

Static response of a transversely isotropic and layered half-space to general dislocation sources

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The static deformation problem of a transversely isotropic and layered half-space by general dislocation sources is solved. The solution is given in the Cartesian system and the cylindrical system of vector functions using multiplications of matrices. The source functions of general dislocations in a transversely isotropic medium are obtained in these two systems using a new approach. Explicit expressions for the surface displacements due to three- and two-dimensional dislocation sources are also obtained in terms of the cylindrical system and the Cartesian system of vector functions, respectively. It is shown that the present solution contains the solutions to the three- and two-dimensional source problems for the corresponding isotropic media, and therefore provides a unified solution for these two problems, which have long been studied separately. The formulation developed can be evaluated numerically to study the effects of anisotropy, as well as of Earth layering, on the static fields.

1. Introduction

The elasticity theory of dislocations was developed and applied by Steketee (1958), Rongved and Frasier (1958) and by Maruyama (1964, 1966). Since then, great progress has been achieved using this theory and its geophysical application (see Okada (1985) and Rybicki (1986) for reviews).

Singh (1970) studied the static response of an isotropic multilayered half-space to three-dimensional sources by using the propagator matrix method in the cylindrical system of vector functions. Sato and Matsu'ura (1973) calculated the static displacement fields of a fault that spreads over several layers in an isotropic layered half-space, but there appear to be numerical instabilities in their computations. Using the theoretical results of Singh (1970), Jovanovich et al. (1974a, b) developed a more general numerical method to compute the displacement and strain fields due to an arbitrary shear dislocation in an isotropic and layered half-space.

The static deformation problem using two-dimensional sources has also been studied by many investigators. Rybicki (1971) studied the effect of a single surface layer on the elastic residual field due to a long strike-slip fault using the image method. Freund and Barnett (1976) obtained a two-dimensional solution of surface deformation due to dip-slip faulting in a uniform half-space, using the theory of analytic functions of a complex variable. Recently, Singh (1985) and Singh and Garg (1985) studied the two-dimensional problem of a long displacement dislocation in an isotropic multilayered half-space in terms of the propagator matrices, and obtained the surface displacements caused by a line source of either dip-slip or strike-slip type of arbitrary dip.

However, it is useful to study the effect of anisotropy on the static field resulting from surface loads or internal sources because the upper part of the Earth is anisotropic (Dziewonski and Anderson, 1981). Small and Booker (1984, 1986) studied the two- and three-dimensional deformations of a transversely isotropic and layered material by surface loads, using double Fourier transforms, Hankel transforms and the finite layer approach. Singh (1986) solved the problem of a transversely isotropic multilayered half-space deformed by surface loads under the assumption of axially symmetric deformation, and the propagator matrix method was introduced to avoid the cumbersome nature of the problem. Garg and Singh (1987) solved the corresponding two-dimensional problem using the same approach. More recently, by introducing two systems of vector functions and using the propagator matrix method, Pan (1989, referred to as Paper I hereafter) solved the corresponding three-dimensional deformation problem, providing a complete and unified solution of the transversely isotropic and layered elastic half-space by general surface loads.

In the present paper, we formulate the static deformation problem of general dislocation sources in a transversely isotropic and layered half-space. The solution is given in terms of the general solutions and layer matrices of Paper I, and in the Cartesian system and cylindrical system of vector functions. The source functions of general point dislocations in a transversely isotropic medium are obtained in these two systems using a unified approach. Whereas the solution for the three-dimensional source problem can be expressed in terms of either the Cartesian system or the cylindrical system of vector functions, the solution for the two-dimensional source problem is obtained in the Cartesian system of vector functions. Solutions for both the three- and two-dimensional source problems in the transversely isotropic media can be directly reduced to the solutions of the corresponding isotropic case.

2. General solution

The model considered is shown schematically in Fig. 1. The layered elastic system consists of $p - 1$ homogeneous and transversely isotropic layers overlying a homogeneous transversely isotropic half-space. We place the origins of the Cartesian coordinates ($x_1 = x$, $x_2 = y$, $x_3 = z$) and cylindrical coordinates (r , θ , z) at the surface. The z -axis is chosen as the axis of symmetry of the transversely isotropic elastic

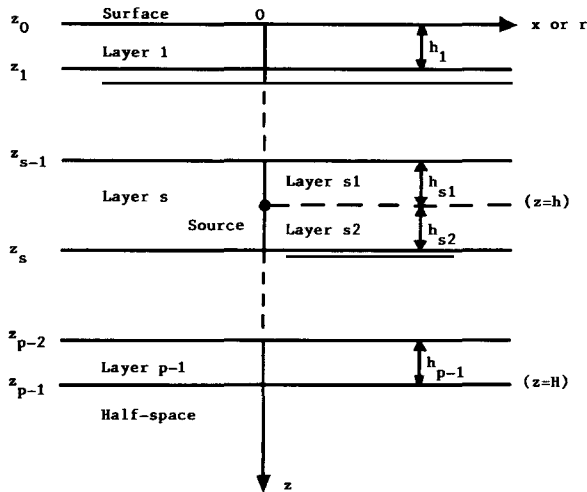


Fig. 1. Geometry of a multilayered half-space. The source is located at $z = h$ in the layer s . The half-space is at a depth H from the surface.

medium and is drawn into it. A source is located on the z -axis at a depth, h , below the surface. The layer interfaces are assumed to be in welded contact.

We will solve the present problem using the approach proposed by Singh (1970) in which the source is removed from the equations of equilibrium so that they become homogeneous, and, instead, is represented in terms of the jumps in the displacement and stress components at the source level. However, the body-force equivalent approach (Burrige and Knopoff, 1964) is also used to obtain the source functions in terms of the Cartesian system and cylindrical system of vector functions.

When body force (or body-force equivalent) is present, eqns. (22) and (23) in Paper I become inhomogeneous. Assuming that the body force per unit volume, $\mathbf{F}(x, y, z) = \mathbf{F}(r, \theta, z)$, can be expressed in terms of the Cartesian system and cylindrical system of vector functions in the form

$$\mathbf{F}(x, y, z) = \iint_{-\infty}^{+\infty} [F_L(z)\mathbf{L}(x, y) + F_M(z)\mathbf{M}(x, y) + F_N(z)\mathbf{N}(x, y)] d\alpha d\beta \quad (2.1a)$$

and

$$\mathbf{F}(r, \theta, z) = \sum_m \int_0^{+\infty} [F_L(z)\mathbf{L}(r, \theta) + F_M(z)\mathbf{M}(r, \theta) + F_N(z)\mathbf{N}(r, \theta)] \lambda d\lambda \quad (2.1b)$$

the equations (22) and (23) in Paper I then become

$$\begin{cases} dU_L/dz = \lambda^2 U_M A_{13}/A_{33} + T_L/A_{33} \\ dU_M/dz = -U_L + T_M/A_{44} \\ dT_L/dz = \lambda^2 T_M - F_L \\ dT_M/dz = \lambda^2 U_M (A_{11}A_{33} - A_{13}^2)/A_{33} - A_{13}T_L/A_{33} - F_M \end{cases} \quad (2.2a)$$

$$\begin{cases} dU_N/dz = T_N/A_{44} \\ dT_N/dz = \lambda^2 U_N A_{66} - F_N \end{cases} \quad (2.2b)$$

In eqns. (2.2a) and (2.2b), the expansion coefficients of the body force F_L , F_M and F_N are determined by eqns. (2.1a) or (2.1b) according to the system of vector functions chosen. It should be noted that these coefficients are not necessarily the same in these two systems for a given body force. We use the same symbols for convenience only.

The homogeneous solutions of eqns. (2.2a) and (2.2b) for any layer, k , are

$$[E(z)] = [Z(z)][K] \quad (2.3a)$$

$$[E^L(z)] = [Z^L(z)][K^L] \quad (2.3b)$$

where the column matrices are defined by

$$[E(z)] = [U_L(z), \lambda U_M(z), T_L(z)/\lambda, T_M(z)]^T \quad (2.4)$$

$$[E^L(z)] = [U_N(z), T_N(z)/\lambda]^T \quad (2.5)$$

$$[K] = [A, B, C, D]^T \quad (2.6)$$

$$[K^L] = [A^L, B^L]^T \quad (2.7)$$

with $[-]^T$ denoting the transpose of the matrix $[-]$. The solution matrix $[Z^L(z)]$ is given by (25) in Paper I with the λ before \bar{s} in the second row being replaced by 1. The elements of solution matrix $[Z(z)]$ for different cases of characteristic roots are given in Appendix A of Paper I. From (2.3a, b) we obtain the propagating relations

$$[E(z_{k-1})] = [a_k][E(z_k)] \quad (2.8a)$$

$$[E^L(z_{k-1})] = [a_k^L][E^L(z_k)] \quad (2.8b)$$

where the propagator matrix $[a_k^L]$ is given by equation (28) in Paper I, and the elements of the propagator matrix $[a_k]$ for different cases of characteristic roots are given in Appendix B of Paper I.

Let a source be situated at depth $z = h$ in the layer s (Fig. 1). We divide the source layer into two sub-layers, $s1$ and $s2$, with identical properties. Because of the presence of the source, the functions $[E(z)]$ and $[E^L(z)]$ may be discontinuous across $z = h$. We put

$$[E_{s2}(h)] - [E_{s1}(h)] = [\Delta E] \quad (2.9a)$$

$$[E_{s2}^L(h)] - [E_{s1}^L(h)] = [\Delta E^L] \quad (2.9b)$$

where the subscript $s1$ ($s2$) is attached to $[E]$ and $[E^L]$ to indicate that they belong to the layer $s1$ (layer $s2$). For a given source, we assume that $[\Delta E]$ and $[\Delta E^L]$ are known. Using the same approach as Singh (1970), we find

$$[E(0)] = [G][K_p] - [Q] \quad (2.10a)$$

$$[E^L(0)] = [G^L][K_p^L] - [Q^L] \quad (2.10b)$$

where

$$[G] = [a_1][a_2] - [a_{p-1}][Z_p(H)] \quad (2.11)$$

$$[G^L] = [a_1^L][a_2^L] - [a_{p-1}^L][Z_p^L(H)] \quad (2.12)$$

$$[Q] = [a_1][a_2] - [a_{s-1}][a_{s1}][\Delta E] \quad (2.13)$$

$$[Q^L] = [a_1^L][a_2^L] - [a_{s-1}^L][a_{s1}^L][\Delta E^L] \quad (2.14)$$

$[K_p]$ and $[K_p^L]$ are the constant column matrices corresponding to the half-space.

Applying the stress-free boundary conditions at $z = 0$ and the finiteness condition of the solution in the half-space, we finally find the expansion coefficients of the surface displacement vector

$$U_L(0) = (G|_{24}{}^{14}Q_3 + G|_{42}{}^{13}Q_4)/G|_{24}{}^{34} - Q_1 \quad (2.15)$$

$$\lambda U_M(0) = (G|_{24}{}^{24}Q_3 + G|_{42}{}^{23}Q_4)/G|_{24}{}^{34} - Q_2 \quad (2.16)$$

$$U_N(0) = G_{12}^L Q_2^L / G_{22}^L - Q_1^L \quad (2.17)$$

where

$$G|_{kl}{}^{ij} = G_{ik}G_{jl} - G_{il}G_{jk}$$

If the discontinuities $[\Delta E]$ and $[\Delta E^L]$ are known, we can thus find the expansion coefficients of the surface displacement vector from eqns. (2.15)–(2.17), and can obtain the surface displacements from eqn. (14) or (16) of Paper I as the system of vector functions may be. The displacements and stresses at any point of the medium can also be obtained from suitable equations of Paper I. To obtain the discontinuities $[\Delta E]$ and $[\Delta E^L]$ or the source functions for the dislocation source in a transversely isotropic medium is complicated; we therefore devote the following section to the derivation of the source functions.

3. Source function

Singh et al. (1973) studied two approaches, in the isotropic case, to the source representation and gave the source functions for various sources commonly used in seismology. In the transversely isotropic case, however, Takeuchi and Saito (1972) listed only the source functions for the dislocation source in terms of the cylindrical system of vector functions, and gave no derivation as they regarded this problem as quite

complicated. In this section we use the two approaches (Singh et al., 1973) simultaneously in the two systems of vector functions (see Paper I) to give a simple derivation of the source functions for a dislocation source in a transversely isotropic medium. We first use the body-force equivalent of dislocation source (Burridge and Knopoff, 1964; Aki and Richards, 1980) to get the body-force representations (2.1a, b) i.e., to find the expansion coefficients $F_L(z)$, $F_M(z)$ and $F_N(z)$ in the Cartesian system and the cylindrical system. For these coefficients, we can find the particular solutions of eqns. (2.2a) and (2.2b) by using the propagator matrix (Gilbert and Backus, 1966). From the particular solutions we can then find the discontinuities of the expansion coefficients of displacement and 'surface' stress vectors, which are equivalent to the source functions. It is noted that this method is quite simple and is easy to extend to the general anisotropic case.

3.1. Three-dimensional source function

It is easy to show that (Aki and Richards, 1980), in elastostatics, the body-force equivalent of an arbitrary discontinuity of a displacement vector across a fault surface Σ is

$$f_p(\boldsymbol{\eta}) = - \iint_{\Sigma} [u_i(\boldsymbol{\xi})] c_{ijpq} v_j \frac{\partial}{\partial \eta_q} \delta(\boldsymbol{\eta} - \boldsymbol{\xi}) d\Sigma(\boldsymbol{\xi}) \quad (3.1)$$

where the summation convention has been used. In eqn. (3.1), v_j is the normal to the surface Σ at point $\boldsymbol{\xi}$ (Fig. 2), c_{ijpq} is the elastic constant in the generalized Hooke's law and $[u_i(\boldsymbol{\xi})]$ represents the displacement discontinuity across the surface Σ at point $\boldsymbol{\xi}$.

If we assume that the dislocation is of a point source type located at (x_0, y_0, h) in the Cartesian coordinate system or (r_0, θ_0, h) in the cylindrical coordinate system, and that the discontinuity in the i direction is given by $[u_i] = \Delta u n_i$, eqn. (3.1) then becomes

$$f_p(x, y, z) = -\Delta u d\Sigma n_i v_j c_{ijpq} \frac{\partial}{\partial x_q} [\delta(x - x_0) \delta(y - y_0) \delta(z - h)] \quad (3.2a)$$

in the Cartesian coordinate system and

$$f_p(r, \theta, z) = -\Delta u d\Sigma n_i v_j c_{ijpq} \frac{\partial}{\partial \eta_q} [\delta(r - r_0) \delta(\theta - \theta_0) \delta(z - h)/r] \quad (3.2b)$$

in the cylindrical coordinate system. In (3.2b), $\eta_1 = r$, $\eta_2 = \theta$ and $\eta_3 = z$.

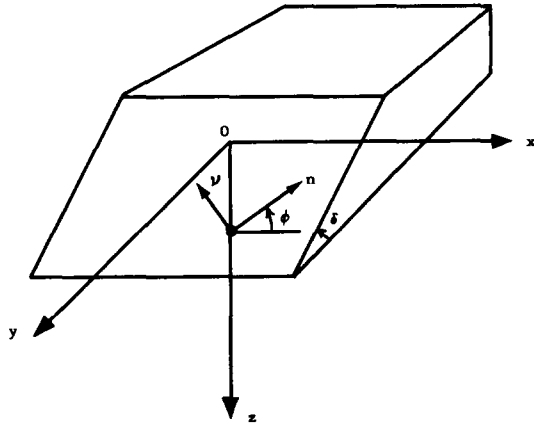


Fig. 2. Geometry of an arbitrary shear dislocation source which is used to represent a fault. The rake is ϕ and the dip is δ . v is the fault normal and n is the unit slip vector. The strike is taken along the x -axis.

Expanding the body-force equivalents (3.2a, b) in terms of the Cartesian system and the cylindrical system respectively, as in (2.1a, b), the coefficients are then given by

$$\begin{aligned} F_L &= \iint_{-\infty}^{+\infty} \mathbf{f} \cdot \mathbf{L}^*(x, y) dx dy \\ F_M &= \lambda^{-2} \iint_{-\infty}^{+\infty} \mathbf{f} \cdot \mathbf{M}^*(x, y) dx dy \\ F_N &= \lambda^{-2} \iint_{-\infty}^{+\infty} \mathbf{f} \cdot \mathbf{N}^*(x, y) dx dy \end{aligned} \quad (3.3a)$$

in the Cartesian system and by

$$\begin{aligned} F_L &= \int_0^{2\pi} \int_0^{+\infty} \mathbf{f} \cdot \mathbf{L}^*(r, \theta) r dr d\theta \\ F_M &= \lambda^{-2} \int_{2\pi}^0 \int_0^{+\infty} \mathbf{f} \cdot \mathbf{M}^*(r, \theta) r dr d\theta \\ F_N &= \lambda^{-2} \int_0^{2\pi} \int_0^{+\infty} \mathbf{f} \cdot \mathbf{N}^*(r, \theta) r dr d\theta \end{aligned} \quad (3.3b)$$

in the cylindrical system. In eqns. (3.3a) and (3.3b), the asterisk indicates the complex conjugate. Using the properties of the Dirac delta function, we can divide the right-hand sides of equations (3.3a) and (3.3b) into the form

$$\begin{aligned} F_L &= F_L^\delta \delta(z-h) + F_L^d \delta'(z-h) \\ F_M &= F_M^\delta \delta(z-h) + F_M^d \delta'(z-h) \\ F_N &= F_N^\delta \delta(z-h) + F_N^d \delta'(z-h) \end{aligned} \quad (3.4)$$

where the prime indicates the derivative with respect to z . The expressions of coefficients $F_L^\delta, F_M^\delta, F_N^\delta, F_L^d, F_M^d$ and F_N^d in the Cartesian system and the cylindrical system are given in Appendices 1 and 2 respectively. It can be shown that (Kennett, 1981) the discontinuities caused by the first part of (3.4) are

$$\begin{aligned} \Delta T_L &= -F_L^\delta & \Delta U_L &= 0 \\ \Delta T_M &= -F_M^\delta & \Delta U_M &= 0 \\ \Delta T_N &= -F_N^\delta & \Delta U_N &= 0 \end{aligned} \quad (3.5)$$

and the discontinuities caused by the second part are

$$\begin{aligned} \Delta T_L &= -\lambda^2 F_M^d & \Delta U_L &= -F_L^d/A_{33} \\ \Delta T_M &= F_L^d A_{13}/A_{33} & \Delta U_M &= -F_M^d/A_{44} \\ \Delta T_N &= 0 & \Delta U_N &= -F_N^d/A_{44} \end{aligned} \quad (3.6)$$

The total discontinuities of the source functions are the sum of eqns. (3.5) and (3.6). Expressing in the components of $[\Delta E]$ and $[\Delta E^L]$, they are

$$\begin{aligned} \Delta U_L &= -F_L^d/A_{33} \\ \lambda \Delta U_M &= -\lambda F_M^d/A_{44} \\ \Delta T_L/\lambda &= -F_L^\delta/\lambda - \lambda F_M^d \\ \Delta T_M &= -F_M^\delta + F_L^d A_{13}/A_{33} \\ \Delta U_N &= -F_N^d/A_{44} \\ \Delta T_N/\lambda &= -F_N^\delta/\lambda \end{aligned} \quad (3.7)$$

Substituting the elastic constants c_{ijpq} of a transversely isotropic medium into Appendices 1 and 2, and calculating these coefficients F_L^δ , F_M^δ , F_N^δ , F_L^d , F_M^d and F_N^d at the point $x_0 = y_0 = r_0 = \theta_0 = 0$, we can finally obtain the discontinuities at the depth $z = h$, which in the Cartesian system are (omitting factor $\Delta u \, d\Sigma/(2\pi)$)

$$\begin{aligned}\Delta U_L &= (n_x v_x + n_y v_y) A_{13}/A_{33} + n_z v_z \\ \lambda \Delta U_M &= i\lambda^{-1} [\alpha(n_x v_z + n_z v_x) + \beta(n_y v_z + n_z v_y)] \\ \Delta T_M &= -(n_x v_x + n_y v_y) A_{13}^2/A_{33} \\ &\quad + \lambda^{-2} [n_x v_x (\alpha^2 A_{11} + \beta^2 A_{12}) + n_y v_y (\alpha^2 A_{12} + \beta^2 A_{11}) + 2(n_x v_y + n_y v_x) \alpha \beta A_{66}] \\ \Delta U_N &= i\lambda^{-2} [\beta(n_x v_z + n_z v_x) - \alpha(n_y v_z + n_z v_y)] \\ \Delta T_N/\lambda &= \lambda^{-3} [(n_x v_y + n_y v_x) (\beta^2 - \alpha^2) A_{66} + (n_x v_x - n_y v_y) \alpha \beta (A_{11} - A_{12})]\end{aligned}\tag{3.8}$$

and in the cylindrical system are (omitting factor $\Delta u \, d\Sigma/(2\pi)^{1/2}$)

$$\begin{aligned}\Delta U_L &= (n_x v_x + n_y v_y) A_{13}/A_{33} + n_z v_z & m = 0 \\ \lambda \Delta U_M &= [\pm (n_x v_z + n_z v_x) - i(n_y v_z + n_z v_y)]/2 & m = \pm 1 \\ \Delta T_M &= [(A_{11} + A_{12})/2 - A_{13}^2/A_{33}] (n_x v_x + n_y v_y) & m = 0 \\ &= [(n_y v_y - n_x v_x) \pm i(n_x v_y + n_y v_x)] A_{66}/2 & m = \pm 2 \\ \Delta U_N &= \lambda^{-1} [-i(n_x v_z + n_z v_x) \mp (n_y v_z + n_z v_y)]/2 & m = \pm 1 \\ \Delta T_N/\lambda &= \lambda^{-1} [(n_x v_y + n_y v_x) \pm i(n_x v_x - n_y v_y)] A_{66}/2 & m = \pm 2\end{aligned}\tag{3.9}$$

In eqns. (3.8) and (3.9), (n_x, n_y, n_z) and (v_x, v_y, v_z) are the (x, y, z) components of the unit vectors \mathbf{n} and \mathbf{v} respectively; components which are not listed in equations (3.8) and (3.9) are zero. In addition, in the derivation of eqns. (3.8) and (3.9), we have chosen the Cartesian coordinates (x, y, z) to express these two unit vectors, and taken $\theta = 0$ along the x -direction and $\theta = \pi/2$ along the y -direction.

Equations (3.8) and (3.9) are the expressions of the source functions for the general three-dimensional point dislocation. Whereas eqn. (3.9) includes the axially symmetric source functions (corresponding to $m = 0$), eqn. (3.8) can be reduced to the corresponding two-dimensional source functions. Noticing the difference between the definitions of the cylindrical system of vector functions given by Singh (1970), Takeuchi and Saito (1972) and the author (see Paper I), it can be shown that eqn. (3.9) is the same as those listed by Takeuchi and Saito (1972), and when eqn. (3.9) is reduced to the isotropic case, the expressions for $m > 0$ are half the value of the corresponding results of Singh (1970). This is due to the fact that Singh uses only the $m \geq 0$ part of the cylindrical system of vector functions. Nevertheless, in the isotropic case, the final expressions for displacements using the present cylindrical system of vector functions and the source functions in eqn. (3.9) are coincident with the corresponding isotropic results (Singh, 1970; Jovanovich et al., 1974a).

3.2. Two-dimensional source

We assume that the two-dimensional deformation is in the (y, z) plane; that is, the long-line source is parallel to the x -axis. As we have pointed out, by replacing 2π by $(2\pi)^{1/2}$ and α by 0 (so $\lambda = |\beta|$, as λ must be positive) in the scalar function (9) of Paper I, we then reduce the three-dimensional deformation directly to the two-dimensional deformation in the (y, z) plane. In addition, as the source type $n_x v_x$ does

not occur in the (y, z) plane problem, it should be omitted from eqn. (3.8). The source functions for the two-dimensional case are therefore (omitting factor $\Delta u \, ds / (2\pi)^{1/2}$, where ds is the line element)

$$\begin{aligned}
\Delta U_L &= n_y v_y A_{13}/A_{33} + n_z v_z \\
\lambda \Delta U_M &= i(n_y v_z + n_z v_y) \text{sign}(\beta) \\
\Delta T_M &= n_y v_y (A_{11} - A_{13}^2/A_{33}) \\
\Delta U_N &= i\lambda^{-1}(n_x v_z + n_z v_x) \text{sign}(\beta) \\
\Delta T_N/\lambda &= \lambda^{-1}(n_x v_y + n_y v_x) A_{66}
\end{aligned} \tag{3.10}$$

where $\text{sign}(\beta)$ is the sign function which has the value of -1 for $\beta < 0$ and $+1$ for $\beta > 0$. Equation (3.10) is the expression of the source functions for the two-dimensional point dislocation. It can be divided into two types, i.e. a plane strain problem (corresponding to type I of Paper I) and an antiplane strain problem (corresponding to type II of Paper I).

3.2.1. Plane strain

In this case, we have three elementary sources. Their source functions are

$$\begin{aligned}
\text{Source}(2,2): \Delta U_L &= A_{13}/A_{33}, \lambda \Delta U_M = 0, \Delta T_L/\lambda = 0, \Delta T_M = (A_{11} - A_{13}^2/A_{33}) \\
\text{Source}(2,3): \Delta U_L &= 0, \lambda \Delta U_M = i \text{sign}(\beta), \Delta T_L/\lambda = 0, \Delta T_M = 0 \\
\text{Source}(3,3): \Delta U_L &= 1, \lambda \Delta U_M = 0, \Delta T_L/\lambda = 0, \Delta T_M = 0
\end{aligned} \tag{3.11a}$$

3.2.2. Antiplane strain

In this case, we have two elementary sources. Their source functions are

$$\begin{aligned}
\text{Source}(1,2): \Delta U_N &= 0, \Delta T_N/\lambda = A_{66}/\lambda \\
\text{Source}(1,3): \Delta U_N &= i \text{sign}(\beta)/\lambda, \Delta T_N/\lambda = 0
\end{aligned} \tag{3.11b}$$

It can be shown that, for the isotropic case, the expressions of source functions will be reduced to those given by Singh and Garg (1985). Therefore we have not only provided the expressions of the source functions for three- and two-dimensional point dislocation in the transversely isotropic medium, but also provided a unified approach to the three- and two-dimensional source problems, which have long been studied separately.

4. Surface displacement

Having obtained the above elementary source functions, we can find the surface displacements of a transversely isotropic and layered half-space resulting from these elementary sources using eqns. (2.15)–(2.17) of this paper and using eqns. (14) or (16) of Paper I according to the system of vector functions chosen. By suitable linear combinations of these fundamental solutions, we can then get the surface displacements due to an arbitrary shear dislocation source.

4.1. Three-dimensional dislocation source

Figure 2 is a scheme of an arbitrary shear dislocation source with the strike along the x -axis. \mathbf{v} is the fault normal, \mathbf{n} is the unit slip vector and is perpendicular to the normal, ϕ is the rake and δ is the dip of the fault. It is easy to show that for this shear dislocation source, we have the dyadic

$$\mathbf{n}\mathbf{v} = \cos \phi [\mathbf{i}_x \mathbf{i}_y \sin \delta - \mathbf{i}_x \mathbf{i}_z \cos \delta] + \sin \phi [\mathbf{i}_y \mathbf{i}_z \cos 2\delta - 2^{-1}(\mathbf{i}_y \mathbf{i}_y - \mathbf{i}_z \mathbf{i}_z) \sin 2\delta] \tag{4.1}$$

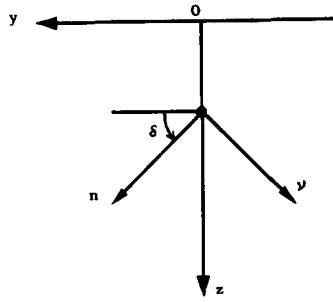


Fig. 3. Geometry of a long shear dislocation source which is used to represent a long fault parallel to the x -axis. δ is the dip, ν is the normal and \mathbf{n} is the unit slip vector.

Using the definition of the elementary dislocation source (Steketee, 1958), we find that the displacements due to an arbitrary shear dislocation source can be expressed as the linear combination of the fundamental displacements due to the elementary dislocation sources, i.e.

$$u_i = \cos \phi [u_i^1 \sin \delta - u_i^4 \cos \delta] + \sin \phi [u_i^2 \cos 2\delta - u_i^3 \sin 2\delta] \quad i = r, \theta, z \quad (4.2)$$

We have chosen the cylindrical system of vector functions for the three-dimensional problem, as many previous researchers have done. However, the fundamental displacements can also be expressed in the Cartesian system of vector functions and it is perhaps more convenient to use this system than the cylindrical system for the commonly studied rectangular fault when the integration over the fault area is required. In eqn. (4.2), u_i^1 represents the displacement due to the source (1,2), or the vertical strike-slip source ($\phi = 0^\circ$, $\delta = 90^\circ$); u_i^2 represents the displacement due to the source (2,3), or the vertical dip-slip source ($\phi = 90^\circ$, $\delta = 90^\circ$); u_i^3 represents the displacement due to the combined source [(2,2)–(3,3)]/2, or the 45° dip-slip source ($\phi = 90^\circ$, $\delta = 45^\circ$); u_i^4 represents the displacement due to the source (1,3), which can be obtained from u_i^2 by replacing θ by $\theta - \pi/2$. To obtain the displacement field due to an arbitrary shear dislocation source, we are therefore required to calculate only two fundamental displacement fields due to the elementary sources (1,2) and (2,3), and one displacement field due to the combined source [(2,2)–(3,3)]/2.

4.2. Two-dimensional dislocation source

Figure 3 is a scheme of an arbitrary two-dimensional shear dislocation source. ν is the fault normal; \mathbf{n} is the unit slip vector and is perpendicular to the normal; δ is the dip of the fault. The two-dimensional dislocation source in fact represents a long line source parallel to the x -axis. We will discuss the plane strain and the antiplane strain problems separately.

4.2.1. Plane strain

This corresponds to the problem of a long dip-slip fault (Singh and Garg, 1985) and its solution is of the type I of Paper I. In this case

$$\mathbf{n} = (0, \cos \delta, \sin \delta)$$

$$\nu = (0, -\sin \delta, \cos \delta)$$

and therefore, we have the dyadic

$$\mathbf{n}\nu = \mathbf{i}_y \mathbf{i}_z \cos 2\delta - 2^{-1} [\mathbf{i}_y \mathbf{i}_y - \mathbf{i}_z \mathbf{i}_z] \sin 2\delta \quad (4.3)$$

Similarly, the displacements due to a long dip-slip source can be expressed as the linear combination of the displacements due to the elementary source (2,3) and the combined source [(2,2)–(3,3)]/2, i.e.

$$u_i = u_i^2 \cos 2\delta - u_i^3 \sin 2\delta \quad i = y, z \quad (4.4)$$

in which u_i^2 and u_i^3 have the same physical meanings as those in eqn. (4.2), except that the former is in the two-dimensional sense. To obtain the displacement field due to a long dip-slip source, we are thus required to calculate only one fundamental displacement field due to the elementary source (2,3) and one displacement field due to the combined source [(2,2)–(3,3)]/2.

4.2.2. Antiplane strain

This corresponds to the problem of a long strike-slip fault (Singh and Garg, 1985) and its solution is of the type II of Paper I. In this case

$$\mathbf{n} = (1, 0, 0)$$

$$\mathbf{v} = (0, -\sin \delta, \cos \delta)$$

and therefore, we have the dyadic

$$\mathbf{n}\mathbf{v} = \mathbf{i}_x \mathbf{i}_z \cos \delta - \mathbf{i}_x \mathbf{i}_y \sin \delta \quad (4.5)$$

The displacement due to a long strike-slip source can thus be expressed by

$$u_x = u_x^4 \cos \delta - u_x^1 \sin \delta \quad (4.6)$$

Again, u_x^4 and u_x^1 have the same physical meanings as those in (4.2), except that the former is in the two-dimensional sense. Consequently, to obtain the displacement due to a long strike-slip source, we are required to calculate only two fundamental displacement fields due to the elementary sources (1,3) and (1,2).

So far we have obtained the expressions of the static displacements due to an arbitrary shear dislocation source in three- and two-dimensional cases. They are expressed by the linear combinations of some fundamental displacements due to some elementary sources, or of some Green's functions. The remaining work therefore is to obtain the Green's functions appearing in eqns. (4.3), (4.4) and (4.6).

4.3. Three-dimensional Green's function

As we have mentioned above, to get the static displacement field due to an arbitrary shear dislocation source in a transversely isotropic and layered half-space, we are actually required to calculate only three Green's functions corresponding to the sources (1,2), (2,3) and [(2,2)–(3,3)]/2. Using eqns. (2.15)–(2.17) and (3.9) of this paper and eqn. (16) of Paper I, they are found to be (omitting factor $\Delta u \, d\Sigma / (2\pi)$)

(1) For source (1,2)

$$\begin{aligned} u_r^1 &= - \int_0^{+\infty} \left[(U_M(0)/i) \frac{\partial}{\partial r} J_2(\lambda r) + 2U_N(0) J_2(\lambda r)/r \right] \lambda \, d\lambda \sin 2\theta \\ u_\theta^1 &= - \int_0^{+\infty} \left[2(U_M(0)/i) J_2(\lambda r)/r + U_N(0) \frac{\partial}{\partial r} J_2(\lambda r) \right] \lambda \, d\lambda \cos 2\theta \\ u_z^1 &= - \int_0^{+\infty} (U_L(0)/i) J_2(\lambda r) \lambda \, d\lambda \sin 2\theta \end{aligned} \quad (4.7)$$

In eqn. (4.7), $J_m(\lambda r)$ is the Bessel function of order m . The expansion coefficients of the surface displacement vector $U_L(0)$, $U_M(0)$ and $U_N(0)$ in the cylindrical system of vector functions are respectively

$$\begin{aligned} U_L(0)/i &= A_{66} \left[(G|_{24}{}^{14} V_{34} + G|_{42}{}^{13} V_{44}) / G|_{24}{}^{34} - V_{14} \right] \\ \lambda U_M(0)/i &= A_{66} \left[(G|_{24}{}^{24} V_{34} + G|_{42}{}^{23} V_{44}) / G|_{24}{}^{34} - V_{24} \right] \\ U_N(0) &= A_{66} (G|_{12}{}^L V_{22}^L / G|_{22}{}^L - V_{12}^L) / \lambda \end{aligned} \quad (4.8)$$

where V_{ij} and V^L_{ij} are the elements of the matrices $[V]$ and $[V^L]$ respectively, which are given by

$$[V] = [a_1][a_2] - [a_{s-1}][a_{s1}]$$

$$[V^L] = [a^L_1][a^L_2] - [a^L_{s-1}][a^L_{s1}]$$

(2) For source (2,3)

$$u_r^2 = - \int_0^{+\infty} \left[(U_M(0)/i) \frac{\partial}{\partial r} J_1(\lambda r) + U_N(0) J_1(\lambda r)/r \right] \lambda \, d\lambda \sin \theta$$

$$u_\theta^2 = - \int_0^{+\infty} \left[(U_M(0)/i) J_1(\lambda r)/r + U_N(0) \frac{\partial}{\partial r} J_1(\lambda r) \right] \lambda \, d\lambda \cos \theta \quad (4.9)$$

$$u_z^2 = - \int_0^{+\infty} (U_L(0)/i) J_1(\lambda r) \lambda \, d\lambda \sin \theta$$

In eqn. (4.9)

$$U_L(0)/i = - \left[(G|_{24}{}^{14} V_{32} + G|_{42}{}^{13} V_{42}) / G|_{24}{}^{34} - V_{12} \right]$$

$$\lambda U_M(0)/i = - \left[(G|_{24}{}^{24} V_{32} + G|_{42}{}^{23} V_{42}) / G|_{24}{}^{34} - V_{22} \right] \quad (4.10)$$

$$U_N(0) = - (G^L_{12} V^L_{21} / G^L_{22} - V^L_{11}) / \lambda$$

(3) For source [(2,2)-(3.3)]/2

$$\left\{ \begin{aligned} u_r^3 &= \int_0^{+\infty} \left\{ U_M^0(0) \frac{\partial}{\partial r} J_0(\lambda r) + \left[(U_M^2(0) \frac{\partial}{\partial r} J_2(\lambda r) - 2(U_N^2(0)/i) J_2(\lambda r)/r \right] \cos 2\theta \right\} \lambda \, d\lambda \\ u_\theta^3 &= \int_0^{+\infty} \left[-2U_M^2(0) J_2(\lambda r)/r + (U_N^2(0)/i) \frac{\partial}{\partial r} J_2(\lambda r) \right] \lambda \, d\lambda \sin 2\theta \\ u_z^3 &= \int_0^{+\infty} [U_L^0(0) J_0(\lambda r) + U_L^2(0) J_2(\lambda r) \cos 2\theta] \lambda \, d\lambda \end{aligned} \right. \quad (4.11)$$

In eqn. (4.11), $U_L^2(0)$, $U_M^2(0)$ and $U_N^2(0)$ are the expansion coefficients of the surface displacement vector corresponding to $m = 2$ and they are given by

$$U_L^2(0) = A_{66} \left[(G|_{24}{}^{14} V_{34} + G|_{42}{}^{13} V_{44}) / G|_{24}{}^{34} - V_{14} \right] / 2$$

$$\lambda U_M^2(0) = A_{66} \left[(G|_{24}{}^{24} V_{34} + G|_{42}{}^{23} V_{44}) / G|_{24}{}^{34} - V_{24} \right] / 2 \quad (4.12)$$

$$U_N^2(0)/i = -A_{66} (G^L_{12} V^L_{22} / G^L_{22} - V^L_{12}) / (2\lambda)$$

$U_L^0(0)$ and $U_M^0(0)$ are the expansion coefficients of the surface displacement vector corresponding to $m = 0$ and they are given by

$$U_L^0(0) = (G|_{24}{}^{14} Q_3 + G|_{42}{}^{13} Q_4) / G|_{24}{}^{34} - Q_1$$

$$\lambda U_M^0(0) = (G|_{24}{}^{24} Q_3 + G|_{42}{}^{23} Q_4) / G|_{24}{}^{34} - Q_2 \quad (4.13)$$

where

$$Q_j = \left\{ (A_{13}/A_{33} - 1) V_{j1} + [(A_{11} + A_{12})/2 - A_{13}^2/A_{33}] V_{j4} \right\} / 2 \quad j = 1, 2, 3, 4$$

4.4. Two-dimensional Green's function

We have shown that it is convenient to use the Cartesian system of vector functions to study the problem of two-dimensional deformation. The two-dimensional solution can be obtained directly from the

corresponding three-dimensional solution by replacing 2π by $(2\pi)^{1/2}$ and α by 0 in the scalar function (2.9) of Paper I. The two-dimensional Green's functions appearing in eqns. (4.4) and (4.6) are derived by this approach and are given in the following.

4.4.1. Plane strain

In this case, the Green's functions corresponding to the elementary source (2,3) and the combined source [(2,2)–(3,3)]/2 are required to obtain the surface displacement field (4,4).

(1) For source (2,3), the expansion coefficients $U_L(0)$ and $U_M(0)$ of the surface displacement vector due to this elementary source are (omitting factor $\Delta u \, ds/(2\pi)^{1/2}$; this factor is also omitted in the following expressions of expansion coefficients)

$$\begin{aligned} U_L(0)/i &= \text{sign}(\beta) \left[(G|_{24}{}^{14}V_{32} + G|_{42}{}^{13}V_{42})/G|_{24}{}^{34} - V_{12} \right] \\ \lambda U_M(0)/i &= \text{sign}(\beta) \left[(G|_{24}{}^{24}V_{32} + G|_{42}{}^{23}V_{42})/G|_{24}{}^{34} - V_{22} \right] \end{aligned} \quad (4.14)$$

Using equation (14) of Paper I, we find that the surface displacement components are (omitting factor $\Delta u \, ds/\pi$; this factor is also omitted in the following expressions of surface displacement components)

$$\begin{aligned} u_y^2 &= \int_0^{+\infty} \left[(G|_{24}{}^{24}V_{32} + G|_{42}{}^{23}V_{42})/G|_{24}{}^{34} - V_{22} \right] \cos \beta y \, d\beta \\ u_z^2 &= \int_0^{+\infty} \left[(G|_{24}{}^{14}V_{32} + G|_{42}{}^{13}V_{42})/G|_{24}{}^{34} - V_{12} \right] \sin \beta y \, d\beta \end{aligned} \quad (4.15)$$

(2) For source [(2,2)–(3,3)]/2, the expansion coefficients $U_L(0)$ and $U_M(0)$ in this case are

$$\begin{aligned} U_L(0) &= (G|_{24}{}^{14}Q_3 + G|_{42}{}^{13}Q_4)/G|_{24}{}^{34} - Q_1 \\ \lambda U_M(0) &= (G|_{24}{}^{24}Q_3 + G|_{42}{}^{23}Q_4)/G|_{24}{}^{34} - Q_2 \end{aligned} \quad (4.16)$$

where

$$Q_j = \left[(A_{13}/A_{33} - 1)V_{j1} + (A_{11} - A_{13}^2/A_{33})V_{j4} \right] / 2 \quad j = 1, 2, 3, 4$$

The surface displacement components are found to be

$$\begin{aligned} u_y^3 &= - \int_0^{+\infty} \left[(G|_{24}{}^{24}Q_3 + G|_{42}{}^{23}Q_4)/G|_{24}{}^{34} - Q_2 \right] \sin \beta y \, d\beta \\ u_z^3 &= \int_0^{+\infty} \left[(G|_{24}{}^{14}Q_3 + G|_{42}{}^{13}Q_4)/G|_{24}{}^{34} - Q_1 \right] \cos \beta y \, d\beta \end{aligned} \quad (4.17)$$

4.4.2. Antiplane strain

In this case, the Green's functions corresponding to the elementary sources (1,2) and (1,3) are required to obtain the surface displacement field (4.6).

(1) For source (1,2), the expansion coefficient $U_N(0)$ is found to be

$$U_N(0) = A_{66} (G_{12}^L V_{22}^L / G_{22}^L - V_{12}^L) / \lambda \quad (4.18)$$

and therefore the surface displacement component is

$$u_x^1 = -A_{66} \int_0^{+\infty} (G_{12}^L V_{22}^L / G_{22}^L - V_{12}^L) \sin \beta y \, d\beta \quad (4.19)$$

(2) For source (1,3), the expansion coefficient $U_N(0)$ is given by

$$U_N(0)/i = \text{sign}(\beta) (G_{12}^L V_{21}^L / G_{22}^L - V_{11}^L) / \lambda \quad (4.20)$$

and the surface displacement component is found to be

$$u_x^4 = \int_0^{+\infty} (G_{12}^L V_{21}^L / G_{22}^L - V_{11}^L) \cos \beta y \, d\beta \quad (4.21)$$

5. Discussion and conclusion

The results of Paper I have been used to solve the corresponding deformation problem by internal sources in a transversely isotropic and layered half-space. The source functions of general point dislocations in a transversely isotropic medium are derived in the Cartesian system and the cylindrical system of vector functions by a unified and simple approach. Explicit expressions for the surface displacements due to three- and two-dimensional dislocation sources are obtained in these two systems of vector functions in terms of propagator matrices. Whereas the same procedure can be applied to get the displacement and stress fields at any point in the medium, the results for a finite fault can be obtained by integration over the fault area. It is shown that the present solution contains the solutions to the three- and two-dimensional source problems for the corresponding isotropic media, and therefore provides a unified solution for these two problems, which have long been studied separately. The formulation developed can be calculated using the numerical procedure of Jovanovich et al. (1974a, b) to study the effects of anisotropy, as well as of Earth layering, on the static fields.

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Appendix 1

In the Cartesian system of vector functions, the coefficients F_L^δ , F_M^δ , F_N^δ , F_L^d , F_M^d and F_N^d in eqn. (3.4) are given respectively by (omitting factor $\Delta u \, d\Sigma$)

$$F_L^\delta = (2\pi)^{-1} n_i v_j \left\{ \left(c_{ijzx} \frac{\partial}{\partial x} + c_{ijzy} \frac{\partial}{\partial y} \right) \exp[i(\alpha x + \beta y)] \right\}_0$$

$$F_M^\delta = \lambda^{-2} n_i v_j \left\{ \left(c_{ijxx} \frac{\partial^2}{\partial x^2} + 2c_{ijxy} \frac{\partial^2}{\partial x \partial y} + c_{ijyy} \frac{\partial^2}{\partial y^2} \right) S^* \right\}_0$$

$$F_N^\delta = -\lambda^{-2} n_i v_j \left\{ \left[c_{ijxy} \left(\frac{\partial^2}{\partial x^2} - \frac{\partial^2}{\partial y^2} \right) + (c_{ijyy} - c_{ijxx}) \frac{\partial^2}{\partial x \partial y} \right] S^* \right\}_0$$

$$F_L^d = -(2\pi)^{-1} n_i v_j c_{ijzz} \{ \exp[i(\alpha x + \beta y)] \}_0$$

$$F_M^d = -\lambda^{-2} n_i v_j \left\{ \left(c_{ijxz} \frac{\partial}{\partial x} + c_{ijyz} \frac{\partial}{\partial y} \right) S^* \right\}_0$$

$$F_N^d = \lambda^{-2} n_i v_j \left\{ \left(c_{ijyz} \frac{\partial}{\partial x} - c_{ijxz} \frac{\partial}{\partial y} \right) S^* \right\}_0$$

where $\{-\}_0$ implies $x = x_0$, $y = y_0$; S^* is the complex conjugate of the scalar function defined by eqn. (9) of Paper I, that is

$$S^* = \exp[i(\alpha x + \beta y)] / (2\pi)$$

Appendix 2

In the cylindrical system of vector functions, the coefficients F_L^δ , F_M^δ , F_N^δ , F_L^d , F_M^d and F_N^d in eqn. (3.4) are given respectively by (omitting factor $\Delta u \, d\Sigma$)

$$F_L^\delta = (2\pi)^{-1/2} \left\{ n_i v_j c_{ijzr} \frac{\partial}{r \partial r} [J_m(\lambda r) r] \exp(-im\theta) + c_{ijz\theta} J_m(\lambda r) \frac{\partial}{r \partial \theta} [n_i v_j \exp(-im\theta)] \right\}_0$$

$$F_M^\delta = \lambda^{-2} \left\{ n_i v_j c_{ijrr} \frac{\partial}{r \partial r} \left(r \frac{\partial S^*}{\partial r} \right) + c_{ijr\theta} \frac{\partial}{r \partial \theta} \left(n_i v_j \frac{\partial S^*}{\partial r} \right) + n_i v_j c_{ij\theta r} \frac{\partial^2 S^*}{r \partial r \partial \theta} + c_{ij\theta\theta} \frac{\partial}{r^2 \partial \theta} \left(n_i v_j \frac{\partial S^*}{\partial \theta} \right) \right\}_0$$

$$F_N^\delta = -\lambda^{-2} \left\{ n_i v_j c_{ijr\theta} \frac{\partial}{r \partial r} \left(r \frac{\partial S^*}{\partial r} \right) + c_{ij\theta\theta} \frac{\partial}{r \partial \theta} \left(n_i v_j \frac{\partial S^*}{\partial r} \right) - n_i v_j c_{ijrr} \frac{\partial^2 S^*}{r \partial r \partial \theta} - c_{ijr\theta} \frac{\partial}{r^2 \partial \theta} \left(n_i v_j \frac{\partial S^*}{\partial \theta} \right) \right\}_0$$

$$F_L^d = -(2\pi)^{-1/2} \left\{ n_i v_j c_{ijzz} J_m(\lambda r) \exp(-im\theta) \right\}_0$$

$$F_M^d = -\lambda^{-2} \left\{ n_i v_j \left(c_{ijrz} \frac{\partial}{\partial r} + c_{ij\theta z} \frac{\partial}{r \partial \theta} \right) S^* \right\}_0$$

$$F_N^d = \lambda^{-2} \left\{ n_i v_j \left(c_{ij\theta z} \frac{\partial}{\partial r} - c_{ijrz} \frac{\partial}{r \partial \theta} \right) S^* \right\}_0$$

where $\{-\}_0$ implies $r = r_0$, $\theta = \theta_0$; $J_m(\lambda r)$ denotes the Bessel function of order m ; S^* is the complex conjugate of the scalar function defined by eqn. (12) of Paper I, that is

$$S^* = J_m(\lambda r) \exp(-im\theta) / (2\pi)^{1/2} \quad m = 0, \pm 1, \pm 2, \dots$$

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