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Dislocation in an infinite poroelastic medium

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Summary. First a generalized Volterra relation for a transient and coupled poroelastic medium of two and three dimensions is derived from a reciprocal theorem, which reveals a direct connection of the solid displacement due to solid and fluid point dislocations with the stress and pore pressure due to a point force. A decomposition technique is then employed to find the complete fundamental solution of fluid and solid point dislocations. The solutions are provided in an exact closed form and are needed as influence functions in the displacement discontinuity method in two- and three-dimensional poroelastic media.

1 Introduction

The displacement discontinuity method (DDM), as a means of solving boundary value problems in elasticity, has become a popular numerical method in the field of geomechanics. The most significant characteristics is its ability of handling rock discontinuities and fractures [1]. The successful application of the DDM, however, relies on the derivation of the fundamental solution for a displacement discontinuity singularity, or mathematically a point dislocation in the infinite space.

Dislocation in a homogeneous, isotropic and linear elastic medium has been studied by many investigators. An early discussion on the Volterra dislocaton in a semi-infinite space of three dimensions was given by Steketee [2]. He found that solutions to six dislocation nuclei are required for a general problem and he constructed the solution to one of them by making use of Volterra relation [2]. Maruyama [3] derived the remaining five solutions, and proved that the Volterra relation is valid not only for a Volterra dislocation, but also for a Somigliana dislocation [3]. The corresponding two-dimensional problem was also studied by the same author [4]. Maruyama's work [3], [4] has been extended by Singh [5] and Singh and Garg [6] to an isotropic and layered half-space of three and two dimensions, respectively and by Pan [7] to the corresponding transversely isotropic medium.

While the solution for the dislocation in a pure elastic medium has been considered in great detail, comparatively little work has been done on similar problems in a poroelastic medium. Recently, Detournay and Cheng [8] derived the poroelastic solution of a point displacement discontinuity in a two dimensional space. The methodology used by the authors is based on a decomposition technique proposed by Biot [9]. This solution has been used by Vandamme et al. [10] in their DDM program to model the opening of a hydraulic fracture in a poroelastic medium. It is noted that in using the result in [8], however, two subsequent coordinate transformations between the local (crack) and the global system are required for an arbitrary crack line, which would make the numerical calculation complicated [10].

The objective of this paper is to derive the fundamental solutions of a point dislocation in two- and three-dimensional poroelastic media. The methodology is based on a generalized Volterra relation from the reciprocal theorem, and on the variable decomposition technique [9].

2 Basic equations

The theory of linearized quasi-static poroelasticity was first introduced by Biot [11], [12] for modelling the response of fluid-saturated porous solids. The isotropic version of the Biot model [11], [12] has been rationalized by Rice and Cleary [13] with a consistent set of five material parameters. As in the original formulation [11], [13], we choose the basic dynamic variables to be the total stress σ_{ij} and the pore pressure p. The corresponding conjugate kinematic quantities are the solid strain e_{ij}, derivable from a solid displacement vector u_i, and the variation of fluid volume per unit reference volume, ξ. The five material parameters are: the shear modulus G, the drained and undrained Poisson's ratio ν and ν_u, Skempton's pore pressure coefficient B and the permeability coefficient k. The equilibrium equation and Darcy's law with body forces can be expressed, respectively, as [13]

$$\sigma_{ij,j} = -F_i \tag{1}$$

$$q_i = -k(p_{,i} - f_i) \tag{2}$$

where the summation convention is implied with commas indicating spatial differentiation; F_i and f_i are body forces per unit volume acting on the mixture (fluid and solid) and fluid, respectively; q_i is the specific discharge, which multiplied by the fluid mass indicates the fluid mass flow rate in the x_i direction [13].

The constitutive relations in the Rice-Cleary formulation of the Biot model can be shown to take the forms [13]

$$\sigma_{ij} = 2Ge_{ij} + [2G\nu/(1-2\nu)]\,\delta_{ij}e - \alpha\delta_{ij}p \tag{3}$$

$$p = -\{2GB(1+\nu_u)/[3(1-2\nu_u)]\}\,e + \{2GB^2(1-2\nu)\,(1+\nu_u)^2/[9(\nu_u-\nu)\,(1-2\nu_u)]\}\,\xi \tag{4}$$

where δ_{ij} is the Kronecker's delta, and α is the Biot coefficient of effective stress [14], defined as

$$\alpha = 3(\nu_u - \nu)/[B(1-2\nu)\,(1+\nu_u)]. \tag{5}$$

The solid strain e_{ij} is related to u_i by

$$e_{ij} = (u_{i,j} + u_{j,i})/2 \tag{6}$$

and $e = e_{ii}$. It is noted that in writing Eq. (4) we have chosen ξ, instead of the variation of fluid mass [13], as the second kinematic quantity.

The final equation needed to complete the poroelasticity theory of Biot is that of fluid-mass conservation, which, on the basis of the present notation, can be shown to be [15]

$$\partial\xi/\partial t + q_{i,i} = \gamma \tag{7}$$

in which γ is the rate of injected fluid volume from fluid source.

Equation (7) can also be expressed in an alternative form by integrating it with respect to time, i.e.

$$\xi - \xi_0 + v_{i,i} = Q \tag{8}$$

where ξ_0 is the initial value of ξ at $t = 0$;

$$v_i = \int_0^t q_i \, dt \tag{9}$$

is the relative fluid displacement, and

$$Q = \int_0^t \gamma \, dt \tag{10}$$

the volume of injected fluid.

In summary, the basic formulation of the theory of Biot for linear, isotropic and quasi-static poroelasticity consists of Eqs. (1)—(7). This set of equations with the selected variables and parameters is quite convenient in some applications [13], and yet, as emphasized by Rice and Cleary [13], does not prohibit the continuum viewpoint, i.e. it is possible to compare the present formulation with those from the continuum theory of mixtures [16], [17]: While the continuity Eq. (7) is obviously equivalent to the linearized mass-conservation law for the fluid constituent [15], the constitutive Eqs. (3) and (4) are coincident with the reduced results of the general porous media models in the present circumstances [16, Eqs. (9.23), (9.24)], apart from minor differences in notation and an error in [16, Eq. (9.24)] probably due to misprint. The governing Eqs. (1) and (2) can also be derived from the theory of mixtures. The simplest approach to do so is to make direct use of some Beskos' linearized results [17, Eqs. (24)—(27), (32)]. But one should add body force terms to and neglect inertia terms in the linear momentum balance Eq. [17], and identify appropriately the variables and parameters involved. We shall, however, omit details of the procedure as they are very straightforward and turn our attention to the reciprocal relations based on the basic equations.

3 Reciprocal theorem and representation relations

We start with a generalized reciprocal theorem of Betti type in poroelasticity that can be expressed as

$$\sigma_{ij}^1 e_{ij}^2 + p^1 \xi^2 = \sigma_{ij}^2 e_{ij}^1 + p^2 \xi^1 \tag{11}$$

where superscript 1 and 2 denote two independent systems of the field quantities. It is shown that this relation is a direct consequence of the constitutive Eqs. (3) and (4). In addition to reciprocity in space, we shall also require the two systems to be reciprocal in time. Following Burridge and Knopoff [18], we define two adjoint systems: while the first adjoint system is identical to the original one, the time variable t is replaced by $-t$ to get the second adjoint system $u_i^2(\boldsymbol{x}, -t)$, $F_i^2(\boldsymbol{x}, -t)$, etc.

Using these two systems and their corresponding Eqs. (1)—(10), we can integrate the reciprocal relation (11), with respect to space and time, to obtain

$$\iint [(\sigma_{ij}^1 \bar{u}_i^2 - \bar{\sigma}_{ij}^2 u_i^1) \, n_j - (p^1 \bar{v}^2 - \bar{p}^2 v^1)] \, dS \, dt$$
$$+ \iint [(F_i^1 \bar{u}_i^2 - \bar{F}_i^2 u_i^1) + (f_i^1 \bar{v}_i^2 - \bar{f}_i^2 v_i^1) + (p^1 \bar{Q}^2 - \bar{p}^2 Q^1)] \, dV \, dt = 0 \tag{12}$$

where an overbar has been used to indicate the second adjoint system, i.e. $\bar{u}_i^2(\boldsymbol{x}, t) = u_i^2 \times (\boldsymbol{x}, -t)$, $\bar{F}_i^2(\boldsymbol{x}, t) = F_i^2(\boldsymbol{x}, -t)$, etc.; n_j is the unit outward normal of the boundary; dS and dV denote the boundary and domain integral, respectively; $v = v_i n_i$ is the normal component of fluid displacement. It is noted that in obtaining Eq. (12), we have assumed that the initial value of the variation of fluid content is zero, i.e. $\xi_0^1 = \bar{\xi}_0^2 = 0$.

If, for the second system, we set $f_i^2 = Q^2 = 0$, and

$$F_i^2(\boldsymbol{x}, t) = \delta_{ij}\delta(\boldsymbol{x}, t; \boldsymbol{y}, -\tau) \equiv \delta_{ij}\delta(x_1 - y_1)\,\delta(x_2 - y_2)\,\delta(x_3 - y_3)\,\delta(t + \tau) \tag{13}$$

where $\boldsymbol{y}$ is a point in V, and δ the Dirac delta function, then $\bar{f}_i^2 = \bar{Q}^2 = 0$,

$$\bar{F}_i^2(\boldsymbol{x}, t) = \delta_{ij}\delta(\boldsymbol{x}, t; \boldsymbol{y}, \tau) \tag{14}$$

and Eq. (12) becomes

$$\begin{aligned} u_j(\boldsymbol{y}, \tau) = \iint dS(\boldsymbol{x})\, dt\{&[\sigma_{ik}(\boldsymbol{x}, t)\, {}^F u_i^j(\boldsymbol{x}, -t; \boldsymbol{y}, -\tau) \\ &- u_i(\boldsymbol{x}, t)\, {}^F\sigma_{ik}^j(\boldsymbol{x}, -t; \boldsymbol{y}, -\tau)]\, n_k(\boldsymbol{x}) + [v(\boldsymbol{x}, t)\, {}^F p^j(\boldsymbol{x}, -t; \boldsymbol{y}, -\tau) \\ &- p(\boldsymbol{x}, t)\, {}^F v^j(\boldsymbol{x}, -t; \boldsymbol{y}, -\tau)]\} + \iint dV(\boldsymbol{x})\, dt[F_i(\boldsymbol{x}, t)\, {}^F u_i^j(\boldsymbol{x}, -t; \boldsymbol{y}, -\tau) \\ &+ f_i(\boldsymbol{x}, t)\, {}^F v_i^j(\boldsymbol{x}, -t; \boldsymbol{y}, -\tau) - Q(\boldsymbol{x}, t)\, {}^F p^j(\boldsymbol{x}, -t; \boldsymbol{y}, -\tau)] \end{aligned} \tag{15}$$

where the superscripts 1 and 2 have been dropped, and the left superscript F has been used to denote the Green's functions. For instance, ${}^F u_i^j(\boldsymbol{x}, -t; \boldsymbol{y}, -\tau)$ is the solid displacement in the i-direction at $(\boldsymbol{x}, -t)$ due to an instantaneous point force of unit impulse in the j-direction at $(\boldsymbol{y}, -\tau)$. The point force acting on the mixture (solid and fluid) will hereafter be referred to as point force for simplicity.

Similar procedure as above can be used to derive other two integral equations related to the Green's functions of a fluid dilation and of a fluid body force [19]. In the following, however, we will devote our effort to the derivation of the representation relations between the Green's functions of a point force and those of a point dislocation.

If we further set $F_i(\boldsymbol{x}, t) = f_i(\boldsymbol{x}, t) = Q(\boldsymbol{x}, t) = 0$, Eq. (15) is then simplified to

$$\begin{aligned} u_j(\boldsymbol{y}, \tau) = \iint dS(\boldsymbol{x})\, dt\{&[\sigma_{ik}(\boldsymbol{x}, t)\, {}^F u_i^j(\boldsymbol{x}, -t; \boldsymbol{y}, -\tau) \\ &- u_i(\boldsymbol{x}, t)\, {}^F\sigma_{ik}^j(\boldsymbol{x}, -t; \boldsymbol{y}, -\tau)]\, n_k(\boldsymbol{x}) + [v(\boldsymbol{x}, t)\, {}^F p^j(\boldsymbol{x}, -t; \boldsymbol{y}, -\tau) \\ &- p(\boldsymbol{x}, t)\, {}^F v^j(\boldsymbol{x}, -t; \boldsymbol{y}, -\tau)]\}\,. \end{aligned} \tag{16}$$

Let it be assumed that we wish to find the response from prescribed discontinuities in the solid and fluid displacements across a surface (or a curve in two-dimensional space)

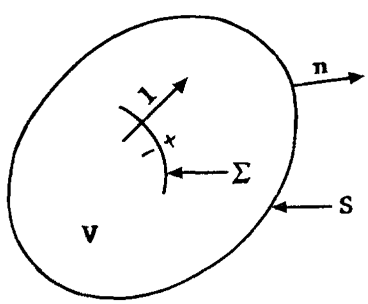


Fig. 1. Schematic diagram of boundaries and domain used in the integration. $\boldsymbol{n}$ is the unit outward normal vector of the boundary S, $\boldsymbol{l}$ the unit normal of the surface Σ imbedded in V. Solid and fluid displacements may be discontinuous across Σ

Σ imbedded in V (Fig. 1). Let $\boldsymbol{l}$ be the unit normal to Σ, and $[u_i]$ $(\boldsymbol{x}, t)$ and $[v]$ $(\boldsymbol{x}, t)$ be the discontinuities in u_i and $v = v_i l_i$ across Σ in the direction of $\boldsymbol{l}$ at $(\boldsymbol{x}, t)$. These discontinuities may have any form, provided that the following relations hold:

$$\begin{aligned} &\sigma_{ij}^{+} l_j^{+} + \sigma_{ij}^{-} l_j^{-} = 0 \\ &p^{+} - p^{-} = 0 \end{aligned} \tag{17}$$

where $-l_j^{+} = l_j^{-} = l_j$. We call the discontinuity satisfying relation (17) the generalized Somigliana dislocation. It is an extension to poroelasticity of the Somigliana dislocation in elasticity [3].

Assume v, u_i, p and σ_{ik} satisfy the same homogeneous boundary condition on S, and apply Eq. (16) to the region bounded internally by Σ and externally by S. ${}^F v^j$ and ${}^F u_i^j$ do not have prescribed discontinuities on Σ. Then the boundary integral over S vanishes and we are left with the surface integral over Σ only:

$$\begin{aligned} u_j(\boldsymbol{y}, \tau) = \iint d\Sigma(\boldsymbol{x})\, dt \{ &[u_i]\,(\boldsymbol{x}, t)\, {}^F\sigma_{ik}^{j}(\boldsymbol{x}, -t; \boldsymbol{y}, -\tau)\, l_k(\boldsymbol{x}) \\ &- [v]\,(\boldsymbol{x}, t)\, {}^F p^j(\boldsymbol{x}, -t; \boldsymbol{y}, -\tau)\}\,. \end{aligned} \tag{18}$$

This is a representation relation that gives the solid displacements due to instantaneous solid and fluid dislocations in terms of the Green's functions of a point force. The variables in the functions should, however, be changed to those in normal sense. It is done by direct observation of the expressions of ${}^F\sigma_{ik}^{j}$ and ${}^F p^j$ in [19]. Equation (18) thus becomes

$$u_j(\boldsymbol{y}, \tau) = \iint d\Sigma(\boldsymbol{x})\, dt\{-[u_i]\,(\boldsymbol{x}, t)\, {}^F\sigma_{ik}^{j}(\boldsymbol{y}, \tau; \boldsymbol{x}, t)\, l_k(\boldsymbol{x}) + [v]\,(\boldsymbol{x}, t)\, {}^F p^j(\boldsymbol{y}, \tau; \boldsymbol{x}, t)\}\,. \tag{19}$$

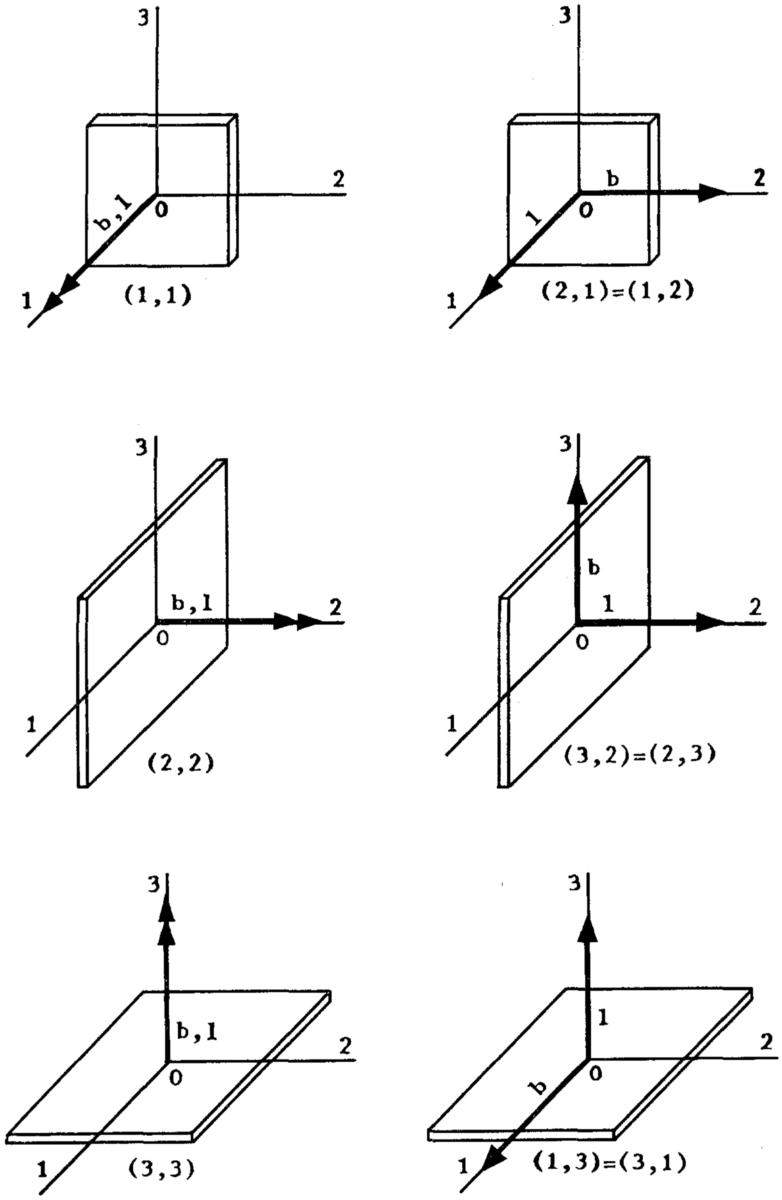
Equation (19) is called a generalized Volterra relation, for it is a direct extension to poroelasticity of the Volterra relation in elasticity [2]. In this equation, ${}^F\sigma_{ik}^{j}(\boldsymbol{y}, \tau; \boldsymbol{x}, t)$ and ${}^F p^j(\boldsymbol{y}, \tau; \boldsymbol{x}, t)$ are the total stress σ_{ik} and the pressure p, respectively, at $(\boldsymbol{y}, \tau)$ due to an instantaneous point force of unit impulse in the j-direction at $(\boldsymbol{x}, t)$. With the point force solution being known [19], we can immediately obtain, from Eq. (19), the solid displacement field due to a point dislocation. That is, the fundamental solid displacement in the j-direction at $(\boldsymbol{y}, \tau)$ due to an instantaneous fluid point dislocation and solid point dislocation (i, k), of unit impulse at $(\boldsymbol{x}, t)$ are

$${}^d u_j(\boldsymbol{y}, \tau; \boldsymbol{x}, t) = {}^F p^j(\boldsymbol{y}, \tau; \boldsymbol{x}, t) \tag{20}$$

and

$${}^d u_j^{ik}(\boldsymbol{y}, \tau; \boldsymbol{x}, t) = -{}^F\sigma_{ik}^{j}(\boldsymbol{y}, \tau; \boldsymbol{x}, t) \tag{21}$$

respectively. While the physical meaning of a fluid point dislocation is apparent (Fig. 2), some explanations for a solid point dislocation are needed. A general solid point dislocation can be described by a dyadic $\boldsymbol{bl}$, where $\boldsymbol{b}$ is the unit vector in the direction of solid dislocation, and $\boldsymbol{l}$ the unit normal to the dislocation surface. Though there are nine elements in $\boldsymbol{bl}$, only six are independent since $\sigma_{ik} = \sigma_{ki}$. These six elements are therefore called elementary dislocations [2], [3]. They are marked as (i, k) and illustrated in Fig. 2. While the first letter in (i, k) indicates the i-direction of the dislocation, the second indicates the k-direction of the normal to the dislocation surface. Notice that by $(i, k) = (k, i)$ is meant that the response from the dislocation (i, k) is identical with that from the dislocation (k, i).



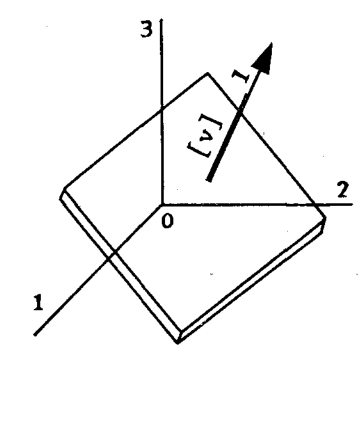


Fig. 2. Definitions of the basic solid and fluid dislocations. $\boldsymbol{b}$ is the unit vector of the solid dislocation $[\boldsymbol{u}]$ and $\boldsymbol{l}$ the unit normal of the dislocation surface. $[v]$ is the fluid dislocation, i.e. the normal discontinuity of the fluid displacement across the dislocation surface

4 Fundamental solutions

We have shown, in the foregoing, that the fundamental solid displacements due to solid and fluid point dislocations can be obtained directly from the total stress and pore pressure solutions of a point force which have been derived in [19]. However, it is seen from the basic equations in Section 2, that other field quantities can not be obtained directly from these equations. We thus employ the decomposition technique [9] to solve this problem.

By combining Eqs. (1)—(7), we obtain the following two equations

$$G\nabla^2 u_i + [G/(1-2\nu_u)]\, e_{,i} - \{2GB(1+\nu_u)/[3(1-2\nu_u)]\}\, \xi_{,i} = -F_i \tag{22}$$

$$\partial\xi/\partial t - c\nabla^2\xi = \{kB(1+\nu_u)/[3(1-\nu_u)]\}\, F_{i,i} - kf_{i,i} + \gamma \tag{23}$$

where

$$c = 2kGB^2(1-\nu)\,(1+\nu_u)^2/[9(1-\nu_u)\,(\nu_u-\nu)]$$

is a generalized consolidation coefficient [13].

We now introduce the following decomposition [9] which allows further uncoupling of the field Eqs. (22) and (23):

$$u_i = u_i^e + \Delta u_i = u_i^e + \{B(1+\nu_u)/[3(1-\nu_u)]\}\, \Phi_{,i}. \tag{24}$$

If we require the first part of the solid displacement field to satisfy the Navier equation of elasticity with undrained coefficient, i.e.,

$$G\nabla^2 u_i^e + [G/(1-2\nu_u)]\, e^e_{,i} = -F_i \tag{25}$$

it can then be proved from Eq. (22) that the second part is governed by

$$\xi = \nabla^2\Phi. \tag{26}$$

Substituting Eqs. (26) into (23), and relaxing a Laplacian leads to a diffusion equation for Φ,

$$\partial\Phi/\partial t - c\nabla^2\Phi = g_1 + g_2 + g_3 + \psi \tag{27}$$

in which,

$$\nabla^2 g_1 = \{kB(1+\nu_u)/[3(1-\nu_u)]\}\, F_{i,i}, \qquad \nabla^2 g_2 = -kf_{i,i},$$
$$\nabla^2 g_3 = \gamma, \qquad \nabla^2\psi = 0. \tag{28}$$

The above results suggest that the solid displacement field can be decomposed into an "undrained" part, u_i^e, satisfying an elasticity equation with undrained coefficient, and an irrotational part derivable from a potential, Φ, that is governed by a diffusion equation. This decomposition technique was originally proposed by Biot [9], and recently used by Cheng and Predeleanu [19] and Detournay and Cheng [8] for the derivation of the solutions of a point force and of a two-dimensional solid point dislocation, respectively.

Since the second part of the solid displacement is of a potential field, we can find the potential by an integral of the irrotational part once the total solid displacement field is known. Then the variation of fluid content can be obtained from Eq. (26). A direct relation between the variation of fluid content and the second part of the solid displacement is therefore found to be

$$\xi = \{3(1-\nu_u)/[B(1+\nu_u)]\}\, \Delta u_{i,i}. \tag{29}$$

The formula needed to derive the relative fluid displacement can be obtained by substituting Eq. (29) into (8)

$$v_{i,i} = Q - \{3(1-\nu_u)/[B(1+\nu_u)]\}\,\Delta u_{i,i}. \tag{30}$$

Again we have assumed $\xi_0 = 0$.

With the above preparation, we can proceed to derive the Green's functions of a fluid and solid dislocation. Because in the antiplane strain case there exists only the elastic field, discussion on it can be neglected in poroelasticity. In the following, therefore, we use the term "two dimensional" exclusively for "plane strain", and superscripts and subscripts in two and three dimensions vary from 1 to 2 and 1 to 3, respectively.

4.1 Fluid point dislocation

The pore pressure due to an instantaneous point force of unit impulse has been derived in two and three dimensions by Cheng and Predeleanu [19]. The solid displacement in the i-direction at $(\boldsymbol{y}, \tau)$ due to an instantaneous fluid point dislocation of unit impulse at $(\boldsymbol{x}, t)$ is thus obtained from the representation relation (20)

$$^{d}u_i(\boldsymbol{y}, \tau; \boldsymbol{x}, t) = \{B(1+\nu_u)\,r_i/[3\pi(2r)^{m-1}\,(1-\nu_u)]\}\,[\delta(\tau - t) - f_m z E(z)/(\tau - t)] \tag{31}$$

where $m = 2$ and 3 correspond to the two- and three-dimensional deformations, respectively:

$$r = |\boldsymbol{y} - \boldsymbol{x}|, \qquad r_i = r_{,i} = \partial r/\partial y_i = (y_i - x_i)/r; \qquad z = r^2/[4c(\tau - t)], \qquad E(z) = \exp(-z);$$

$$f_2 = 1, \qquad f_3 = r/[\pi c(\tau - t)]^{1/2}.$$

It is apparent that the first (proportional to $\delta(\tau - t)$) and second terms on the right-hand side of Eq. (31) correspond to the elastic and potential displacements, respectively. The variation of fluid content is therefore obtained by substituting the above potential displacement into Eq. (29):

$$^{d}\xi(\boldsymbol{y}, \tau; \boldsymbol{x}, t) = -f_m E(z)\,(m - 2z)/[8\pi(2r)^{m-2}\,c(\tau - t)^2] \tag{32}$$

and the pore pressure from the constitutive Eq. (4):

$$^{d}p(\boldsymbol{y}, \tau; \boldsymbol{x}, t) = -f_m E(z)\,(m - 2z)/[8\pi(2r)^{m-2}\,k(\tau - t)^2]. \tag{33}$$

The normal component of the relative fluid displacement $v_i l_i$ is derived from Eq. (30)

$$^{d}v(\boldsymbol{y}, \tau; \boldsymbol{x}, t) = r_n f_m z E(z)/[\pi(2r)^{m-1}\,(\tau - t)] \tag{34}$$

where $r_n = \partial r/\partial l = r_i l_i$ is the normal differentiation.

Finally, we obtain the total stress from Eq. (3):

$$^{d}\sigma_{ij}(\boldsymbol{y}, \tau; \boldsymbol{x}, t) = \{GB(1+\nu_u)/[3\pi(2r)^{m-2}\,(1-\nu_u)\,r^2]\}\,\{(\delta_{ij} - m r_i r_j)\,\delta(\tau - t)$$
$$+ f_m z E(z)\,[(m-1)\,\delta_{ij} - 2z(\delta_{ij} - r_i r_j)]/(\tau - t)\}. \tag{35}$$

Notice that, except for the expression of v, the present Green's functions of a fluid point dislocation are the same as those of a fluid dilation derived in [19]. However, the expressions of v would be also identical if the term proportional to $\delta(\tau - t)$ were not included by mistake in the expression of v in [19]. The identity of these two Green's functions implies an equivalence of a fluid point dislocation with a fluid dilation.

4.2 Solid point dislocation

The total stress caused by an instantaneous point force of unit impulse is also available [19], we therefore have the solid displacement in the i-direction at $(\boldsymbol{y}, \tau)$ due to an instantaneous solid point dislocation (k, l) of unit impulse at $(\boldsymbol{x}, t)$, from the representation relation (21):

$$\begin{aligned}{}^{d}u_i^{kl}(\boldsymbol{y}, \tau; \boldsymbol{x}, t) = {} & \{1/[4\pi(2r)^{m-2}(1-\nu_u)\, r]\}\, [(1-2\nu_u)(-\delta_{kl}r_i + \delta_{ik}r_l + \delta_{il}r_k) \\ & + m r_i r_k r_l]\, \delta(\tau - t) + \{c(\nu_u - \nu)\, f_m/[2\pi(2r)^{m-2}(1-\nu_u)(1-\nu)]\} \\ & \times \big[\big(\delta_{kl}r_i + \delta_{ik}r_l + \delta_{il}r_k - (m+2)\, r_i r_k r_l\big)\, g_m \\ & - zE(z)\, r_i(\delta_{kl} - r_k r_l)/\big(cr(\tau - t)\big)\big] \end{aligned} \tag{36}$$

where

$$g_2 = 2[1 - E(z) - zE(z)]/r^3$$

$$g_3 = \Big\{-E(z) + (3/r^3)\int_0^r x^2 \exp\big[-x^2/\big(4c(\tau - t)\big)\big]\, dx\Big\}/[2cr(\tau - t)].$$

Proceeding the same way as for the fluid point dislocation case, we have found other field quantities needed in boundary integral formulation:

$$\begin{aligned}{}^{d}\xi^{kl}(\boldsymbol{y}, \tau; \boldsymbol{x}, t) = {} & \{3(\nu_u - \nu)\, f_m E(z)/[8\pi Bc(2r)^{m-2}(1+\nu_u)(1-\nu)\, t^2]\} \\ & \times [-(m-1)\,\delta_{kl} + 2z(\delta_{kl} - r_k r_l)] \end{aligned} \tag{37}$$

$$\begin{aligned}{}^{d}p^{kl}(\boldsymbol{y}, \tau; \boldsymbol{x}, t) = {} & \{GB(1+\nu_u)/[3\pi(1-\nu_u)]\}\, \Big\{\big[1/\big(r^2(2r)^{m-2}\big)\big] \\ & \times (-\delta_{kl} + m r_k r_l)\, \delta(\tau - t) + \big[f_m E(z)/\big(4c(2r)^{m-2}(\tau - t)^2\big)\big] \\ & \times [-(m-1)\,\delta_{kl} + 2z(\delta_{kl} - r_k r_l)]\Big\} \end{aligned} \tag{38}$$

$$\begin{aligned}{}^{d}v^{kl}(\boldsymbol{y}, \tau; \boldsymbol{x}, t) = {} & \{-kGB(1+\nu_u)\, f_m/[3\pi(2r)^{m-2}(1-\nu_u)]\} \\ & \times \big[\big(\delta_{kl}r_n + l_k r_l + l_l r_k - (m+2)\, r_n r_k r_l\big)\, g_m \\ & - zE(z)\, r_n(\delta_{kl} - r_k r_l)/\big(cr(\tau - t)\big)\big] \end{aligned} \tag{39}$$

$$\begin{aligned}{}^{d}\sigma_{ij}^{kl}(\boldsymbol{y}, \tau; \boldsymbol{x}, t) = {} & \{G/[2\pi(2r)^{m-2}(1-\nu_u)\, r^2]\}\, [-m(m+2)\, r_i r_j r_k r_l \\ & + m\nu_u(\delta_{il}r_j r_k + \delta_{jl}r_i r_k + \delta_{ik}r_j r_l + \delta_{jk}r_i r_l) + m(1-2\nu_u)(\delta_{ij}r_k r_l + \delta_{kl}r_i r_j) \\ & + (1-2\nu_u)(\delta_{il}\delta_{jk} + \delta_{ik}\delta_{jl}) - (1-4\nu_u)\,\delta_{ij}\delta_{kl}]\, \delta(\tau - t) \\ & + \{cG(\nu_u - \nu)\, f_m/[\pi(1-\nu)(1-\nu_u)(2r)^{m-2}]\}\, \{[\delta_{ij}r_k r_l + \delta_{kl}r_i r_j + \delta_{ik}r_j r_l \\ & + \delta_{jk}r_i r_l + \delta_{il}r_j r_k + \delta_{jl}r_i r_k + (m-2)\,\delta_{ij}\delta_{kl} - (m+4)\, r_i r_j r_k r_l \\ & - 2z(\delta_{ij} - r_i r_j)(\delta_{kl} - r_k r_l)]\, E(z)/[4c^2(\tau - t)^2] \\ & - [(m+2)(\delta_{ij}r_k r_l + \delta_{kl}r_i r_j + \delta_{ik}r_j r_l + \delta_{jk}r_i r_l + \delta_{il}r_j r_k + \delta_{jl}r_i r_k) \\ & - \delta_{ij}\delta_{kl} - \delta_{ik}\delta_{jl} - \delta_{jk}\delta_{il} - (m+2)(m+4)\, r_i r_j r_k r_l]\, g_m/r\}. \end{aligned} \tag{40}$$

It is noted that the first part of the displacement and stress expressions above, proportional to $\delta(\tau - t)$, is identical to the elastic Green's function of a point dislocation with undrained Poisson's ratio [3], [4], [6]. Also we note that for the two-dimensional fundamental solution ($m = 2$), if we let $l = 2$, i.e. fix the normal of the dislocation surface, the present result is then reduced to that obtained in [8], with a sign difference caused by the different definitions for the positive dislocation [8].

5 Boundary integral equation

The fundamental solutions obtained in the preceding section of an instantaneous fluid and solid dislocation can be distributed on the locus of a fracture (crack) to generate a desirable solution field. In particular, the following integral equations can be exploited for the numerical solution of a boundary value problem:

$$\sigma_{ij}(\boldsymbol{y}, t) = \int_0^t \int_\Sigma d\Sigma(\boldsymbol{x})\, d\tau \{[u_k]\, (\boldsymbol{x}, \tau)\, {}^d\sigma_{ij}^{kq}(\boldsymbol{y}, t; \boldsymbol{x}, \tau)\, l_q(\boldsymbol{x}) + [v]\, (\boldsymbol{x}, \tau)\, {}^d\sigma_{ij}(\boldsymbol{y}, t; \boldsymbol{x}, \tau)\} \tag{41}$$

$$p(\boldsymbol{y}, t) = \int_0^t \int_\Sigma d\Sigma(\boldsymbol{x})\, d\tau \{[u_k]\, (\boldsymbol{x}, \tau)\, {}^d p^{kq}(\boldsymbol{y}, t; \boldsymbol{x}, \tau)\, l_q(\boldsymbol{x}) + [v]\, (\boldsymbol{x}, \tau)\, {}^d p(\boldsymbol{y}, t; \boldsymbol{x}, \tau)\}\,. \tag{42}$$

Equations (41) and (42) can be discretized, integrated numerically in both time and spatial coordinates, and collocated for the known stress and pressure boundary conditions along the crack surface. These operations result in the formulation of a linear system of algebraic equations that need to be solved for the fluid and solid displacement jumps at each time step. Once these dislocations along the crack surface are known, displacement, stress, etc. in the poroelastic medium can be evaluated by using equations similar to (41) and (42).

6 Conclusion

The fundamental solutions of fluid and solid point dislocations in two- and three-dimensional poroelastic media have been derived. A generalized Volterra relation that provides the relation between the Green's functions of a point dislocation and those of a point force is first obtained from a reciprocal theorem. The variable decomposition technique suggested by Biot [9] is then employed to find the entire Green's functions in two and three dimensions.

As it has been shown that, in the two dimensional case, the Green's functions provided here will be reduced to those given in [8] if we let one superscript be 2. A direct examination shows that the present result is more general and yet convenient, since the direction of displacement discontinuity and that of the dislocation (crack) surface can be arbitrary. When it is applied to the DDM, no transformation is required between the local (crack) and the global coordinate system [8]. It is therefore convenient for us to handle cracks of any shape.

Since many problems in applied mechanics are not necessarily confined to two-dimensional space, the three-dimensional Green's functions given herein are thus much-needed in the DDM. They can be used in a boundary element procedure to solve general boundary value problems governed by poroelasticity. This numerical technique is particularly appealing for solving problems involving fractures and discontinuities [8], [10].

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