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Mode selection of guided waves for ultrasonic inspection of gas pipelines with thick coating

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Dedicated to Prof. Dr. Franz Ziegler on his 60th birthday

Abstract

Mode selection of guided waves in an elastic steel plate with thick coating layer has been studied here. Effect of both thick elastic coating and damping of the coating layer on the dispersion and mode shapes has been investigated and modes that are less affected by elastic coating and damping have been identified. For modeling the viscoelastic behavior of the coating layer, the standard linear solid is adopted and two damping factors are introduced into the Lamé constants. Viscoelastic property of the coating layer causes all wave numbers to be complex. In order to find the complex wave numbers for a given frequency, a Rayleigh–Ritz type method is used to get approximate values of the complex wave numbers and then the IMSL subroutine *zanlyt* is employed to refine roots. It is found that a thick elastic coating introduces new dispersive modes and also, in general, changes considerably the modes of the bare steel plate, and that damping of the coating layer has significant effect of the dispersion and mode shapes. However, we found that there are certain modes of the bare steel plate that are less affected by the thick elastic coating and damping. These modes could be selected for the ultrasonic inspection of stress-corrosion cracks in thin-walled gas pipelines with protective coal-tar coating.

Keywords: Guided wave; Dispersion; Mode shape; Gas pipeline; Viscoelastic coating; Ultrasonic inspection

1. Introduction

Guided waves in layered plates have been studied extensively in recent years. The interest in this area comes from the need to characterize defects in, and mechanical properties of, layered structures. Ultrasonic technique is highly suitable for such characterization. For successful use of this technique, however, a considerable amount of analytical and experimental work needs to be done in order to develop devices for characterizing material degradation and sizing defects.

For an isotropic or anisotropic and homogeneous plate, the dispersion relation for guided waves was studied by Mindlin and co-workers (Mindlin, 1960; Newman and Mindlin, 1957; Kaul and Mindlin, 1962a, b). For a bilayered isotropic plate, Jones (1964) studied phase velocities vs. wave numbers for various thicknesses of the top layer. Bratton and Datta (1992) discussed further the guided waves in such a bilayered plate and identified new modes dependent on the thickness of the coating layer. Dispersion of waves in alu-

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minum/polymer bilayers was investigated both theoretically and experimentally by Laperre and Thys (1993). Wave propagation in other multilayered structures has also been studied. References can be found in the monographs edited by Mal and Ting (1988) and Datta et al. (1990) and in the review article by Chimenti (1997). Among the theoretical studies, mention may be made of those by Datta et al. (1988), Mal (1988a, b), Karunasena et al. (1991a, b, 1994), and Shull et al. (1994).

Most studies mentioned above are limited to the pure elastic case or when the damping is small. For laminated composites made of polymers or for the earth foundation, however, the effect of material damping can be large. Wave propagation was studied by Aki and Richards (1981) and Carcione et al. (1988) for frequency-dependent viscoelastic media and by Naciri et al. (1994a, b) for viscoelastic and heterogeneous media. The effect of material damping on guided waves was discussed by Tanaka and Kon-No (1980) for a homogenous plate described by a standard linear viscoelastic model and Nkemzi and Green (1992, 1994) studied SH, as well as P-SV motion in a sandwich plate with a viscoelastic core described by a standard linear solid. It may be added that the leaky Lamb wave method for obtaining material attenuation properties of bonded structural composites has been used in recent years (Mal et al., 1989; Xu et al., 1990).

The purpose of this paper is to present a detailed study and identify the modes that are less affected by thick elastic coating with or without damping so that they can be used for ultrasonic inspection of stress-corrosion cracks in gas pipelines. The model considered is a bilayered plate, namely, an elastic steel plate with a thick coating layer, which can be viscoelastic. This is motivated by the need to model guided waves in gas pipelines, which have thick tar-coating. While wave propagation along the longitudinal direction can be analyzed in terms of cylindrical Lamb waves (see, e.g., Alleyne and Cawley, 1996), for guided waves along the circumferential direction a thin-walled pipe can be modeled as a plate when the transmitter/receiver angle of separation is not large (Fig. 1). In this study, the focus is on circumferential waves because they are sensitive to stress-

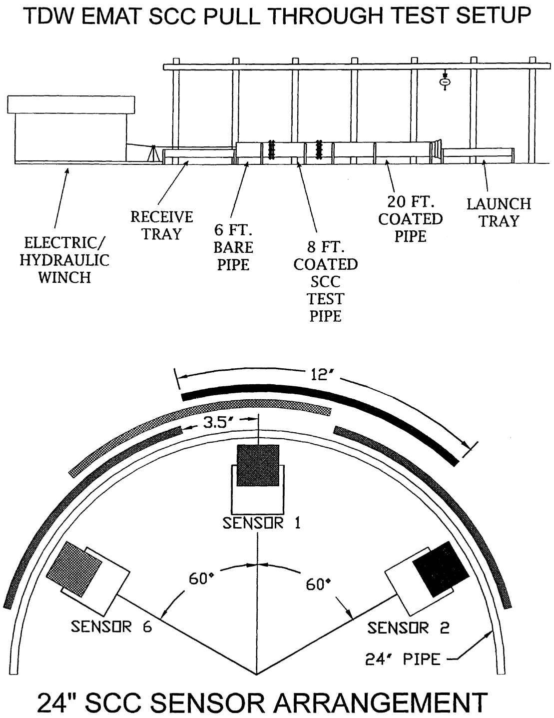

Fig. 1. Sensor arrangement on a gas pipe in the EMAT test set-up.

corrosion cracks, which elongate along the pipe-axis and penetrate the pipe-wall along the radial direction. Longitudinal waves along the pipe would not be sensitive to such defects. To model damping of the tar-coating, the standard linear solid is adopted and two damping factors are introduced into the Lamé constants in such a way that the Poisson's ratio is frequency-independent. Although the coating material (coal-tar) may not be accurately described by a standard linear solid, this model forms the building block (Aki and Richards, 1981) for more complex models. This model has the correct low and high frequency behavior and it satisfies the Kramers–Krönig relation (Aki and Richards, 1981) that must be satisfied by any physical model. In the literature, it has often been the practice to assume frequency independent viscoelastic constants. This is rather ad hoc and may describe the material behavior in some finite range of frequency, but it cannot be physical over the entire range of frequency. Since there is no available model to describe coal-tar, we have chosen to describe it by a standard linear solid. The objective is to see if the modes of the bare steel plate that are not significantly affected by a thick elastic coating do not suffer large changes in the presence of damping.

Based on the formulation derived in this paper, programs have been written to perform parametric study of dispersion of Lamb waves and the modal displacements in a bilayered plate. Comparing the solutions to the steel plate with and without thick elastic coating, we found that thick coating causes new dispersive modes to emerge and changes most mode shapes of the bare steel plate. However, there are certain modes of the steel plate that are much less affected by a thick elastic coating. The effect of damping on these identified modes has also been investigated. We found that its effect on these modes is, in general, to increase slightly the amplitude of displacements in the elastic layer; but some of the mode shapes are preserved even in the case of high damping. These unaffected steel plate modes are particularly useful for the design of ultrasonic electromagnetic acoustic transducer (EMAT) for inspection of stress-corrosion cracks in gas pipelines. The design of such an EMAT based system for gas pipeline inspection is underway at TD Williamson. Fig. 1 shows the schematic of the system.

2. Dispersion relation

In this formulation, we follow the notation of Bratton and Datta (1992). We let the positive x-axis extend to the right along the interface and positive z-axis to extend upward perpendicular to the interface. The top and bottom layers are the coating, which may be viscoelastic, and steel plate, respectively. Using subscripts 1 and 2 for layer 1 (coating) and layer 2 (steel), respectively, the waves propagating in the x–z plane can be described by two scalar potentials for plane strain. In terms of these two potentials, the displacements can be derived as (Bratton and Datta, 1992):

$$u_x^j = \mathrm{i}k[A_j \cos q_j(z \mp h_j) + B_j \sin q_j(z \mp h_j)] - p_j[-C_j \sin p_j(z \mp h_j) + D_j \cos p_j(z \mp h_j)], \tag{1a}$$

$$u_z^j = q_j[-A_j \sin q_j(z \mp h_j) + B_j \cos q_j(z \mp h_j)] + \mathrm{i}k[C_j \cos p_j(z \mp h_j) + D_j \sin p_j(z \mp h_j)], \tag{1b}$$

where $j = 1, 2$ refers to layers 1 and 2, respectively. For $j = 1$, and 2, z belongs to the region $0 \leqslant z \leqslant h_1$ and $-h_2 \leqslant z \leqslant 0$, respectively. Also in Eqs. (1a) and (1b), the sign − (+) is taken when $j = 1$ (2); A_j, B_j, C_j and D_j are coefficients to be determined; k is the wave number; h_1 and h_2 denote the thickness of layers 1 and 2, respectively; and finally q_j and p_j are given by:

$$q_j = \sqrt{\omega^2/C_{\mathrm{L}j}^2 - k^2}, \qquad p_j = \sqrt{\omega^2/C_{\mathrm{T}j}^2 - k^2} \tag{2}$$

with the requirement that $\mathrm{Im}(q_j, p_j) \geqslant 0$. In Eq. (2), ω is the circular frequency and $C_{\mathrm{L}j}$ and $C_{\mathrm{T}j}$ are, longitudinal and transverse velocities, respectively, and given by

$$C_{\mathrm{L}j} = \sqrt{(\lambda_j + 2\mu_j)/\rho_j}, \quad C_{\mathrm{T}j} = \sqrt{\mu_j/\rho_j} \tag{3}$$

with ρ_j being the density, λ_j and μ_j the Lamé constants, of layer j.

The expressions for stress components in both layers can be obtained from the displacement (1) by using the constitutive relation (Bratton and Datta, 1992). Then, the boundary and interface conditions are enforced to derive the dispersion equation. For the current problem, the boundary conditions which must be satisfied are: zero tractions at surfaces $z = +h_1$ and $-h_2$, and the continuity of displacements and tractions at the interface $z = 0$. Application of these conditions yields eight equations with eight unknowns. Expressing four of these unknowns in terms of the other four, the eight equations can be reduced to a system of four equations as expressed in the following way (Bratton and Datta, 1992):

$$\begin{bmatrix} A_{11} & A_{12} & A_{13} & A_{14} \\ A_{21} & A_{22} & A_{23} & A_{24} \\ A_{31} & A_{32} & A_{33} & A_{34} \\ A_{41} & A_{42} & A_{43} & A_{44} \end{bmatrix} \begin{bmatrix} D_1 \\ B_1 \\ D_2 \\ B_2 \end{bmatrix} = \mathbf{0}, \tag{4}$$

where the elements of the matrix $[\mathbf{A}]$ are given in Appendix A for the sake of completeness. Equating the determinant of $[\mathbf{A}]$ to zero gives the dispersion equation relating the frequency to the wave number. This equation is complicated and can only be solved numerically. In this paper, the subroutine in IMSL library, *zanlyt* (IMSL, 1980) is used to find the roots (wave numbers) for a given frequency. Close estimates of the roots were obtained first by the stiffness method employed by Karunasena et al. (1991a). They were then refined by using the exact Eq. (4).

3. Results and discussion

Our analysis is carried out in two steps: step one is to identify the modes that are less affected by a thick elastic coating; and step two is to find, among them, the modes that remain less affected by the damping of the coating layer.

Our formulation and programs are first used to identify the modes of the bare steel plate that preserve their mode shapes when a thick elastic coating is applied to the steel plate.

For a single and a bilayered elastic plate, the parameters are taken from Bratton and Datta (1992) with layers 1 and 2 corresponding to the coating and steel layers, respectively (Table 1). The material properties in the coating layer give a longitudinal velocity $C_{L1} = 2.50$ km/s and a transverse velocity $C_{T1} = 1.00$ km/s while those in the elastic (steel) plate give $C_{L2} = 5.96$ km/s and $C_{T2} = 3.23$ km/s. The dispersion curves are shown in Fig. 2 for a 1 cm elastic (steel) with and without a 0.3 cm elastic coating layer. Results are presented in terms of the normalized wave number $k' = kh_2$ and frequency $\omega' = \omega h_2 / C_{T2}$. While Fig. 2(a) is for the range of $0 \leqslant \omega' \leqslant 15$ and $0 \leqslant k' \leqslant 15$, Fig. 2(b) is for $10 \leqslant \omega' \leqslant 15$ and $0 \leqslant k' \leqslant 60$. It is seen from these figures that the dispersion curves of the bare steel plate are significantly modified when a thick coating layer is applied to the steel plate. It must be borne in mind that guided waves in a coated plate are neither symmetric nor antisymmetric. These modes of the bare steel plate are now coupled. Thus the branches do not cross, but they come very close at certain frequencies and wavelengths, a phenomenon discussed before (Bratton and Datta, 1992). It is also interesting to note that new modes are seen to appear because of the coating. For example, at $\omega' = 5$ and 13, there are 3 and 8 modes in the bare steel plate; for the coating case, there are 6 and 14

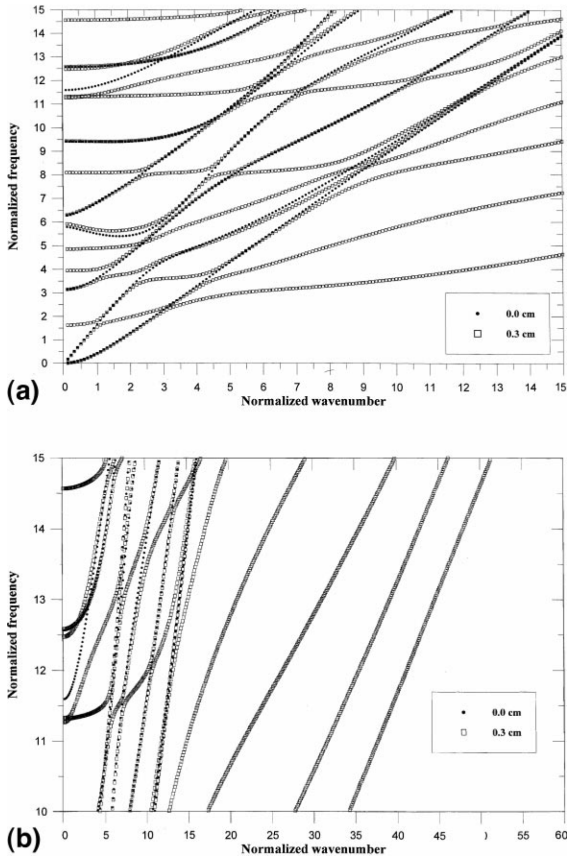

Fig. 2. The dispersion of a 1 cm steel plate with and without a 0.3 cm elastic coating. Range of $0 \leqslant \omega' \leqslant 15$ and $0 \leqslant k' \leqslant 15$ in (a) and of $10 \leqslant \omega' \leqslant 15$ and $0 \leqslant k' \leqslant 60$ in (b).

Table 1
Material properties of the layers

Layer	Thickness (cm)	Density ($\times 10^3$ kg/m^3)	λ (GPa)	μ (GPa)	Damping τ_1
Coating	0.30	1.20	5.100	1.200	0–0.5
Steel	1.00	7.86	115.19	82.003	0

modes. Clearly, these features pose a serious problem for using ultrasonic waves to characterize corrosion cracks in gas pipelines having thick elastic protective coatings. The problem is compounded by the fact that the elastic and damping properties of coating are difficult to measure. So one is interested in finding the modes in the steel plate that are not significantly modified by the elastic coating layer. As we will show, there are such modes even at moderately high frequencies.

Closer examination of Fig. 2(a) and (b) reveals that certain modes of the steel plate persist even in the presence of the thick elastic coating. For example, Table 2 lists the wave numbers when ω' is equal to 10 and 13 having values that are very close to those of the steel plate with and without elastic coating. In order to see which of these modes in the bare steel plate preserve their shapes, mode shapes are plotted in Figs. 3 and 4. It is seen from Fig. 3 that for the bare steel plate the six values of k' at $\omega' = 10$ correspond to the branches a_2, s_2, s_1, a_1, s_0 and a_0. When a thick elastic coating is applied then the mode shopes of a_2, s_2, s_1, a_1 and a_0 are remarkably close while the shape of the s_0 mode of the steel plate has been considerably modified. For higher frequency, i.e., at $\omega' = 13$, it is found that the branches s_1, a_1 and a_0 more or less preserve their shapes. On the other hand, a_2 and s_2 modes suffer substantial changes. Therefore, if the thick coating is purely elastic, the modes corresponding to the branches a_2, s_2, s_1, a_1, and a_0 at $\omega' = 10$ and those corresponding to s_1, a_1, a_0 at $\omega' = 13$ can be used for the ultrasonic inspection of a gas pipeline. The effect of damping in the coating on these modes is analyzed in the following.

After having identified the modes that are less affected by a thick elastic coating, we now come to the second step, namely, to find among them, the

Table 2
Wave numbers k' for frequency $\omega' = 10$ and 13 of a 1 cm steel plate with and without a 0.3 cm elastic coating

ω'	h_1	k'
10	0.0	3.98, 4.26, 5.76, 7.89, 10.66, 10.89
10	0.3	4.00, 4.29, 5.76, 7.88, 10.47, 10.82
13	0.0	6.79, 7.06, 8.96, 11.77, 13.99, 14.07
13	0.3	6.19, 6.82, 7.33, 9.22, 11.79, 13.30, 14.34

modes that are still not appreciably affected by the damping properties of the coating layer. Since the damping properties of the coating are not easily measured and they also are variable (depending on the environment), it is assumed here that the coating behaves like a standard linear solid (Nkemzi and Green, 1992, 1994). While a standard linear solid may not be able to represent the true behavior of real viscoelastic materials, it does, however, exhibit the four most common features of a viscoelastic solid: instantaneous elasticity, creep, stress relaxation, and creep strain. For the standard linear solid, the Lamé constants in the 0.3 cm coating layer are assumed to be

$$\overline{\lambda_1}(\omega') = \lambda_1 \frac{1 - \mathrm{i}\omega'\tau_1}{1 - \mathrm{i}\omega'\tau_2}, \quad \overline{\mu_1}(\omega') = \mu_1 \frac{1 - \mathrm{i}\omega'\tau_1}{1 - \mathrm{i}\omega'\tau_2}, \qquad (5)$$

where λ_1 and μ_1 are the elastic moduli in the absence of damping (Table 1). τ_1 and τ_2 are normalized damping factors, i.e., $\tau_j = t_j C_{T2}/h_2$ with t_1 and t_2 being the creep and stress relaxation times, respectively (Nkemzi and Green, 1992, 1994). This model implies that the Poisson's ratio is frequency-independent. While this assumption is not necessary in the analysis, this is made to keep the number of parameters minimum.

In order to perform a parametric study, we varied $\tau_1 = 0.000$, 0.005, 0.025, 0.050, and 0.500, while τ_2 was taken as $\tau_1/5$ (Nkemzi and Green, 1992). [1] The first case corresponds to the pure elastic behavior. It is worth noting that introduction of the complex moduli means that all wave numbers, solutions of Eq. (4) for a given frequency, are complex. Therefore, we need both the real and imaginary parts of the wave numbers to determine the dispersion characteristics.

The effect of damping was studied for all the modes that were identified in the foregoing to be much less affected by the elastic coating. It is found that in general the imaginary part of k' ($\mathrm{Im}(k')$) increases with increasing damping τ_1. For instance, Table 3 shows the effects of damping on the branch 3 (a_1) when $\omega' = 13$. For this particular pair of frequency/wave number, the real part of k'

[1] This choice is arbitrary and is consistent with the requirement that $\tau_1 > \tau_2$.

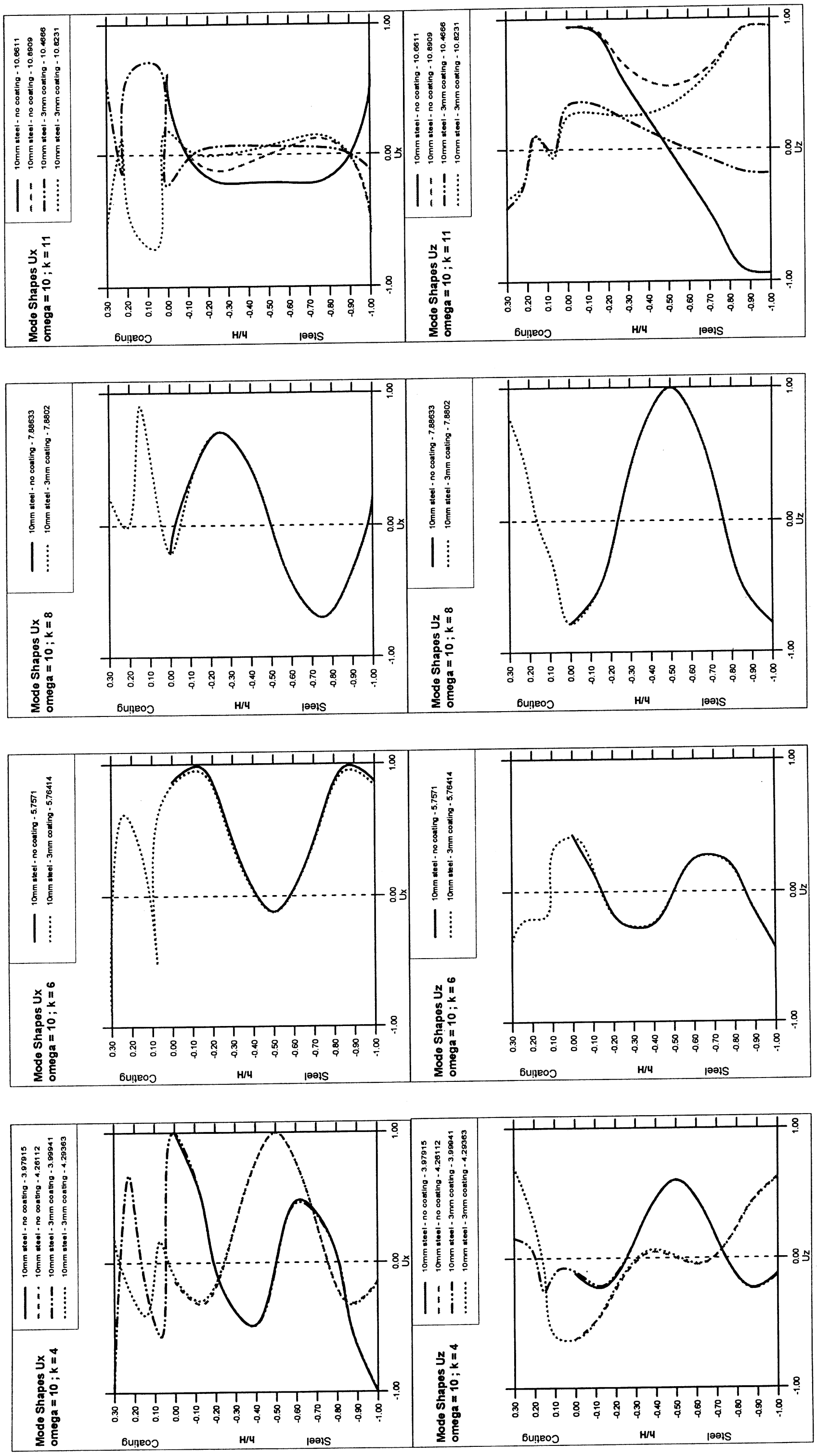

Fig. 3. Mode shapes for a 1 cm steel plate with and without a 0.3 cm elastic coating. The normalized frequency is $\omega' = 10$ and k' values correspond to those listed in Table 2. In this and the following figures, h and H stand, respectively, for h_1 and h_2 defined in the text. The k values indicated outside the box at the top of Figs. 3, 4 and 6 are to help identify the grouping of wave numbers around those values. Note that k is the same as k'.

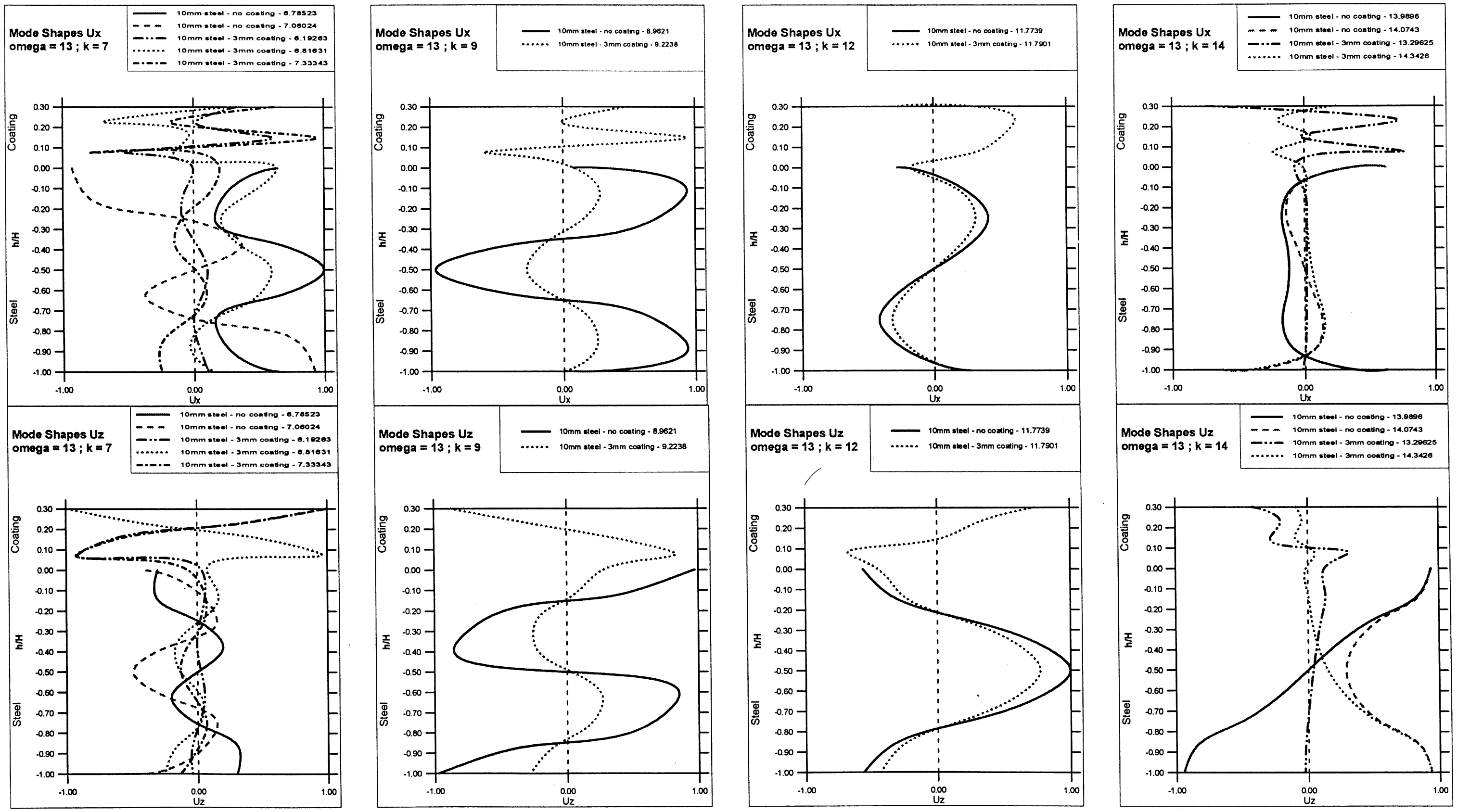

Fig. 4. Mode shapes for a 1 cm steel plate with and without a 0.3 cm elastic coating. The normalized frequency is $\omega' = 13$ and k' values correspond to those listed in Table 2.

Table 3
Wave number k' on branch 3 (a_1) for frequency $\omega' = 13$ of a 1 cm steel plate with a 0.3 cm viscoelastic coating

Damping	Re(k')	Im(k')
0.000	11.7933	0.0000
0.005	11.7945	0.0255
0.025	11.7948	0.0470
0.050	11.7966	0.0504
0.500	11.8048	0.0513

(Re(k')) also increases with increasing damping τ_1. It is quite interesting to note that for the foregoing identified modes, especially for those at $\omega' = 13$, the amplitude of the mode displacement in the steel layer generally increases with increasing damping. However, damping does not modify the mode shapes much.

As an illustration, Fig. 5 shows the variation of the mode shape for relatively low damping (i.e., $\tau_1 = 0.000, 0.005, 0.025, 0.050$) for the a_1 mode at $\omega' = 13$, while Fig. 6 shows comparison of the a_1 mode shapes at $\omega' = 13$ for a higher damping case (i.e., $\tau_1 = 0.5$). As can be observed from these figures, the a_1 mode remains similar. It was found that s_1 and a_0 modes were also relatively unaffected

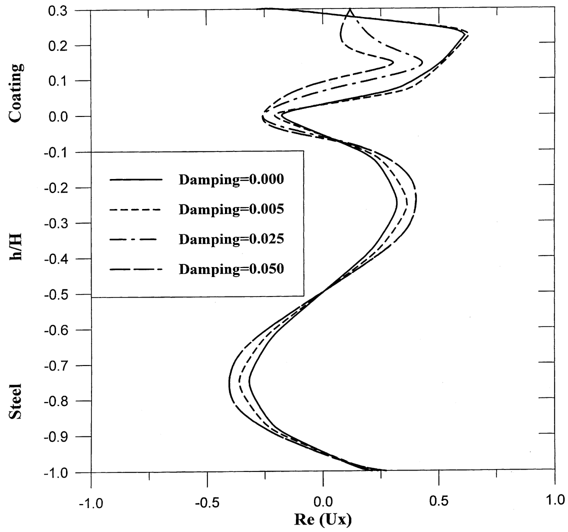

Fig. 5. Mode shapes (a_1) of real (u_x) for a 1 cm steel plate with a 0.3 cm viscoelastic coating. The normalized frequency is $\omega' = 13$ and k' values correspond to those listed in Table 3. The damping factor τ_1 varies as 0.000, 0.005, 0.025, 0.050.

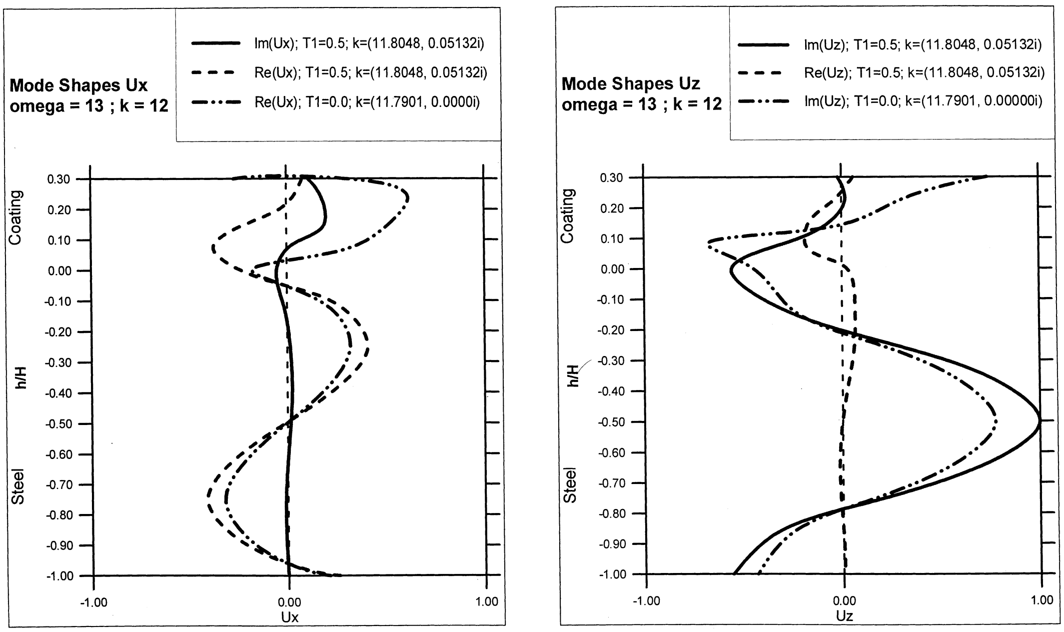

Fig. 6. Mode shapes (a_1) of u_x and u_z for a 1 cm steel plate with and without a 0.3 cm viscoelastic coating. The normalized frequency is $\omega' = 13$ and k' values correspond to those listed in Table 3. The damping factor τ_1 is taken to be 0.0 and 0.5.

by damping. This study indicates that at high frequency, the a_0, a_1 and s_1 modes would be the choice for inspection of stress-corrosion cracks in gas pipelines.

4. Conclusion

A parametric study of the effect of thick elastic coating with and without damping on the guided wave modes in a bilayered plate was conducted. Although both layers could be modeled as viscoelastic media, this study focused on the case where only the coating layer was viscoelastic. Since the damping properties of the coating are not easily measured and they also are variable, the standard linear model was used to describe the viscoelastic properties of the coating layer. Two damping factors were introduced into the Lamé constants in such a way that the Poisson's ratio was taken to be frequency-independent. The relevant dispersion equation was derived and programs based on it were written. In order to find the complex wave numbers for a given frequency, the stiffness method coupled with the IMSL subroutine *zanlyt* was employed.

For the steel plate with a thick elastic coating, our results show that a thick elastic coating would introduce new modes, which are mainly confined to the coating layer, and also affect most mode shapes of the bare steel plate. However, certain modes are unaffected by the thick elastic coating. It was found that in the range $10 \leqslant \omega' \leqslant 13$, s_1, a_1, and a_0 modes of the bare steel plate are similar in shape to those of the steel plate with a thick elastic coating layer. Thus, these modes would be most suitable for ultrasonic inspection of gas pipelines with thick elastic coating. Furthermore, if the coating layer is viscoelastic, the material damping then affects some of the identified modes, but not significantly. Thus, it would be possible to use these modes for ultrasonic characterization of stress-corrosion cracks in the steel layer of a coated plate.

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Appendix A

Elements of the matrix [**A**] in Eq. (4):

$$A(1,1) = -\frac{2k^2 p_1}{p_1^2 - k^2}\cos(q_1 h_1) - p_1\cos(p_1 h_1),$$

$$A(1,2) = -\mathrm{i}k\sin(q_1 h_1) + \frac{2\mathrm{i}kq_1 p_1}{p_1^2 - k^2}\sin(p_1 h_1),$$

$$A(1,3) = \frac{2k^2 p_2}{p_2^2 - k^2}\cos(q_2 h_2) + p_2\cos(p_2 h_2),$$

$$A(1,4) = -\mathrm{i}k\sin(q_2 h_2) + \frac{2\mathrm{i}kq_2 p_2}{p_2^2 - k^2}\sin(p_2 h_2),$$

$$A(2,1) = \frac{2\mathrm{i}kq_1 p_1}{p_1^2 - k^2}\sin(q_1 h_1) - \mathrm{i}k\sin(p_1 h_1),$$

$$A(2,2) = q_1\cos(q_1 h_1) + \frac{2k^2 q_1}{p_1^2 - k^2}\cos(p_1 h_1),$$

$$A(2,3) = \frac{2\mathrm{i}kq_2 p_2}{p_2^2 - k^2}\sin(q_2 h_2) - \mathrm{i}k\sin(p_2 h_2),$$

$$A(2,4) = -q_2\cos(q_2 h_2) - \frac{2k^2 q_2}{p_2^2 - k^2}\cos(p_2 h_2),$$

$$A(3,1) = -\mu_1\left[\frac{4k^2 q_1 p_1}{p_1^2 - k^2}\sin(q_1 h_1) + (p_1^2 - k^2)\sin(p_1 h_1)\right]$$

$$A(3,2) = 2\mathrm{i}kq_1\mu_1[\cos(q_1 h_1) - \cos(p_1 h_1)],$$

$$A(3,3) = -\mu_2\left[\frac{4k^2 q_2 p_2}{p_2^2 - k^2}\sin(q_2 h_2) + (p_2^2 - k^2)\sin(p_2 h_2)\right],$$

$$A(3,4) = -2\mathrm{i}kq_2\mu_2[\cos(q_2 h_2) - \cos(p_2 h_2)],$$

$$A(4,1) = -2\mathrm{i}kp_1\mu_1[\cos(q_1 h_1) - \cos(p_1 h_1)],$$

$$A(4,2) = \mu_1\left[(p_1^2 - k^2)\sin(q_1 h_1) + \frac{4k^2 q_1 p_1}{p_1^2 - k^2}\sin(p_1 h_1)\right],$$

$$A(4,3) = 2ikp_2\mu_2[\cos(q_2h_2) - \cos(p_2h_2)],$$

$$A(4,4) = \mu_2\left[(p_2^2 - k^2)\sin(q_2h_2) + \frac{4k^2q_2p_2}{p_2^2 - k^2}\sin(p_2h_2)\right].$$

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