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Pyroelectric and pyromagnetic effects are important for applications of multiferroic materials in elevated temperature environments. In this paper, we derive exact closed-form electromagnetic Green's function expressions for polarization fields in uniaxial multiferroic materials and bimaterials induced by a steady point heat source. The pyroelectric and pyromagnetic effects as a result of temperature change in multiferroic materials are incorporated in this study. The degenerate and nondegenerate cases, which pertain to whether the heat conduction characteristic constant is equal to one of the two electromagnetic characteristic constants, are discussed in detail. The Green's functions for a bimaterial composed of two perfectly bonded uniaxial multiferroic half-spaces subjected to a point heat source are further obtained by means of the image method.

1 Introduction Multiferroic materials simultaneously possess both ferroelectric and ferromagnetic (or antiferromagnetic) order in the same phase. They hold great potential for applications as the multiferroic coupling allows switching of the magnetic state by an electric field and likewise switching of the ferroelectric polarization by a magnetic field [1–6]. Significantly, multiferroics could lead to a new generation of memory and microwave devices that can be controlled both electrically and magnetically [3, 6].

The Green's functions in multiferroic materials can be utilized to tailor the magnetoelectric effect [7, 8] and to investigate inclusions of various shapes with spontaneous polarization and magnetization [9, 10]. Li and Li [9] obtained Green's functions for the uniaxial multiferroic material induced by a point electric or magnetic charge. The Green's functions for a uniaxial multiferroic half-space and bimaterial were addressed very recently [11]. The corresponding Green's functions for exponentially graded uniaxial multiferroic materials were also derived [12]. However, the aforementioned works on Green's functions in multiferroic materials [9, 11] were confined to the isothermal case in which the pyroelectric and pyromagnetic effects, which have been observed [13, 14] and which have found many applications both in science and technology [15, 16], were not taken into consideration. We also point out that thermal source is important in smart materials, as was discussed for piezoelectric [17] and magnetoelectroelastic [18] materials under the thermal source/loading in two dimension. A thermomagnetoelastic model was even proposed for earthquake source mechanism study [19].

Obtained in this research are the induced electromagnetic fields for a uniaxial multiferroic material and bimaterial subjected to a steady point heat source. In the course of elaborating our method we establish electromagnetic characteristic constants, λ_1 and λ_2, to parameterize the multiferroic behavior. Both the nondegenerate case, in which the heat conduction characteristic constant λ_0 (the ratio of the transverse and axial thermal conductivity tensor elements) is different from the two electromagnetic characteristic constants λ_1 and λ_2, and the degenerate case, in which the heat conduction characteristic constant λ_0 is equal to one of the two electromagnetic characteristic constants λ_1 and λ_2, are addressed. Once the Green's functions for a multiferroic full-space are known, the corresponding Green's functions for two bonded multiferroic half-spaces are obtained by the image method, with the twelve unknown constants being determined by inverting a simple 4×4 matrix [11].

2 A steady point heat source in a homogeneous uniaxial multiferroic material The constitutive equations for a uniaxial multiferroic material with its unique axis along the x_3-axis can be written as

$$\begin{bmatrix} D_1 \\ B_1 \end{bmatrix} = \begin{bmatrix} \varepsilon_{11} & \alpha_{11} \\ \alpha_{11} & \mu_{11} \end{bmatrix} \begin{bmatrix} E_1 \\ H_1 \end{bmatrix}, \quad \begin{bmatrix} D_2 \\ B_2 \end{bmatrix} = \begin{bmatrix} \varepsilon_{11} & \alpha_{11} \\ \alpha_{11} & \mu_{11} \end{bmatrix} \begin{bmatrix} E_2 \\ H_2 \end{bmatrix}, \quad \begin{bmatrix} D_3 \\ B_3 \end{bmatrix} = \begin{bmatrix} \varepsilon_{33} & \alpha_{33} \\ \alpha_{33} & \mu_{33} \end{bmatrix} \begin{bmatrix} E_3 \\ H_3 \end{bmatrix} + \begin{bmatrix} p \\ m \end{bmatrix} T, \tag{1}$$

where D_i and B_i ($i = 1, 2, 3$) are the electric displacement and magnetic flux components (in the x_1-, x_2-, and x_3-directions); E_i and H_i are electric field and magnetic field components; T is the temperature change; ε_{11} and ε_{33} are the two dielectric permittivity constants in the x_1- and x_3-directions, respectively; α_{11} and α_{33} are the two magnetoelectric constants (in the x_1- and x_3-directions); μ_{11} and μ_{33} are the two magnetic permeability constants (in the x_1- and x_3-directions); and p and m are, respectively, the pyroelectric and pyromagnetic constants (in the x_3-direction).

The electric and magnetic fields are related to the electric potential ϕ and magnetic potential ψ through the following 2×1 column matrix relation

$$\begin{bmatrix} E_i \\ H_i \end{bmatrix} = -\begin{bmatrix} \phi_{,i} \\ \psi_{,i} \end{bmatrix}, \tag{2}$$

where the subscript comma "," followed by the index i ($i = 1, 2, 3$) denotes the derivative of the potential with respect to the coordinate x_i.

In the absence of free electric and magnetic charges, the electric displacement D_i and magnetic flux B_i satisfy the following Gauss' equations

$$\frac{\partial D_1}{\partial x_1} + \frac{\partial D_2}{\partial x_2} + \frac{\partial D_3}{\partial x_3} = 0, \quad \frac{\partial B_1}{\partial x_1} + \frac{\partial B_2}{\partial x_2} + \frac{\partial B_3}{\partial x_3} = 0. \tag{3}$$

Substituting Eq. (2) into Eq. (1), and then the results into Eq. (3), we finally arrive at the following set of inhomogeneous partial differential equations for ϕ and ψ

$$\begin{bmatrix} \varepsilon_{11} & \alpha_{11} \\ \alpha_{11} & \mu_{11} \end{bmatrix} \begin{bmatrix} \left(\frac{\partial^2}{\partial x_1^2} + \frac{\partial^2}{\partial x_2^2} \right) \phi \\ \left(\frac{\partial^2}{\partial x_1^2} + \frac{\partial^2}{\partial x_2^2} \right) \psi \end{bmatrix} + \begin{bmatrix} \varepsilon_{33} & \alpha_{33} \\ \alpha_{33} & \mu_{33} \end{bmatrix} \begin{bmatrix} \frac{\partial^2 \phi}{\partial x_3^2} \\ \frac{\partial^2 \psi}{\partial x_3^2} \end{bmatrix} = \begin{bmatrix} p \\ m \end{bmatrix} \frac{\partial T}{\partial x_3}. \tag{4}$$

In addition we assume that a steady point heat source of strength Q is located at the origin of a uniaxial multiferroic space. As a result the temperature T should satisfy the following 3D Poisson's equation

$$\frac{\partial^2 T}{\partial x_1^2} + \frac{\partial^2 T}{\partial x_2^2} + \frac{1}{\lambda_0} \frac{\partial^2 T}{\partial x_3^2} = -\frac{Q}{k_{11}} \delta(x_1)\, \delta(x_2)\, \delta(x_3), \tag{5}$$

where $\delta()$ is the Dirac delta function; $\lambda_0 = k_{11}/k_{33}$ is the dimensionless heat conduction characteristic constant; and k_{11} and k_{33} are two heat conductivity constants (in the x_1- and x_3-directions). It is obvious that for an isotropic thermal material, the heat conduction characteristic constant $\lambda_0 = 1$, whilst it can be larger or smaller than 1, depending whether the strong direction of the heat conduction is along x_1- or x_3-direction. Equation (5) can be further expressed in the following standard form

$$\frac{\partial^2 T}{\partial x_1^2} + \frac{\partial^2 T}{\partial x_2^2} + \frac{\partial^2 T}{\partial \left(\sqrt{\lambda_0}\, x_3 \right)^2} = -\frac{Q}{\tilde{k}} \delta(x_1)\, \delta(x_2)\, \delta\left(\sqrt{\lambda_0}\, x_3 \right), \tag{6}$$

where $\tilde{k} = \sqrt{k_{11} k_{33}}$ can be considered as the effective heat conductivity. The solution to Eq. (6) can be expediently given by

$$T = \frac{Q}{4\pi \tilde{k}} \frac{1}{\sqrt{x_1^2 + x_2^2 + \lambda_0 x_3^2}}. \tag{7}$$

Inserting the above expression for the temperature T into Eq. (4), we arrive at

$$\begin{bmatrix} \varepsilon_{11} & \alpha_{11} \\ \alpha_{11} & \mu_{11} \end{bmatrix} \begin{bmatrix} \left(\frac{\partial^2}{\partial x_1^2} + \frac{\partial^2}{\partial x_2^2} \right) \phi \\ \left(\frac{\partial^2}{\partial x_1^2} + \frac{\partial^2}{\partial x_2^2} \right) \psi \end{bmatrix} + \begin{bmatrix} \varepsilon_{33} & \alpha_{33} \\ \alpha_{33} & \mu_{33} \end{bmatrix} \begin{bmatrix} \frac{\partial^2 \phi}{\partial x_3^2} \\ \frac{\partial^2 \psi}{\partial x_3^2} \end{bmatrix} = -\frac{Q \sqrt{\lambda_0}\, x_3}{4\pi k_{33} (x_1^2 + x_2^2 + \lambda_0 x_3^2)^{3/2}} \begin{bmatrix} p \\ m \end{bmatrix}. \tag{8}$$

In the following we will decouple the coupled inhomogeneous partial differential equations (8) using the eigenvalue approach [11]. We first consider the following eigenvalue problem [11]

$$\left(\begin{bmatrix} \varepsilon_{11} & \alpha_{11} \\ \alpha_{11} & \mu_{11} \end{bmatrix} - \lambda \begin{bmatrix} \varepsilon_{33} & \alpha_{33} \\ \alpha_{33} & \mu_{33} \end{bmatrix} \right) \boldsymbol{v} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}. \tag{9}$$

The two eigenvalues λ_1 and λ_2, which are termed the electromagnetic characteristic constants, are given by [11]

$$\begin{aligned}\lambda_1 &= \frac{\mu_{33}\varepsilon_{11}+\mu_{11}\varepsilon_{33}-2\alpha_{11}\alpha_{33}+\sqrt{(\mu_{11}\varepsilon_{33}-\mu_{33}\varepsilon_{11})^2+4(\alpha_{11}\mu_{33}-\alpha_{33}\mu_{11})(\alpha_{11}\varepsilon_{33}-\alpha_{33}\varepsilon_{11})}}{2(\mu_{33}\varepsilon_{33}-\alpha_{33}^2)}\,,\\ \lambda_2 &= \frac{\mu_{33}\varepsilon_{11}+\mu_{11}\varepsilon_{33}-2\alpha_{11}\alpha_{33}-\sqrt{(\mu_{11}\varepsilon_{33}-\mu_{33}\varepsilon_{11})^2+4(\alpha_{11}\mu_{33}-\alpha_{33}\mu_{11})(\alpha_{11}\varepsilon_{33}-\alpha_{33}\varepsilon_{11})}}{2(\mu_{33}\varepsilon_{33}-\alpha_{33}^2)}\,,\end{aligned} \tag{10}$$

and the two eigenvectors associated with λ_1 and λ_2 are

$$\boldsymbol{v}_1 = \begin{bmatrix}-\alpha_{11}+\lambda_1\alpha_{33}\\ \varepsilon_{11}-\lambda_1\varepsilon_{33}\end{bmatrix}, \qquad \boldsymbol{v}_2 = \begin{bmatrix}-\alpha_{11}+\lambda_2\alpha_{33}\\ \varepsilon_{11}-\lambda_2\varepsilon_{33}\end{bmatrix}. \tag{11}$$

Since the two matrices $\begin{bmatrix}\varepsilon_{11} & \alpha_{11}\\ \alpha_{11} & \mu_{11}\end{bmatrix}$ and $\begin{bmatrix}\varepsilon_{33} & \alpha_{33}\\ \alpha_{33} & \mu_{33}\end{bmatrix}$ are real and symmetric, it can be easily verified that the following orthogonal relationships with respect to the two symmetric matrices hold [11]

$$\begin{aligned}&\begin{bmatrix}\boldsymbol{v}_1^T\\ \boldsymbol{v}_2^T\end{bmatrix}\begin{bmatrix}\varepsilon_{33} & \alpha_{33}\\ \alpha_{33} & \mu_{33}\end{bmatrix}[\boldsymbol{v}_1 \quad \boldsymbol{v}_2] = \begin{bmatrix}\delta_1 & 0\\ 0 & \delta_2\end{bmatrix},\\ &\begin{bmatrix}\boldsymbol{v}_1^T\\ \boldsymbol{v}_2^T\end{bmatrix}\begin{bmatrix}\varepsilon_{11} & \alpha_{11}\\ \alpha_{11} & \mu_{11}\end{bmatrix}[\boldsymbol{v}_1 \quad \boldsymbol{v}_2] = \begin{bmatrix}\lambda_1\delta_1 & 0\\ 0 & \lambda_2\delta_2\end{bmatrix},\end{aligned} \tag{12}$$

where

$$\begin{aligned}\delta_1 &= \alpha_{11}^2\varepsilon_{33}+\varepsilon_{11}^2\mu_{33}-2\alpha_{11}\alpha_{33}\varepsilon_{11}\\ &\quad+(\mu_{33}\varepsilon_{33}-\alpha_{33}^2)(\lambda_1^2\varepsilon_{33}-2\lambda_1\varepsilon_{11})\,,\\ \delta_2 &= \alpha_{11}^2\varepsilon_{33}+\kappa_{11}^2\mu_{33}-2\alpha_{11}\alpha_{33}\varepsilon_{11}\\ &\quad+(\mu_{33}\varepsilon_{33}-\alpha_{33}^2)(\lambda_2^2\varepsilon_{33}-2\lambda_2\varepsilon_{11})\,.\end{aligned} \tag{13}$$

We now introduce two new functions f and g, which are related to ϕ and ψ through

$$\begin{bmatrix}\phi\\ \psi\end{bmatrix} = \boldsymbol{\Phi}\begin{bmatrix}f\\ g\end{bmatrix}, \tag{14}$$

where $\boldsymbol{\Phi} = [\boldsymbol{v}_1\, \boldsymbol{v}_2]$.

In view of Eqs. (8), (12) and (14), the two new functions f and g satisfy the following two independent 3D Poisson's equations

$$\begin{aligned}\left(\frac{\partial^2}{\partial x_1^2}+\frac{\partial^2}{\partial x_2^2}+\frac{1}{\lambda_1}\frac{\partial^2}{\partial x_3^2}\right)f &= \frac{c_1\sqrt{\lambda_0}\,x_3}{(x_1^2+x_2^2+\lambda_0 x_3^2)^{3/2}}\,,\\ \left(\frac{\partial^2}{\partial x_1^2}+\frac{\partial^2}{\partial x_2^2}+\frac{1}{\lambda_2}\frac{\partial^2}{\partial x_3^2}\right)g &= \frac{c_2\sqrt{\lambda_0}\,x_3}{(x_1^2+x_2^2+\lambda_0 x_3^2)^{3/2}}\,,\end{aligned} \tag{15}$$

where the two constants c_1 and c_2 are given by

$$\begin{aligned}c_1 &= \frac{Q[p(\alpha_{11}-\lambda_1\alpha_{33})-m(\varepsilon_{11}-\lambda_1\varepsilon_{33})]}{4\pi k_{33}\delta_1\lambda_1}\,,\\ c_2 &= \frac{Q[p(\alpha_{11}-\lambda_2\alpha_{33})-m(\varepsilon_{11}-\lambda_2\varepsilon_{33})]}{4\pi k_{33}\delta_2\lambda_2}\,.\end{aligned} \tag{16}$$

In the following we will discuss the solutions to Eq. (15) according to whether the heat conduction characteristic constant λ_0 is equal to one of the two electromagnetic characteristic constants λ_1 and λ_2. We assume that the two electromagnetic characteristic constants are distinct (which is true for a uniaixial material) in order to simplify our discussion. In the case of isotropy where $\lambda_1 = \lambda_2$, a small perturbation can be utilized to separate the two roots so that the solutions presented in this paper can still be utilized with neglected errors [20].

2.1 The nondegenerate case: $\lambda_1 \neq \lambda_2 \neq \lambda_0$.

When $\lambda_1 \neq \lambda_2 \neq \lambda_0$, it can be easily checked that the solutions to Eq. (15) can be written as

$$\begin{aligned}f &= \operatorname{sign}(x_3)\left[d_1 \ln R_1^* + \frac{\lambda_1 c_1}{\lambda_1-\lambda_0}\ln R_0^*\right],\\ g &= \operatorname{sign}(x_3)\left[d_2 \ln R_2^* + \frac{\lambda_2 c_2}{\lambda_2-\lambda_0}\ln R_0^*\right],\end{aligned} \tag{17}$$

where $R_i^* = R_i + \sqrt{\lambda_i}\,|x_3|$ with $R_i = \sqrt{r^2+\lambda_i x_3^2}$, and $r^2 = x_1^2 + x_2^2$ $(i = 0, 1, 2)$; d_1 and d_2 are two unknown constants; and the sign function is defined as follows

$$\operatorname{sign}(x_3) = \begin{cases}1 & \text{when} \quad x_3 > 0\\ -1 & \text{when} \quad x_3 < 0\end{cases}. \tag{18}$$

Due to the fact that the electric and magnetic potentials ϕ and ψ should be *continuous* across the plane $x_3 = 0$, then we have $\phi = \psi = 0$ (or equivalently $f = g = 0$) on $x_3 = 0$ in view of the fact that f and g are odd functions of x_3. As a result it follows from Eq. (17) that the two unknown constants d_1 and d_2 can be uniquely determined to be

$$d_1 = -\frac{\lambda_1 c_1}{\lambda_1-\lambda_0}, \qquad d_2 = -\frac{\lambda_2 c_2}{\lambda_2-\lambda_0}. \tag{19}$$

Consequently the expressions of f and g can be finally given by

$$\begin{aligned}f &= \frac{\lambda_1 c_1}{\lambda_1-\lambda_0}\operatorname{sign}(x_3)\ln\frac{R_0^*}{R_1^*}\,,\\ g &= \frac{\lambda_2 c_2}{\lambda_2-\lambda_0}\operatorname{sign}(x_3)\ln\frac{R_0^*}{R_2^*}\,.\end{aligned} \tag{20}$$

The expressions of the electric and magnetic potentials ϕ and ψ are thus given by

$$\begin{bmatrix}\phi\\ \psi\end{bmatrix} = \boldsymbol{\Phi}\begin{bmatrix}\dfrac{\lambda_1 c_1}{\lambda_1-\lambda_0}\operatorname{sign}(x_3)\ln\dfrac{R_0^*}{R_1^*}\\[2ex] \dfrac{\lambda_2 c_2}{\lambda_2-\lambda_0}\operatorname{sign}(x_3)\ln\dfrac{R_0^*}{R_2^*}\end{bmatrix}, \tag{21}$$

which can be written more explicitly as

$$\begin{aligned}\phi &= \frac{\lambda_1 c_1(\lambda_1\alpha_{33}-\alpha_{11})}{\lambda_1-\lambda_0}\operatorname{sign}(x_3)\ln\frac{R_0^*}{R_1^*} + \frac{\lambda_2 c_2(\lambda_2\alpha_{33}-\alpha_{11})}{\lambda_2-\lambda_0}\operatorname{sign}(x_3)\ln\frac{R_0^*}{R_2^*},\\ \psi &= \frac{\lambda_1 c_1(\varepsilon_{11}-\lambda_1\varepsilon_{33})}{\lambda_1-\lambda_0}\operatorname{sign}(x_3)\ln\frac{R_0^*}{R_1^*} + \frac{\lambda_2 c_2(\varepsilon_{11}-\lambda_2\varepsilon_{33})}{\lambda_2-\lambda_0}\operatorname{sign}(x_3)\ln\frac{R_0^*}{R_2^*}.\end{aligned} \tag{22}$$

The electric and magnetic fields induced by the point heat source can then be determined as

$$\begin{aligned}E_1 &= \frac{\lambda_1 c_1(\alpha_{11}-\lambda_1\alpha_{33})}{\lambda_1-\lambda_0}\operatorname{sign}(x_3)\left(\frac{x_1}{R_0R_0^*}-\frac{x_1}{R_1R_1^*}\right) + \frac{\lambda_2 c_2(\alpha_{11}-\lambda_2\alpha_{33})}{\lambda_2-\lambda_0}\operatorname{sign}(x_3)\left(\frac{x_1}{R_0R_0^*}-\frac{x_1}{R_2R_2^*}\right),\\ E_2 &= \frac{\lambda_1 c_1(\alpha_{11}-\lambda_1\alpha_{33})}{\lambda_1-\lambda_0}\operatorname{sign}(x_3)\left(\frac{x_2}{R_0R_0^*}-\frac{x_2}{R_1R_1^*}\right) + \frac{\lambda_2 c_2(\alpha_{11}-\lambda_2\alpha_{33})}{\lambda_2-\lambda_0}\operatorname{sign}(x_3)\left(\frac{x_2}{R_0R_0^*}-\frac{x_2}{R_2R_2^*}\right),\\ E_3 &= \frac{\lambda_1 c_1(\alpha_{11}-\lambda_1\alpha_{33})}{\lambda_1-\lambda_0}\left(\frac{\sqrt{\lambda_0}}{R_0}-\frac{\sqrt{\lambda_1}}{R_1}\right) + \frac{\lambda_2 c_2(\alpha_{11}-\lambda_2\alpha_{33})}{\lambda_2-\lambda_0}\left(\frac{\sqrt{\lambda_0}}{R_0}-\frac{\sqrt{\lambda_2}}{R_2}\right),\end{aligned} \tag{23}$$

$$\begin{aligned}H_1 &= \frac{\lambda_1 c_1(\lambda_1\varepsilon_{33}-\varepsilon_{11})}{\lambda_1-\lambda_0}\operatorname{sign}(x_3)\left(\frac{x_1}{R_0R_0^*}-\frac{x_1}{R_1R_1^*}\right) + \frac{\lambda_2 c_2(\lambda_2\varepsilon_{33}-\varepsilon_{11})}{\lambda_2-\lambda_0}\operatorname{sign}(x_3)\left(\frac{x_1}{R_0R_0^*}-\frac{x_1}{R_2R_2^*}\right),\\ H_2 &= \frac{\lambda_1 c_1(\lambda_1\varepsilon_{33}-\varepsilon_{11})}{\lambda_1-\lambda_0}\operatorname{sign}(x_3)\left(\frac{x_2}{R_0R_0^*}-\frac{x_2}{R_1R_1^*}\right) + \frac{\lambda_2 c_2(\lambda_2\varepsilon_{33}-\varepsilon_{11})}{\lambda_2-\lambda_0}\operatorname{sign}(x_3)\left(\frac{x_2}{R_0R_0^*}-\frac{x_2}{R_2R_2^*}\right),\\ H_3 &= \frac{\lambda_1 c_1(\lambda_1\varepsilon_{33}-\varepsilon_{11})}{\lambda_1-\lambda_0}\left(\frac{\sqrt{\lambda_0}}{R_0}-\frac{\sqrt{\lambda_1}}{R_1}\right) + \frac{\lambda_2 c_2(\lambda_2\varepsilon_{33}-\varepsilon_{11})}{\lambda_2-\lambda_0}\left(\frac{\sqrt{\lambda_0}}{R_0}-\frac{\sqrt{\lambda_2}}{R_2}\right).\end{aligned} \tag{24}$$

It is observed from the above two expressions that the horizontal electric and magnetic fields E_1, E_2, H_1, H_2 are odd functions of x_3, and are zero on the horizontal plane $x_3 = 0$. On the other hand, the vertical electric and magnetic fields E_3 and H_3 are even functions of x_3, inversely proportional to $r = \sqrt{x_1^2 + x_2^2}$ on the horizontal plane $x_3 = 0$, and are zero on the x_3-axis excluding the origin. The electric displacements and magnetic fluxes can be determined by using Eq. (1) and the above two expressions.

2.2 The degenerate case: $\lambda_1 = \lambda_0$ $(\lambda_1 \neq \lambda_2)$. Next we address the degenerate case $\lambda_1 = \lambda_0$ $(\lambda_1 \neq \lambda_2)$. Applying the L'Hospital's rule to Eq. (21) when $\lambda_1 \to \lambda_0$ yields the expressions of the electric potential ϕ and magnetic potential ψ as follows

$$\begin{bmatrix}\phi\\ \psi\end{bmatrix} = \boldsymbol{\Phi}\begin{bmatrix}-\dfrac{c_1\sqrt{\lambda_0}\,x_3}{2R_0}\\[2ex] \dfrac{\lambda_2 c_2}{\lambda_2-\lambda_0}\operatorname{sign}(x_3)\ln\dfrac{R_0^*}{R_2^*}\end{bmatrix}, \tag{25}$$

which can be written more explicitly as

$$\begin{aligned}\phi &= \frac{\sqrt{\lambda_0}\,c_1(\alpha_{11}-\lambda_0\alpha_{33})x_3}{2R_0} + \frac{\lambda_2 c_2(\lambda_2\alpha_{33}-\alpha_{11})}{\lambda_2-\lambda_0}\operatorname{sign}(x_3)\ln\frac{R_0^*}{R_2^*},\\ \psi &= \frac{\sqrt{\lambda_0}\,c_1(\lambda_0\varepsilon_{33}-\varepsilon_{11})x_3}{2R_0} + \frac{\lambda_2 c_2(\varepsilon_{11}-\lambda_2\varepsilon_{33})}{\lambda_2-\lambda_0}\operatorname{sign}(x_3)\ln\frac{R_0^*}{R_2^*}.\end{aligned} \tag{26}$$

The electric and magnetic fields induced by the point heat source can then be determined as

$$\begin{aligned}
E_1 &= \frac{\sqrt{\lambda_0}\, c_1(\alpha_{11}-\lambda_0\alpha_{33})\, x_1 x_3}{2R_0^3} + \frac{\lambda_2 c_2(\alpha_{11}-\lambda_2\alpha_{33})}{\lambda_2-\lambda_0}\,\mathrm{sign}\,(x_3)\left(\frac{x_1}{R_0R_0^*}-\frac{x_1}{R_2R_2^*}\right),\\
E_2 &= \frac{\sqrt{\lambda_0}\, c_1(\alpha_{11}-\lambda_0\alpha_{33})\, x_2 x_3}{2R_0^3} + \frac{\lambda_2 c_2(\alpha_{11}-\lambda_2\alpha_{33})}{\lambda_2-\lambda_0}\,\mathrm{sign}\,(x_3)\left(\frac{x_2}{R_0R_0^*}-\frac{x_2}{R_2R_2^*}\right),\\
E_3 &= \frac{\sqrt{\lambda_0}\, c_1(\lambda_0\alpha_{33}-\alpha_{11})\, r^2}{2R_0^3} + \frac{\lambda_2 c_2(\alpha_{11}-\lambda_2\alpha_{33})}{\lambda_2-\lambda_0}\left(\frac{\sqrt{\lambda_0}}{R_0}-\frac{\sqrt{\lambda_2}}{R_2}\right),
\end{aligned} \tag{27}$$

$$\begin{aligned}
H_1 &= \frac{\sqrt{\lambda_0}\, c_1(\lambda_0\varepsilon_{33}-\varepsilon_{11})\, x_1 x_3}{2R_0^3} + \frac{\lambda_2 c_2(\lambda_2\varepsilon_{33}-\varepsilon_{11})}{\lambda_2-\lambda_0}\,\mathrm{sign}\,(x_3)\left(\frac{x_1}{R_0R_0^*}-\frac{x_1}{R_2R_2^*}\right),\\
H_2 &= \frac{\sqrt{\lambda_0}\, c_1(\lambda_0\varepsilon_{33}-\varepsilon_{11})\, x_2 x_3}{2R_0^3} + \frac{\lambda_2 c_2(\lambda_2\varepsilon_{33}-\varepsilon_{11})}{\lambda_2-\lambda_0}\,\mathrm{sign}\,(x_3)\left(\frac{x_2}{R_0R_0^*}-\frac{x_2}{R_2R_2^*}\right),\\
H_3 &= \frac{\sqrt{\lambda_0}\, c_1(\varepsilon_{11}-\lambda_0\varepsilon_{33})\, r^2}{2R_0^3} + \frac{\lambda_2 c_2(\lambda_2\varepsilon_{33}-\varepsilon_{11})}{\lambda_2-\lambda_0}\left(\frac{\sqrt{\lambda_0}}{R_0}-\frac{\sqrt{\lambda_2}}{R_2}\right).
\end{aligned} \tag{28}$$

Similar to the nondegenerate case, we also observe that, for the degenerate case, the horizontal electric and magnetic fields are odd functions of x_3, and are zero on the horizontal plane $x_3 = 0$. On the other hand, the vertical electric and magnetic fields are even functions of x_3, are inversely proportional to r when $x_3 = 0$, and are zero on the x_3 axis excluding the origin. The electric displacements and magnetic fluxes can be similarly determined by using Eq. (1) and the above two expressions.

Before ending this section, we add that the other degenerate case $\lambda_1 = \lambda_0$ ($\lambda_1 \neq \lambda_2$) can be discussed similarly. The results obtained in this section can be further applied to derive the electromagnetic Green's functions for a uniaxial multiferroic bimaterial induced by a steady point heat source, which will be discussed in detail in the ensuing section.

3 A steady point heat source in a homogeneous uniaxial multiferroic bimaterial

In this section we investigate the electromagnetic fields in two bonded multiferroic half-spaces induced by a steady point heat source. We assume that both half-spaces are uniaxial multiferroic materials having the unique axis along the x_3-axis, and that the interface $x_3 = 0$ of the two multiferroic half-spaces are perfect. Namely, the temperature, electric potential, magnetic potential, normal heat flux, normal electric displacement and normal magnetic flux are all *continuous* across the interface $x_3 = 0$. Without loss of generality, a steady point heat source of strength Q is assumed to be located at $x_1 = x_2 = 0$, $x_3 = h$ ($h > 0$) in the upper half-space of the multiferroic bimaterial. In the following the subscripts 1 and 2 to vectors or matrices and the superscripts (1) and (2) to scalars are used to identify the quantities in the upper and lower half spaces, respectively. In addition, we only consider the nondegenerate case for the two multiferroic half-spaces in which the heat conduction characteristic constant is different from the electromagnetic characteristic constants, i.e., $\lambda_1^{(i)} \neq \lambda_2^{(i)} \neq \lambda_0^{(i)}$, $(i = 1, 2)$.

First, making use of the image method [11] and enforcing the continuity conditions of temperature and normal heat flux across the interface $x_3 = 0$, we arrive at the temperature field in the uniaxial bimaterial as follows

$$\begin{aligned}
T^{(1)} &= \frac{Q}{4\pi\tilde{k}^{(1)}\sqrt{x_1^2+x_2^2+\lambda_0^{(1)}(x_3-h)^2}} + \frac{Q(\tilde{k}^{(1)}-\tilde{k}^{(2)})}{4\pi\tilde{k}^{(1)}(\tilde{k}^{(1)}+\tilde{k}^{(2)})\sqrt{x_1^2+x_2^2+\lambda_0^{(1)}(x_3+h)^2}}\,, \quad (x_3>0)\\
T^{(2)} &= \frac{Q}{2\pi(\tilde{k}^{(1)}+\tilde{k}^{(2)})\sqrt{x_1^2+x_2^2+(\sqrt{\lambda_0^{(2)}}\,x_3-\sqrt{\lambda_0^{(1)}}\,h)^2}}\,, \quad (x_3<0)\,.
\end{aligned} \tag{29}$$

It can be found that the electric and magnetic potentials in the bimaterial, induced by the temperature field (29), take the following forms

$$\begin{bmatrix}\phi^{(1)}\\ \psi^{(1)}\end{bmatrix} = \boldsymbol{\Phi}_1 \begin{bmatrix} L_{10}\ln R_{10}^+ + L_{11}\ln R_{11}^+ + L_{12}\ln R_{12}^+ + \dfrac{\lambda_1^{(1)}c_1}{\lambda_1^{(1)}-\lambda_0^{(1)}}\,\mathrm{sign}\,(x_3-h)\ln\dfrac{R_0^*}{R_1^*} + \dfrac{\lambda_1^{(1)}e_1}{\lambda_1^{(1)}-\lambda_0^{(1)}}\ln R_{00}^+ \\[2ex] L_{20}\ln R_{20}^+ + L_{21}\ln R_{21}^+ + L_{22}\ln R_{22}^+ + \dfrac{\lambda_2^{(1)}c_2}{\lambda_2^{(1)}-\lambda_0^{(1)}}\,\mathrm{sign}\,(x_3-h)\ln\dfrac{R_0^*}{R_2^*} + \dfrac{\lambda_2^{(1)}e_2}{\lambda_2^{(1)}-\lambda_0^{(1)}}\ln R_{00}^+ \end{bmatrix}, \quad (x_3>0) \tag{30}$$

$$\begin{bmatrix}\phi^{(2)}\\ \psi^{(2)}\end{bmatrix} = \boldsymbol{\Phi}_2 \begin{bmatrix} L_{30}\ln R_{10}^- + L_{31}\ln R_{11}^- + L_{32}\ln R_{12}^- + \dfrac{\lambda_1^{(2)}e_3}{\lambda_1^{(2)}-\lambda_0^{(2)}}\ln R_{00}^- \\ L_{40}\ln R_{20}^- + L_{41}\ln R_{21}^- + L_{42}\ln R_{22}^- + \dfrac{\lambda_2^{(2)}e_4}{\lambda_2^{(2)}-\lambda_0^{(2)}}\ln R_{00}^- \end{bmatrix}, \quad (x_3 < 0) \tag{31}$$

where L_{ij} ($i = 1-4, j = 0, 1, 2$) are the unknown coefficients to be determined, and

$$\begin{aligned} R_i^* &= \sqrt{x_1^2 + x_2^2 + \lambda_i^{(1)}(x_3 - h)^2} + \sqrt{\lambda_i^{(1)}}\,|x_3 - h|\,, \\ R_{ij}^+ &= \sqrt{x_1^2 + x_2^2 + (\sqrt{\lambda_i^{(1)}}\,x_3 + \sqrt{\lambda_j^{(1)}}\,h)^2} + \sqrt{\lambda_i^{(1)}}\,x_3 + \sqrt{\lambda_j^{(1)}}\,h\,, \quad (i, j = 0, 1, 2) \\ R_{ij}^- &= \sqrt{x_1^2 + x_2^2 + (\sqrt{\lambda_i^{(2)}}\,x_3 - \sqrt{\lambda_j^{(1)}}\,h)^2} + \sqrt{\lambda_j^{(1)}}\,h - \sqrt{\lambda_i^{(2)}}\,x_3\,. \end{aligned} \tag{32}$$

Furthermore, c_1, c_2, e_1, e_2, e_3, e_4 in Eq. (30) are given by

$$\begin{aligned} c_1 &= -\frac{Q(p^{(1)}\Phi_{11}^{(1)} + m^{(1)}\Phi_{21}^{(1)})}{4\pi k_{33}^{(1)}\delta_1^{(1)}\lambda_1^{(1)}}\,, \\ c_2 &= -\frac{Q(p^{(1)}\Phi_{12}^{(1)} + m^{(1)}\Phi_{22}^{(1)})}{4\pi k_{33}^{(1)}\delta_2^{(1)}\lambda_2^{(1)}}\,, \end{aligned} \tag{33a}$$

$$\begin{aligned} e_1 &= \frac{Q(\tilde{k}^{(2)} - \tilde{k}^{(1)})\,(p^{(1)}\Phi_{11}^{(1)} + m^{(1)}\Phi_{21}^{(1)})}{4\pi k_{33}^{(1)}\delta_1^{(1)}\lambda_1^{(1)}(\tilde{k}^{(1)} + \tilde{k}^{(2)})}\,, \\ e_2 &= \frac{Q(\tilde{k}^{(2)} - \tilde{k}^{(1)})\,(p^{(1)}\Phi_{12}^{(1)} + m^{(1)}\Phi_{22}^{(1)})}{4\pi k_{33}^{(1)}\delta_2^{(1)}\lambda_2^{(1)}(\tilde{k}^{(1)} + \tilde{k}^{(2)})}\,, \end{aligned} \tag{33b}$$

$$\begin{aligned} e_3 &= \frac{Q\sqrt{\lambda_0^{(2)}}\,(p^{(2)}\Phi_{11}^{(2)} + m^{(2)}\Phi_{21}^{(2)})}{2\pi\delta_1^{(2)}\lambda_1^{(2)}(\tilde{k}^{(1)} + \tilde{k}^{(2)})}\,, \\ e_4 &= \frac{Q\sqrt{\lambda_0^{(2)}}\,(p^{(2)}\Phi_{12}^{(2)} + m^{(2)}\Phi_{22}^{(2)})}{2\pi\delta_2^{(2)}\lambda_2^{(2)}(\tilde{k}^{(1)} + \tilde{k}^{(2)})}\,, \end{aligned} \tag{33c}$$

with $\Phi_{11}^{(i)} = \lambda_1^{(i)}\alpha_{33}^{(i)} - \alpha_{11}^{(i)}$, $\Phi_{12}^{(i)} = \lambda_2^{(i)}\alpha_{33}^{(i)} - \alpha_{11}^{(i)}$, $\Phi_{21}^{(i)} = \varepsilon_{11}^{(i)} - \lambda_1\varepsilon_{33}^{(i)}$, $\Phi_{22}^{(i)} = \varepsilon_{11}^{(i)} - \lambda_2\varepsilon_{33}^{(i)}$ being the four components of Φ_i.

Then, by enforcing the continuity conditions of the electric and magnetic potentials as well as the normal electric displacement and normal magnetic flux across the interface $x_3 = 0$, the twelve unknowns L_{ij} ($i = 1–4, j = 0, 1, 2$) can be uniquely determined to be

$$\begin{bmatrix} L_{10} & L_{11} & L_{12} \\ L_{20} & L_{21} & L_{22} \\ L_{30} & L_{31} & L_{32} \\ L_{40} & L_{41} & L_{42} \end{bmatrix} = \begin{bmatrix} \Phi_{11}^{(1)} & \Phi_{12}^{(1)} & -\Phi_{11}^{(2)} & -\Phi_{12}^{(2)} \\ \Phi_{21}^{(1)} & \Phi_{22}^{(1)} & -\Phi_{21}^{(2)} & -\Phi_{22}^{(2)} \\ J_{11}^{(1)} & J_{12}^{(1)} & J_{11}^{(2)} & J_{12}^{(2)} \\ J_{21}^{(1)} & J_{22}^{(1)} & J_{21}^{(2)} & J_{22}^{(2)} \end{bmatrix}^{-1} \begin{bmatrix} \xi_{10} & \xi_{11} & \xi_{12} \\ \xi_{20} & \xi_{21} & \xi_{22} \\ \xi_{30} & \xi_{31} & \xi_{32} \\ \xi_{40} & \xi_{41} & \xi_{42} \end{bmatrix}, \tag{34}$$

where

$$\begin{aligned} J_{11}^{(i)} &= \sqrt{\lambda_1^{(i)}}\,(\varepsilon_{33}^{(i)}\Phi_{11}^{(i)} + \alpha_{33}^{(i)}\Phi_{21}^{(i)})\,, & J_{12}^{(i)} &= \sqrt{\lambda_2^{(i)}}\,(\varepsilon_{33}^{(i)}\Phi_{12}^{(i)} + \alpha_{33}^{(i)}\Phi_{22}^{(i)})\,, \\ J_{21}^{(i)} &= \sqrt{\lambda_1^{(i)}}\,(\alpha_{33}^{(i)}\Phi_{11}^{(i)} + \mu_{33}^{(i)}\Phi_{21}^{(i)}), & J_{22}^{(i)} &= \sqrt{\lambda_2^{(i)}}\,(\alpha_{33}^{(i)}\Phi_{12}^{(i)} + \mu_{33}^{(i)}\Phi_{22}^{(i)})\,, \end{aligned} \quad (i = 1, 2) \tag{35}$$

and

$$\begin{bmatrix} \xi_{10} \\ \xi_{20} \\ \xi_{30} \\ \xi_{40} \end{bmatrix} = \begin{bmatrix} \dfrac{\Phi_{11}^{(1)}\lambda_1^{(1)}(c_1 - e_1)}{\lambda_1^{(1)} - \lambda_0^{(1)}} + \dfrac{\Phi_{12}^{(1)}\lambda_2^{(1)}(c_2 - e_2)}{\lambda_2^{(1)} - \lambda_0^{(1)}} + \dfrac{\Phi_{11}^{(2)}\lambda_1^{(2)}e_3}{\lambda_1^{(2)} - \lambda_0^{(2)}} + \dfrac{\Phi_{12}^{(2)}\lambda_2^{(2)}e_4}{\lambda_2^{(2)} - \lambda_0^{(2)}} \\ \dfrac{\Phi_{21}^{(1)}\lambda_1^{(1)}(c_1 - e_1)}{\lambda_1^{(1)} - \lambda_0^{(1)}} + \dfrac{\Phi_{22}^{(1)}\lambda_2^{(1)}(c_2 - e_2)}{\lambda_2^{(1)} - \lambda_0^{(1)}} + \dfrac{\Phi_{21}^{(2)}\lambda_1^{(2)}e_3}{\lambda_1^{(2)} - \lambda_0^{(2)}} + \dfrac{\Phi_{22}^{(2)}\lambda_2^{(2)}e_4}{\lambda_2^{(2)} - \lambda_0^{(2)}} \\ \dfrac{J_{11}^{(1)}\sqrt{\lambda_0^{(1)}\lambda_1^{(1)}}\,(c_1 + e_1)}{\lambda_0^{(1)} - \lambda_1^{(1)}} + \dfrac{J_{12}^{(1)}\sqrt{\lambda_0^{(1)}\lambda_2^{(1)}}\,(c_2 + e_2)}{\lambda_0^{(1)} - \lambda_2^{(1)}} + \dfrac{J_{11}^{(2)}\sqrt{\lambda_0^{(2)}\lambda_1^{(2)}}\,e_3}{\lambda_0^{(2)} - \lambda_1^{(2)}} + \dfrac{J_{12}^{(2)}\sqrt{\lambda_0^{(2)}\lambda_2^{(2)}}\,e_4}{\lambda_0^{(2)} - \lambda_2^{(2)}} + \dfrac{Q(p^{(2)} - p^{(1)})}{2\pi(\tilde{k}^{(1)} + \tilde{k}^{(2)})} \\ \dfrac{J_{21}^{(1)}\sqrt{\lambda_0^{(1)}\lambda_1^{(1)}}\,(c_1 + e_1)}{\lambda_0^{(1)} - \lambda_1^{(1)}} + \dfrac{J_{22}^{(1)}\sqrt{\lambda_0^{(1)}\lambda_2^{(1)}}\,(c_2 + e_2)}{\lambda_0^{(1)} - \lambda_2^{(1)}} + \dfrac{J_{21}^{(2)}\sqrt{\lambda_0^{(2)}\lambda_1^{(2)}}\,e_3}{\lambda_0^{(2)} - \lambda_1^{(2)}} + \dfrac{J_{22}^{(2)}\sqrt{\lambda_0^{(2)}\lambda_2^{(2)}}\,e_4}{\lambda_0^{(2)} - \lambda_2^{(2)}} + \dfrac{Q(m^{(2)} - m^{(1)})}{2\pi(\tilde{k}^{(1)} + \tilde{k}^{(2)})} \end{bmatrix}, \tag{36}$$

and

$$\begin{bmatrix} \xi_{11} & \xi_{12} \\ \xi_{21} & \xi_{22} \\ \xi_{31} & \xi_{32} \\ \xi_{41} & \xi_{42} \end{bmatrix} = \begin{bmatrix} -\Phi_{11}^{(1)} & -\Phi_{21}^{(1)} \\ -\Phi_{12}^{(1)} & -\Phi_{22}^{(1)} \\ J_{11}^{(1)} & J_{21}^{(1)} \\ J_{12}^{(1)} & J_{22}^{(1)} \end{bmatrix} \begin{bmatrix} \dfrac{\lambda_1^{(1)} c_1}{\lambda_1^{(1)} - \lambda_0^{(1)}} & 0 \\ 0 & \dfrac{\lambda_2^{(1)} c_2}{\lambda_2^{(1)} - \lambda_0^{(1)}} \end{bmatrix}. \tag{37}$$

It is observed from Eq. (34) that the twelve unknowns L_{ij} can be simply determined by inverting a *single* 4×4 matrix. This concise procedure is similar to that for the corresponding isothermal case [11]. Once the electric and magnetic potentials in the multiferroic bimaterial are obtained, the electric and magnetic fields as well as electric displacements and magnetic fluxes can be found by taking the derivatives of the electric and magnetic potentials.

4 Conclusions The three-dimensional electromagnetic Green's function solutions for a steady-state point heat source in a uniaxial multiferroic material and bimaterial are derived. The Green's function expressions for a multiferroic full-space are given in Eqs. (22)–(24) for the nondegenerate case $\lambda_1 \neq \lambda_2 \neq \lambda_0$ and in Eqs. (26)–(28) for the degenerate case $\lambda_1 = \lambda_0$ ($\lambda_1 \neq \lambda_2$). The electromagnetic fields induced by a steady point heat source at the origin of a uniaxial multiferroic full-space with the x_3-axis being its uniaxial axis exhibit the following properties: (1) The electric and magnetic potentials as well as the horizontal electric and magnetic fields (and as a result the horizontal electric displacements and magnetic fluxes) are odd functions of x_3, and are zero on the horizontal plane $x_3 = 0$; (2) The vertical electric and magnetic fields (and as a result the vertical electric displacements and magnetic fluxes) are even functions of x_3, and are inversely proportional to r when $x_3 = 0$, and are zero on the x_3-axis excluding the origin. The Green's function solutions for two bonded multiferroic half-spaces are presented in Eqs. (30)–(31) with the twelve constants L_{ij} being determined by Eq. (34). We further remark that by making use of the image method discussed in Section 3, the point heat source induced electromagnetic Green's functions in a multiferroic half-space with various surface electromagnetic boundary conditions [11] can also be derived, and that the influence of the temperature on the electric and magnetic fields will be pursued using the developed Green's functions.

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