can be a few orders larger than the intrinsic ME effect observed in natural single-phase multiferroic materials.[4–6] It has been identified that the interface in multiferroic composites is critical in achieving the ME effect, and that any kind of imperfection at the interface would lead to a reduction in the ME effect.[5,7–9]

At elevated working temperatures exceeding about one-third of the homologous temperature, mass transport becomes important along high diffusivity path such as an interface or grain boundary.[10] Experimental data of Srinivasan *et al.*[5] have demonstrated that the interfacial diffusion of metal ions between manganites and PZT will degrade the ME effect in multilayer composites. Raj and Ashby[11] and Ashby[12] suggested that the microscopically mass diffusion-controlled mechanism on a length scale comparable to the size of the asperity of the interface can be macroscopically described by the linear law for a viscous interface: $\dot{\delta}=\tau/\eta$, where $\dot{\delta}$ is the sliding velocity (i.e., the differentiation of the relative sliding with respect to time t), τ is the interfacial shear stress, and η is the interfacial viscosity which can be determined experimentally and theoretically.[11–14]

Motivated by the importance of the interface in the ME effect of multiferroic composites, we propose, in this paper, a theoretical framework to study how the interfacial diffusion characterized by a viscous interface influences the ME effect in the multiferroic composite. For simplicity, the possible contribution of the current density due to the flow of ions at the interface is not considered, which renders only a scalar magnetic potential into the governing equations. This paper is organized as follows: In Sec. II, the problem of an isolated fiber in an infinite matrix is solved in detail; In Sec. III, we derive the time-dependent effective moduli of the multiferroic composite based on the Mori–Tanaka mean-field method; a numerical example of the ME effect is given in Sec. IV; and conclusions are drawn in Sec. V.

II. ISOLATED FIBER IN AN INFINITE MATRIX

We first consider an isolated multiferroic fiber with a circular cross section (phase 2) of radius R centered at the origin embedded in an infinite multiferroic matrix (phase 1) (Fig. 1). Both the fiber and matrix are 6 mm material symmetry about the fiber axis. At infinity, the matrix is subjected to the antiplane shear stresses σ_{zx}^{∞} and σ_{zy}^{∞}, and the in-plane electric displacements D_x^{∞} and D_y^{∞}, and magnetic fluxes B_x^{∞} and B_y^{∞}. Thus the two-phase composite system is in a state of antiplane deformation described by

$$u_x=u_y=0,\quad u_z=w(x,y,t),$$

a)Electronic mail: pan2@uakron.edu.
b)Present address: Department of Mechanical Engineering, Center for Composite Materials, University of Delaware, Newark, Delaware 19716, USA.

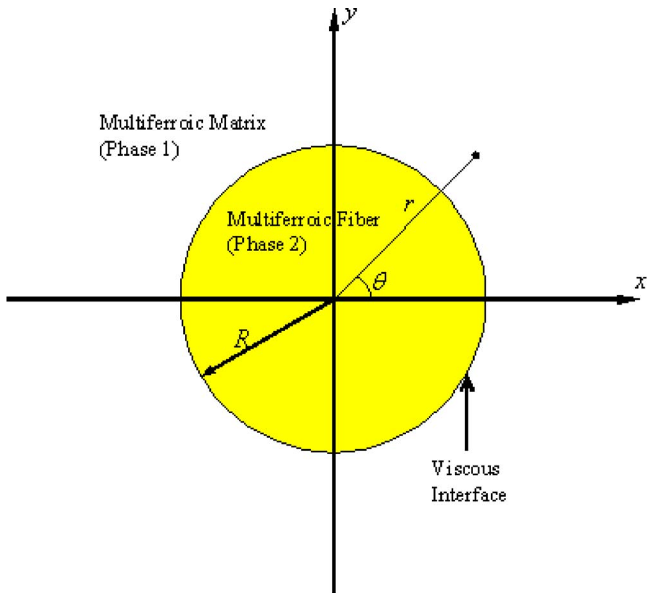

FIG. 1. (Color online) A multiferroic circular cylindrical fiber of radius R bonded to an infinite multiferroic matrix through a viscous interface. The polarization and magnetization direction is along the fiber axis.

$$\phi=\phi(x,y,t),\quad \varphi=\varphi(x,y,t). \tag{1}$$

where u_x, u_y, and u_z denote the elastic displacement components in the x, y, and z directions, respectively; ϕ and φ are the electric and magnetic potentials; and t is the time.

Thus the constitutive equations reduce to

$$\sigma_{zx}=c_{44}\frac{\partial w}{\partial x}+e_{15}\frac{\partial \phi}{\partial x}+q_{15}\frac{\partial \varphi}{\partial x},$$

$$\sigma_{zy}=c_{44}\frac{\partial w}{\partial y}+e_{15}\frac{\partial \phi}{\partial y}+q_{15}\frac{\partial \varphi}{\partial y},$$

$$D_x=e_{15}\frac{\partial w}{\partial x}-\varepsilon_{11}\frac{\partial \phi}{\partial x}-\alpha_{11}\frac{\partial \varphi}{\partial x},$$

$$D_y=e_{15}\frac{\partial w}{\partial y}-\varepsilon_{11}\frac{\partial \phi}{\partial y}-\alpha_{11}\frac{\partial \varphi}{\partial y},$$

$$B_x=q_{15}\frac{\partial w}{\partial x}-\alpha_{11}\frac{\partial \phi}{\partial x}-\mu_{11}\frac{\partial \varphi}{\partial x},$$

$$B_y=q_{15}\frac{\partial w}{\partial y}-\alpha_{11}\frac{\partial \phi}{\partial y}-\mu_{11}\frac{\partial \varphi}{\partial y}, \tag{2}$$

where σ_{kz}, D_k, B_k, $(k=x,y)$, c_{44}, e_{15}, q_{15}, ε_{11}, α_{11}, and μ_{11} are the stresses, electric displacements, magnetic fluxes (i.e., magnetic inductions), elastic modulus, piezoelectric coefficient, piezomagnetic coefficient, dielectric permittivity, ME coefficient, and the magnetic permeability, respectively.

Under this antiplane deformation, the governing equations are simplified to

$$c_{44}\nabla^2 w+e_{15}\nabla^2\phi+q_{15}\nabla^2\varphi=0,$$

$$e_{15}\nabla^2 w-\varepsilon_{11}\nabla^2\phi-\alpha_{11}\nabla^2\varphi=0,$$

$$q_{15}\nabla^2 w-\alpha_{11}\nabla^2\phi-\mu_{11}\nabla^2\varphi=0, \tag{3}$$

where $\nabla^2=\partial^2/\partial x^2+\partial^2/\partial y^2$ is the two-dimensional Laplace operator. In Eq. (3) we have ignored the inertia effect in the fiber and matrix.

The general solution to Eq. (3) can be given by

$$\mathbf{u}=[w\quad \phi\quad \varphi]^T=\mathbf{f}(z,t), \tag{4}$$

where $z=x+\mathrm{i}y=r\exp(\mathrm{i}\theta)$ is the complex variable. The appearance of the real time variable t in Eq. (4) comes from the influence of the viscous interface.

Furthermore, the strains, stresses, electric fields, electric displacements, magnetic fields, and magnetic fluxes can also be concisely expressed in terms of $\mathbf{f}(z,t)$ as follows:

$$\begin{bmatrix}\gamma_{zy}+\mathrm{i}\gamma_{zx}\\ -E_y-\mathrm{i}E_x\\ -H_y-\mathrm{i}H_x\end{bmatrix}=\mathbf{f}'(z,t),\qquad \begin{bmatrix}\sigma_{zy}+\mathrm{i}\sigma_{zx}\\ D_y+\mathrm{i}D_x\\ B_y+\mathrm{i}B_x\end{bmatrix}=\mathbf{L}\mathbf{f}'(z,t), \tag{5}$$

where the superscript comma stands for the differentiation with respect to the complex variable z, and $\mathbf{L}$ is a real and symmetric 3×3 matrix given by

$$\mathbf{L}=\begin{bmatrix}c_{44} & e_{15} & q_{15}\\ e_{15} & -\varepsilon_{11} & -\alpha_{11}\\ q_{15} & -\alpha_{11} & -\mu_{11}\end{bmatrix}. \tag{6}$$

The strains γ_{zx} and γ_{zy}, electric fields E_x and E_y, and the magnetic fields H_x and H_y in Eq. (5) are related to w, ϕ and φ through

$$\gamma_{zx}=w_{,x},\quad \gamma_{zy}=w_{,y},$$

$$E_x=-\phi_{,x},\quad E_y=-\phi_{,y},$$

$$H_x=-\varphi_{,x},\quad H_y=-\varphi_{,y}. \tag{7}$$

The boundary conditions on the viscous interface (or time-dependent sliding interface) between the fiber and matrix can be written as

$$\sigma_{zr}^{(1)}=\sigma_{zr}^{(2)},\quad D_r^{(1)}=D_r^{(2)},\quad B_r^{(1)}=B_r^{(2)},$$

$$\phi^{(1)}=\phi^{(2)},\quad \varphi^{(1)}=\varphi^{(2)},\quad r=R\ \text{and}\ t>0$$

$$\sigma_{zr}^{(2)}=\eta(\dot{w}^{(1)}-\dot{w}^{(2)}), \tag{8}$$

where the overdot denotes differentiation with respect to time t.

At the initial moment the interface is a perfect one on which

$$\sigma_{zr}^{(1)}=\sigma_{zr}^{(2)},\quad D_r^{(1)}=D_r^{(2)},\quad B_r^{(1)}=B_r^{(2)},$$

$$w^{(1)}=w^{(2)},\quad \phi^{(1)}=\phi^{(2)},\quad \varphi^{(1)}=\varphi^{(2)},\quad r=R\ \text{and}\ t=0 \tag{9}$$

In view of the above initial conditions, Eq. (8) can also be expressed more concisely as

$$\begin{bmatrix}\sigma_{zr}^{(1)}\\ D_r^{(1)}\\ B_r^{(1)}\end{bmatrix}=\begin{bmatrix}\sigma_{zr}^{(2)}\\ D_r^{(2)}\\ B_r^{(2)}\end{bmatrix},\qquad \begin{bmatrix}\dot{w}^{(1)}-\dot{w}^{(2)}\\ \dot{\phi}^{(1)}-\dot{\phi}^{(2)}\\ \dot{\varphi}^{(1)}-\dot{\varphi}^{(2)}\end{bmatrix}=R\boldsymbol{\Lambda}\begin{bmatrix}\sigma_{zr}^{(2)}\\ D_r^{(2)}\\ B_r^{(2)}\end{bmatrix},$$

$$r=R \text{ and } t>0 \tag{10}$$

where $\mathbf{\Lambda}$ is a 3×3 diagonal matrix defined by

$$\mathbf{\Lambda}=\frac{1}{\eta R}\mathrm{diag}[1 \quad 0 \quad 0]. \tag{11}$$

The interface conditions in Eq. (10) can also be expressed in terms of two analytic function vectors, $\mathbf{f}_1(z,t)$ defined in the matrix and $\mathbf{f}_2(z,t)$ defined in the fiber, as follows

$$\mathbf{L}_2\mathbf{f}_2^+(z,t)+\mathbf{L}_2\bar{\mathbf{f}}_2^-\left(\frac{R^2}{z},t\right)=\mathbf{L}_1\mathbf{f}_1^-(z,t)+\mathbf{L}_1\bar{\mathbf{f}}_1^+\left(\frac{R^2}{z},t\right), \tag{12a}$$

$$\dot{\mathbf{f}}_1^-(z,t)-\dot{\bar{\mathbf{f}}}_1^+\left(\frac{R^2}{z},t\right)-\dot{\mathbf{f}}_2^+(z,t)+\dot{\bar{\mathbf{f}}}_2^-\left(\frac{R^2}{z},t\right)$$
$$=\mathbf{\Lambda}\mathbf{L}_2\left[z\mathbf{f}_2'^+(z,t)-\frac{R^2}{z}\bar{\mathbf{f}}_2'^-\left(\frac{R^2}{z},t\right)\right],$$

$$|z|=R \text{ and } t>0. \tag{12b}$$

It follows from Eq. (12a) that

$$\mathbf{f}_1(z,t)=\mathbf{L}_1^{-1}\mathbf{L}_2\bar{\mathbf{f}}_2\left(\frac{R^2}{z},t\right)+\mathbf{k}z-\bar{\mathbf{k}}\frac{R^2}{z},$$

$$\bar{\mathbf{f}}_1\left(\frac{R^2}{z},t\right)=\mathbf{L}_1^{-1}\mathbf{L}_2\mathbf{f}_2(z,t)-\mathbf{k}z+\bar{\mathbf{k}}\frac{R^2}{z}, \tag{13}$$

where the vector $\mathbf{k}$ is related to the remote loading through the following:

$$\mathbf{k}=\mathbf{L}_1^{-1}\begin{bmatrix}\sigma_{zy}^\infty+i\sigma_{zx}^\infty\\ D_y^\infty+iD_x^\infty\\ B_y^\infty+iB_x^\infty\end{bmatrix}. \tag{14}$$

Substituting Eq. (13) into Eq. (12b) and eliminating $\mathbf{f}_1^-(z)$ and $\bar{\mathbf{f}}_1^+(R^2/z)$, we finally arrive at the following set of first-order partial differential equations:

$$z\mathbf{\Lambda}\mathbf{L}_2\mathbf{f}_2'(z,t)+\mathbf{H}\mathbf{L}_2\dot{\mathbf{f}}_2(z,t)=\mathbf{0}, \quad (|z|<R), \tag{15}$$

where $\mathbf{H}$ is a real and symmetric matrix given by

$$\mathbf{H}=\mathbf{H}^T=\begin{bmatrix}H_{11} & H_{12} & H_{13}\\ H_{12} & H_{22} & H_{23}\\ H_{13} & H_{23} & H_{33}\end{bmatrix}=\mathbf{L}_1^{-1}+\mathbf{L}_2^{-1}. \tag{16}$$

In order to solve Eq. (15), we first consider the following eigenvalue problem:

$$(\mathbf{\Lambda}-\lambda\mathbf{H})\mathbf{v}=\mathbf{0}, \tag{17}$$

The three eigenvalues λ_i, $(i=1-3)$ of this eigenvalue problem can be explicitly determined as

$$\lambda_1=\frac{H_{22}H_{33}-H_{23}^2}{\eta R|\mathbf{H}|}>0, \quad \lambda_2=\lambda_3=0. \tag{18}$$

The eigenvectors associated with these eigenvalues are

$$\mathbf{v}_1=\begin{bmatrix}H_{22}H_{33}-H_{23}^2\\ H_{13}H_{23}-H_{12}H_{33}\\ H_{12}H_{23}-H_{13}H_{22}\end{bmatrix}, \quad \mathbf{v}_2=\begin{bmatrix}0\\1\\0\end{bmatrix}, \quad \mathbf{v}_3=\begin{bmatrix}0\\ -H_{23}\\ H_{22}\end{bmatrix}. \tag{19}$$

It can be proved that the following orthogonal relationships with respect to the two real and symmetric matrices $\mathbf{H}$ and $\mathbf{\Lambda}$ hold

$$\mathbf{\Phi}^T\mathbf{H}\mathbf{\Phi}=\mathrm{diag}[\delta_1 \quad \delta_2 \quad \delta_3],$$

$$\mathbf{\Phi}^T\mathbf{\Lambda}\mathbf{\Phi}=\lambda_1\delta_1\,\mathrm{diag}[1 \quad 0 \quad 0], \tag{20}$$

where

$$\mathbf{\Phi}=[\mathbf{v}_1 \quad \mathbf{v}_2 \quad \mathbf{v}_3], \tag{21a}$$

and

$$\delta_1=\mathbf{v}_1^T\mathbf{H}\mathbf{v}_1=\lambda_1^{-1}\mathbf{v}_1^T\mathbf{\Lambda}\mathbf{v}_1=|\mathbf{H}|(H_{22}H_{33}-H_{23}^2),$$

$$\delta_2=\mathbf{v}_2^T\mathbf{H}\mathbf{v}_2=H_{22},$$

$$\delta_3=\mathbf{v}_3^T\mathbf{H}\mathbf{v}_3=H_{22}(H_{22}H_{33}-H_{23}^2). \tag{21b}$$

We now introduce a new vector function $\mathbf{\Omega}(z,t)=[\Omega_1(z,t) \;\; \Omega_2(z,t) \;\; \Omega_3(z,t)]^T$ defined by

$$\mathbf{L}_2\mathbf{f}_2(z,t)=\mathbf{\Phi}\mathbf{\Omega}(z,t), \tag{22}$$

In view of Eqs. (20) and (22), the original coupled set of differential Eqs. (15) can be decoupled as follows:

$$\dot{\Omega}_1(z,t)+\lambda_1 z\Omega_1'(z,t)=0,$$

$$\dot{\Omega}_2(z,t)=0, \quad (|z|<R),$$

$$\dot{\Omega}_3(z,t)=0, \tag{23}$$

whose solutions can be expediently given by

$$\Omega_1(z,t)=\Omega_1(\exp(-\lambda_1 t)z,0),$$

$$\Omega_2(z,t)=\Omega_2(z,0), \quad (|z|<R),$$

$$\Omega_3(z,t)=\Omega_3(z,0). \tag{24}$$

It is of interest to observe that the two component functions $\Omega_2(z,t)$ and $\Omega_3(z,t)$ are in fact *time independent*. Due to the fact that at the time $t=0$ the interface is a perfect one, then we arrive at the following initial state of $\mathbf{\Omega}(z,t)$:

$$\mathbf{\Omega}(z,0)=\mathbf{\Phi}^{-1}\mathbf{L}_2\mathbf{f}_2(z,0)=2\,\mathrm{diag}\left[\frac{1}{\delta_1} \quad \frac{1}{\delta_2} \quad \frac{1}{\delta_3}\right]\mathbf{\Phi}^T\mathbf{k}z. \tag{25}$$

During the above derivation we have utilized the first orthogonal relationship in Eq. (20) and the following expression:

$$\mathbf{f}_2(z,0)=2\mathbf{L}_2^{-1}\mathbf{H}^{-1}\mathbf{k}z. \tag{26}$$

It follows from Eqs. (24) and (25) that the explicit solution of $\mathbf{\Omega}(z,t)$ is

$$\mathbf{\Omega}(z,t) = 2\,\mathrm{diag}\left[\frac{\exp(-\lambda_1 t)}{\delta_1} \quad \frac{1}{\delta_2} \quad \frac{1}{\delta_3}\right]\mathbf{\Phi}^T\mathbf{k}z. \tag{27}$$

Consequently $\mathbf{f}_1(z,t)$ defined in the matrix and $\mathbf{f}_2(z,t)$ defined in the fiber are given by

$$\mathbf{f}_1(z,t) = \left[2\mathbf{L}_1^{-1}\mathbf{\Phi}\,\mathrm{diag}\left[\frac{\exp(-\lambda_1 t)}{\delta_1} \quad \frac{1}{\delta_2} \quad \frac{1}{\delta_3}\right]\mathbf{\Phi}^T - \mathbf{I}\right] \times\bar{\mathbf{k}}R^2 z^{-1} + \mathbf{k}z,$$

$$\mathbf{f}_2(z,t) = 2\mathbf{L}_2^{-1}\mathbf{\Phi}\,\mathrm{diag}\left[\frac{\exp(-\lambda_1 t)}{\delta_1} \quad \frac{1}{\delta_2} \quad \frac{1}{\delta_3}\right]\mathbf{\Phi}^T\mathbf{k}z, \tag{28}$$

which indicates that the internal magnetoelectroelastic field such as stresses, electric displacements and magnetic inductions inside the multiferroic fiber is uniform but *time dependent*.

III. TIME-DEPENDENT EFFECTIVE MODULI

We assume that the aligned circular multiferroic fibers of same radius are randomly distributed in the x-y plane; then the fiber-reinforced multiferroic composite is transversely isotropic with the x-y plane being the isotropic plane. The overall constitutive law for the fibrous multiferroic composite can be represented by

$$\begin{bmatrix}\langle\sigma_{zy}\rangle\\ \langle D_y\rangle\\ \langle B_y\rangle\end{bmatrix} = \mathbf{L}_c\begin{bmatrix}\langle\gamma_{zy}\rangle\\ \langle -E_y\rangle\\ \langle -H_y\rangle\end{bmatrix}, \quad \begin{bmatrix}\langle\sigma_{zx}\rangle\\ \langle D_x\rangle\\ \langle B_x\rangle\end{bmatrix} = \mathbf{L}_c\begin{bmatrix}\langle\gamma_{zx}\rangle\\ \langle -E_x\rangle\\ \langle -H_x\rangle\end{bmatrix}, \tag{29}$$

where $\langle\,^*\rangle$ stands for the average value, and $\mathbf{L}_c$ is the effective moduli of the fibrous composite. In the following we will apply the Mori–Tanaka mean-field method[9,14–17] to derive the expression for the effective moduli $\mathbf{L}_c$.

In order to describe the overall behavior of the multiferroic composite, we focus on a representative volume element (RVE). In addition we assume that the RVE is subjected to the loadings σ_{zy}^∞, D_y^∞, and B_y^∞. Then, the volume-averaged physical quantities within the RVE can be proved to be[9,14,17]

$$\begin{bmatrix}\langle\sigma_{zy}\rangle\\ \langle D_y\rangle\\ \langle B_y\rangle\end{bmatrix} = (1-c_2)\begin{bmatrix}\langle\sigma_{zy}\rangle_1\\ \langle D_y\rangle_1\\ \langle B_y\rangle_1\end{bmatrix} + c_2\begin{bmatrix}\langle\sigma_{zy}\rangle_2\\ \langle D_y\rangle_2\\ \langle B_y\rangle_2\end{bmatrix}, \tag{30}$$

$$\begin{bmatrix}\langle\gamma_{zy}\rangle\\ \langle -E_y\rangle\\ \langle -H_y\rangle\end{bmatrix} = (1-c_2)\begin{bmatrix}\langle\gamma_{zy}\rangle_1\\ \langle -E_y\rangle_1\\ \langle -H_y\rangle_1\end{bmatrix} + c_2\begin{bmatrix}\langle\gamma_{zy}\rangle_2\\ \langle -E_y\rangle_2\\ \langle -H_y\rangle_2\end{bmatrix} + \frac{c_2}{\pi R^2}\begin{bmatrix}\oint_l (w^{(1)}-w^{(2)})\hat{n}_2\mathrm{d}l\\ 0\\ 0\end{bmatrix}, \tag{31}$$

where $<>_1$ and $<>_2$ refer to the averages over volumes of the matrix and fiber, respectively; c_2 is the volume fraction of the multiferroic fibers; the line integral is taken along the perimeter l of a typical fiber and $\hat{n}_2$ the y component of the unit normal vector on the interface in the outward direction with respect to the fiber. In addition, $\langle\sigma_{zy}\rangle=\sigma_{zy}^\infty$, $\langle D_y\rangle=D_y^\infty$, $\langle B_y\rangle=B_y^\infty$. Here the Mori–Tanaka mean-field approximation is adopted to evaluate $\langle\sigma_{zy}\rangle_2$, $\langle D_y\rangle_2$, and $\langle B_y\rangle_2$. Under this approximation $\langle\sigma_{zy}\rangle_2$, $\langle D_y\rangle_2$, and $\langle B_y\rangle_2$ are equal to the corresponding values of σ_{zy}, D_y, and B_y in an isolated fiber embedded in an infinitely extended matrix that is subjected to $\langle\sigma_{zy}\rangle_1$, $\langle D_y\rangle_1$, and $\langle B_y\rangle_1$ at infinity.

By employing the results of Eq. (28), it is found that

$$\begin{bmatrix}\langle\sigma_{zy}\rangle_2\\ \langle D_y\rangle_2\\ \langle B_y\rangle_2\end{bmatrix} = 2\mathbf{\Phi}\,\mathrm{diag}\left[\frac{\exp(-\lambda_1 t)}{\delta_1} \quad \frac{1}{\delta_2} \quad \frac{1}{\delta_3}\right] \times\mathbf{\Phi}^T\mathbf{L}_1^{-1}\begin{bmatrix}\langle\sigma_{zy}\rangle_1\\ \langle D_y\rangle_1\\ \langle B_y\rangle_1\end{bmatrix}. \tag{32}$$

Substituting these expressions into Eq. (30), we can find

$$\begin{bmatrix}\langle\sigma_{zy}\rangle_1\\ \langle D_y\rangle_1\\ \langle B_y\rangle_1\end{bmatrix} = \left\{2c_2\mathbf{\Phi}\,\mathrm{diag}\left[\frac{\exp(-\lambda_1 t)}{\delta_1} \quad \frac{1}{\delta_2} \quad \frac{1}{\delta_3}\right]\mathbf{\Phi}^T\mathbf{L}_1^{-1} + (1-c_2)\mathbf{I}\right\}^{-1}\begin{bmatrix}\sigma_{zy}^\infty\\ D_y^\infty\\ B_y^\infty\end{bmatrix}, \tag{33}$$

In addition, we have

$$\begin{bmatrix}\langle\gamma_{zy}\rangle_1\\ \langle -E_y\rangle_1\\ \langle -H_y\rangle_1\end{bmatrix} = \mathbf{L}_1^{-1}\begin{bmatrix}\langle\sigma_{zy}\rangle_1\\ \langle D_y\rangle_1\\ \langle B_y\rangle_1\end{bmatrix}, \quad \begin{bmatrix}\langle\gamma_{zy}\rangle_2\\ \langle -E_y\rangle_2\\ \langle -H_y\rangle_2\end{bmatrix} = \mathbf{L}_2^{-1}\begin{bmatrix}\langle\sigma_{zy}\rangle_2\\ \langle D_y\rangle_2\\ \langle B_y\rangle_2\end{bmatrix}, \tag{34}$$

and the surface integral in Eq. (31) can be carried out as follows:

$$\frac{1}{\pi R^2}\begin{bmatrix}\oint_l (w^{(1)}-w^{(2)})\hat{n}_2\mathrm{d}l\\ 0\\ 0\end{bmatrix} = \frac{2[1-\exp(-\lambda_1 t)]}{\lambda_1\delta_1}\mathbf{\Lambda\Phi}\,\mathrm{diag}\begin{bmatrix}1 & 0 & 0\end{bmatrix} \times\mathbf{\Phi}^T\mathbf{L}_1^{-1}\begin{bmatrix}\langle\sigma_{zy}\rangle_1\\ \langle D_y\rangle_1\\ \langle B_y\rangle_1\end{bmatrix}. \tag{35}$$

By virtue of Eqs. (33)–(35), Eq. (31) can be further written into

$$\begin{bmatrix}\langle\gamma_{zy}\rangle\\ \langle -E_y\rangle\\ \langle -H_y\rangle\end{bmatrix} = \mathbf{L}_1^{-1}\left\{\begin{aligned}&(1-c_2)\mathbf{L}_1 + 2c_2\mathbf{L}_1\mathbf{L}_2^{-1}\mathbf{\Phi}\ \mathrm{diag}\begin{bmatrix}\frac{\exp(-\lambda_1 t)}{\delta_1} & \frac{1}{\delta_2} & \frac{1}{\delta_3}\end{bmatrix}\mathbf{\Phi}^T\\ &+\frac{2c_2[1-\exp(-\lambda_1 t)]}{\lambda_1\delta_1}\mathbf{L}_1\mathbf{\Lambda\Phi}\ \mathrm{diag}\begin{bmatrix}1 & 0 & 0\end{bmatrix}\mathbf{\Phi}^T\end{aligned}\right\} \times\left\{(1-c_2)\mathbf{L}_1 + 2c_2\mathbf{\Phi}\ \mathrm{diag}\begin{bmatrix}\frac{\exp(-\lambda_1 t)}{\delta_1} & \frac{1}{\delta_2} & \frac{1}{\delta_3}\end{bmatrix}\mathbf{\Phi}^T\right\}^{-1}\begin{bmatrix}\sigma_{zy}^\infty\\ D_y^\infty\\ B_y^\infty\end{bmatrix}. \quad (36)$$

In addition, in view of the orthogonal relations in Eq. (20), the following identity can be established:

$$(1-c_2)\mathbf{L}_1 + 2c_2\mathbf{L}_1\mathbf{L}_2^{-1}\mathbf{\Phi}\ \mathrm{diag}\begin{bmatrix}\frac{\exp(-\lambda_1 t)}{\delta_1} & \frac{1}{\delta_2} & \frac{1}{\delta_3}\end{bmatrix}\mathbf{\Phi}^T + \frac{2c_2[1-\exp(-\lambda_1 t)]}{\lambda_1\delta_1}\mathbf{L}_1\mathbf{\Lambda\Phi}\ \mathrm{diag}\begin{bmatrix}1 & 0 & 0\end{bmatrix}\mathbf{\Phi}^T = (1+c_2)\mathbf{L}_1 - 2c_2\mathbf{\Phi}\ \mathrm{diag}\begin{bmatrix}\frac{\exp(-\lambda_1 t)}{\delta_1} & \frac{1}{\delta_2} & \frac{1}{\delta_3}\end{bmatrix}\mathbf{\Phi}^T. \quad (37)$$

Consequently we can simplify Eq. (36) as

$$\begin{bmatrix}\langle\gamma_{zy}\rangle\\ \langle -E_y\rangle\\ \langle -H_y\rangle\end{bmatrix} = \mathbf{L}_1^{-1}\left\{(1+c_2)\mathbf{L}_1 - 2c_2\mathbf{\Phi} \times\mathrm{diag}\begin{bmatrix}\frac{\exp(-\lambda_1 t)}{\delta_1} & \frac{1}{\delta_2} & \frac{1}{\delta_3}\end{bmatrix}\mathbf{\Phi}^T\right\} \times\left\{(1-c_2)\mathbf{L}_1 + 2c_2\mathbf{\Phi} \times\mathrm{diag}\begin{bmatrix}\frac{\exp(-\lambda_1 t)}{\delta_1} & \frac{1}{\delta_2} & \frac{1}{\delta_3}\end{bmatrix}\mathbf{\Phi}^T\right\}^{-1}\begin{bmatrix}\sigma_{zy}^\infty\\ D_y^\infty\\ B_y^\infty\end{bmatrix}. \quad (38)$$

Comparison of Eq. (29) with Eq. (38) immediately leads to the time-dependent effective moduli as

$$\mathbf{L}_c = \mathbf{L}_c^T = \mathbf{L}_1\left\{(1+c_2)\mathbf{L}_1 - 2c_2\mathbf{\Phi} \times\mathrm{diag}\begin{bmatrix}\frac{\exp(-\lambda_1 t)}{\delta_1} & \frac{1}{\delta_2} & \frac{1}{\delta_3}\end{bmatrix}\mathbf{\Phi}^T\right\}^{-1} \times\left\{(1-c_2)\mathbf{L}_1 + 2c_2\mathbf{\Phi} \times\mathrm{diag}\begin{bmatrix}\frac{\exp(-\lambda_1 t)}{\delta_1} & \frac{1}{\delta_2} & \frac{1}{\delta_3}\end{bmatrix}\mathbf{\Phi}^T\right\}. \quad (39)$$

By using the orthogonal relations in Eq. (20), the above expression can be simplified as

$$\mathbf{L}_c(\tilde{t}) = \mathbf{L}_1[(1+c_2)\mathbf{L}_1 - 2c_2\{\mathbf{H}^{-1}\exp(-\tilde{t}) + \mathbf{M}[1-\exp(-\tilde{t})]\}]^{-1} \times [(1-c_2)\mathbf{L}_1 + 2c_2\{\mathbf{H}^{-1}\exp(-\tilde{t}) + \mathbf{M}[1-\exp(-\tilde{t})]\}], \quad (40)$$

where $\tilde{t}=\lambda_1 t$ is a dimensionless time and

$$\mathbf{M} = \mathbf{M}^T = \frac{1}{H_{22}H_{33}-H_{23}^2}\begin{bmatrix}0 & 0 & 0\\ 0 & H_{33} & -H_{23}\\ 0 & -H_{23} & H_{22}\end{bmatrix}. \quad (41)$$

Equation (40) is a very concise expression for the effective magnetoelectroelastic moduli in the multiferroic fibrous composite with a viscous interface. It reveals that while these moduli are constant in space, they are time dependent. Furthermore, one can easily show that at initial time $t=0$ these effective moduli are reduced to

$$\mathbf{L}_c(0) = \mathbf{L}_1[(1+c_2)\mathbf{L}_1 - 2c_2\mathbf{H}^{-1}]^{-1}[(1-c_2)\mathbf{L}_1 + 2c_2\mathbf{H}^{-1}], \quad (42)$$

which are just the results for a perfect interface;[4,9] on the other hand, when $t\to\infty$ these effective moduli become

$$\mathbf{L}_c(\infty) = \mathbf{L}_1[(1+c_2)\mathbf{L}_1 - 2c_2\mathbf{M}]^{-1}[(1-c_2)\mathbf{L}_1 + 2c_2\mathbf{M}], \quad (43)$$

which are the result for a free-sliding interface on which $\sigma_{zr}=0$. This reduced result is actually interesting: For a multiferroic composite composed of piezoelectric fiber and magnetostrictive matrix (or vice versa), we have $H_{23}\equiv 0$; consequently, it is observed from Eq. (43) that there is no ME coupling (effect) when the time approaches infinity.

IV. TIME-DECAYING ME EFFECTS OF $BaTiO_3$ FIBERS REINFORCED IN $CoFe_2O_4$ MATRIX

As a numerical example, here we consider a typical multiferroic fibrous composite consisting of the magnetostrictive $CoFe_2O_4$ matrix reinforced by the piezoelectric $BaTiO_3$ fibers. The pertinent material properties of $BaTiO_3$ are: $c_{44}=43\times 10^9\ \mathrm{N/m^2}$, $e_{15}=11.6\ \mathrm{C/m^2}$, $\varepsilon_{11}=11.2\times 10^{-9}\ \mathrm{C^2/Nm^2}$, $\mu_{11}=5\times 10^{-6}\ \mathrm{Ns^2/C^2}$; while those of $CoFe_2O_4$ are: $c_{44}=45.3\times 10^9\ \mathrm{N/m^2}$, $q_{15}=550\ \mathrm{m/A}$, $\varepsilon_{11}=0.08\times 10^{-9}\ \mathrm{C^2/Nm^2}$, $\mu_{11}=590\times 10^{-6}\ \mathrm{Ns^2/C^2}$. Figure 2 demonstrates the ME coefficient α_{11} as a function of the $BaTiO_3$ volume fraction c_2 at four different normalized time

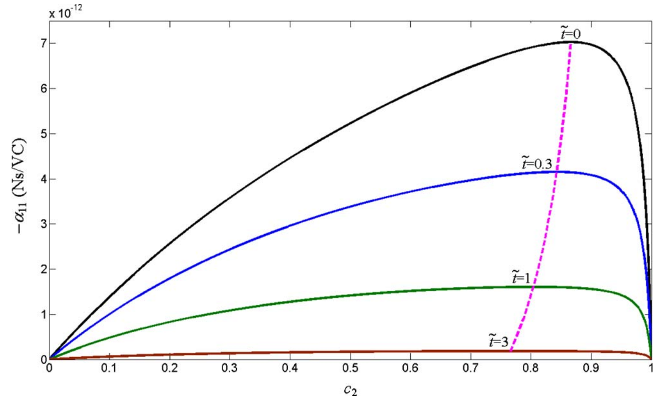

FIG. 2. (Color online) The ME coefficient α_{11} as a function of the $BaTiO_3$ volume fraction c_2 at four different normalized times $\tilde{t}=0$, 0.3, 1, and 3.

$\tilde{t}=0$, 0.3, 1, 3 ($\tilde{t}=\lambda_1 t$). In this figure the dashed line indicates the trace of the locations of the transient maximum ME effect at different times. It is observed from Fig. 2 that: (i) the magnitude of α_{11} monotonically decreases as the time increases; (ii) the ME coefficient α_{11} is *ignorable* when the time t is equal to or greater than three times of the relaxation time $t_0=1/\lambda_1$; (iii) the optimal value of the $BaTiO_3$ volume fraction, at which the transient maximum ME effect at a fixed time occurs, decreases as the time evolves.

V. CONCLUSIONS

The Mori–Tanaka mean-field approach is applied to obtain a closed-form expression of the time-dependent effective moduli of multiferroic fibrous composites with a viscous interface. This expression is considerably interesting since the important ME effect in multiferroic composites is based on the composite product property with the interface as a bridge. While at the initial moment the effective moduli are the largest, just as those for a perfect interface, they are reduced (or deteriorated) with time, and eventually decreased to those for a free-sliding interface as time approaches infinity. Our numerical results for the multiferroic composite of $BaTiO_3$ fibers reinforced in $CoFe_2O_4$ matrix further demonstrate that the viscosity of the interface will cause a time-decaying ME effect. We finally point out that while the responses of piezoelectric fibrous composites with a viscous interface were considered previously,[17] a concise expression of the effective moduli as given in Eq. (40) has never been reported in the literature, not to mention the complex multiferroic composites presented in this article.

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[1]H. Zheng, J. Wang, S. E. Lofland, Z. Ma, L. Mohaddes-Ardabili, T. Zhao, L. Salamanca-Riba, S. R. Shinde, S. B. Ogale, F. Bai, D. Viehland, Y. Jia, D. G. Schlom, M. Wuttig, A. Roytburd, and R. Ramesh, Science **303**, 661 (2004).
[2]M. Fiebig, J. Phys. D **38**, R123 (2005).
[3]N. A. Spaldin and M. Fiebig, Science **309**, 391 (2005).
[4]Y. Benveniste, Phys. Rev. B **51**, 16424 (1995).
[5]G. Srinivasan, E. T. Rasmussen, B. J. Levin, and R. Hayes, Phys. Rev. B **65**, 134402 (2002).
[6]C. W. Nan, N. Cai, Z. Shi, J. Zhai, G. Liu, and Y. Lin, Phys. Rev. B **71**, 014102 (2005).
[7]M. I. Bichurin, V. M. Petrov, and G. Srinivasan, Phys. Rev. B **68**, 054402 (2003).
[8]C. W. Nan, G. Liu, and Y. H. Lin, Appl. Phys. Lett. **83**, 4366 (2003).
[9]X. Wang and E. Pan, Phys. Rev. B **76**, 214107 (2007).
[10]B. T. Chen, C. T. Hu, and S. Lee, Int. J. Eng. Sci. **36**, 1011 (1998).
[11]R. Raj and M. F. Ashby, Metall. Trans. **2**, 1113 (1971).
[12]M. F. Ashby, Surf. Sci. **31**, 498 (1972).
[13]J. V. Funn and I. Dutta, Acta Mater. **47**, 149 (1998).
[14]L. H. He and C. W. Lim, Compos. Sci. Technol. **61**, 579 (2001).
[15]T. Mori and K. Tanaka, Acta Metall. **21**, 571 (1973).
[16]Y. Benveniste, Mech. Mater. **6**, 147 (1987).
[17]L. H. He and C. W. Lim, Composites, Part B **34**, 373 (2003).