Reflection and Transmission Coefficients of Plane Waves in Magnetoelectroelastic Layered Structures

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Reflection and transmission coefficients of plane waves with oblique incidence to a multilayered system of piezomagnetic and/or piezoelectric materials are investigated in this paper. The general Christoffel equation is derived from the coupled constitutive and balance equations, which is further employed to solve the elastic displacements and electric and magnetic potentials. Based on these solutions, the reflection and transmission coefficients in the corresponding layered structures are subsequently obtained by virtue of the propagator matrix method. Two layered examples are selected to verify and illustrate our solutions. One is the purely elastic layered system composed of aluminum and organic glass materials. The other layered system is composed of the novel magnetoelectroelastic material and the organic glass. Numerical results are presented to demonstrate the variation of the reflection and transmission coefficients with different incident angles, frequencies, and boundary conditions, which could be useful to nondestructive evaluation of this novel material structure based on wave propagations.
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Keywords: reflection and transmission coefficients, magnetoelectroelastic material, layered structure, wave propagation

1 Introduction

Starting from the work by Suchtelen [1] on magnetoelectroelastic materials, some mechanics problems in structures made of these multiphase materials have generated great interests in recent years, including static deformation, vibration, and fracture [2–4]. More recently, behaviors of ultrasonic plane waves in piezomagnetic and/or piezoelectric plates have also attracted wide attention [5–7].

As the basic parameters of ultrasonic waves, reflection and transmission coefficients for plane waves, which are obliquely incident to the multilayered elastic or piezoelectric plates, have been extensively studied for some time. For example, Thomson derived the formulation for plane wave propagation in a multilayered solid structure in terms of the continuity of stresses and particle velocities across the interfaces [8]. Haskell extended Thomson's work to the more general multilayered case [9], with later contribution by Brekhovskikh [10]. With the application of acoustic transducers in underwater sonar equipments, reflection and transmission of elastic waves at the boundary of and/or interface between piezoelectric and elastic materials and fluid were discussed carefully in a variety of papers and books [11–18], including also those for electromagnetic materials [19,20]. More recently, sound propagation and power transmission issues [21,22] were investigated, and a spectral finite element model was also proposed for wave analysis in laminated composite [23]. To the best knowledge of the authors, however, reflection and transmission of wave incidence on the multilayered magnetoelectroelastic structure have not been investigated so far, which motivates the present study. Wave propagation feature in the novel magnetoelectroelastic material system is very important and it needs to be fully understood before the material system's real application in practice.

In order to simplify our discussion, only the two dimensional (2D) problem is considered. That is, the plane wave incident on every layer is limited to the same vertical plane. The general Christoffel equation is first derived by combining the coupled constitutive equations and equilibrium equations. It is then employed to yield the elastic displacements and electric and magnetic potentials. Based on these solutions, the reflection and transmission coefficients are subsequently solved for the corresponding layered structure by virtue of the propagator matrix method along with the given interface and boundary conditions. Finally, two numerical examples are used to verify and illustrate our formulation. One is a purely elastic layered system composed of aluminum and organic glass materials. The other layered model is composed of magnetoelectroelastic materials and organic glass. Our numerical results show clearly the variation of the reflection and transmission coefficients with different incident angles, frequencies, and boundary conditions.

2 Governing Equations

Figure 1 shows a structure composed of a semi-infinite homogeneous elastic base and a magnetoelectroelastic and multilayered plate. Layers 1 to $N-1$ are all made of magnetoelectroelastic materials with hexagon crystal structure of class 6 mm and the Nth layer is the semi-infinite elastic base. The Cartesian coordinate system is attached to the layered structures with x_3 being the symmetry axis of the crystal. Since only 2D deformation in the x_1ox_3 plane is considered, there is no displacement in the x_2 direction. The elastic displacements in the $x_1\mathrm{o}x_3$ plane are expressed as follows:

$$u_1 = u_1(x_1,x_3,t) \quad u_3 = u_3(x_1,x_3,t) \tag{1}$$

Similarly, the electric and magnetic potentials can also be assumed to be

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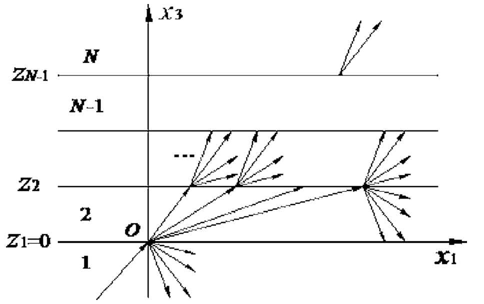

Fig. 1 A semi-infinite elastic base (*N*th layer) bonded to a layered magnetoelectroelastic plate (Layer 1 to Layer *N*−1)

$$\phi = \phi(x_1, x_3, t) \quad \psi = \psi(x_1, x_3, t) \tag{2}$$

Assuming that there is no body force and no electric and magnetic charge in the magnetoelectroelastic system, the general governing equations are then given by [2]

$$\sigma_{ij,j} = \rho \frac{\partial^2 u_i}{\partial t^2} \quad D_{j,j} = 0 \quad B_{j,j} = 0 \tag{3}$$

where σ_{ij}, D_j, and B_j are, respectively, the stresses, electric displacements, and magnetic inductions; and ρ and t are, respectively, the density of the magnetoelectroelastic material and the time variable. For 2D deformation in the $x_1 o x_3$ plane with the symmetry axis of the material along the x_3 direction, the coupling constitutive equations in Ref. [2] can then be simplified as

$$\begin{aligned}
\sigma_{11} &= c_{11}\gamma_{11} + c_{13}\gamma_{33} - e_{31}E_3 - q_{31}H_3 \\
\sigma_{33} &= c_{13}\gamma_{11} + c_{33}\gamma_{33} - e_{33}E_3 - q_{33}H_3 \\
\sigma_{13} &= c_{55}\gamma_{13} - e_{15}E_1 - q_{15}H_1 \\
D_1 &= e_{15}\gamma_{13} - \varepsilon_{11}E_1 - d_{11}H_1 \\
D_3 &= e_{31}\gamma_{11} + e_{33}\gamma_{33} - \varepsilon_{33}E_3 - d_{33}H_3 \\
B_1 &= q_{15}\gamma_{13} - d_{11}E_1 - \mu_{11}H_1 \\
B_3 &= q_{31}\gamma_{11} + q_{33}\gamma_{33} - d_{33}E_3 - \mu_{33}H_3
\end{aligned} \tag{4}$$

where c_{ij}, ε_{ij}, μ_{ij}, e_{ij}, q_{ij}, and d_{ij} are the elastic stiffness, dielectric coefficients, magnetic permeability, and piezoelectric, piezomagnetic, and magnetoelectric constants; and γ_{ij}, and E_j and H_j are the elastic strain, and the electric and magnetic fields, respectively. They are related to the elastic displacement (u_i) and the electric (ϕ) and magnetic (ψ) potentials as

$$\begin{aligned}
&\gamma_{11} = u_{1,1} \quad \gamma_{13} = u_{1,3} + u_{3,1} \quad \gamma_{33} = u_{3,3} \quad E_1 = -\phi_{,1} \\
&E_3 = -\phi_{,3} \quad H_1 = -\psi_{,1} \quad H_3 = -\psi_{,3}
\end{aligned} \tag{5}$$

Substituting Eqs. (4) and (5) into Eq. (3), the elastic displacements and electric and magnetic potentials are found to satisfy the following magnetoelectroelastic coupling equations of motion:

$$\begin{aligned}
&c_{11}u_{1,11} + c_{55}u_{1,33} + (c_{13} + c_{55})u_{3,13} + (e_{31} + e_{15})\phi_{,13} + (q_{31} + q_{15})\psi_{,13} = \rho \ddot{u}_1 \\
&(c_{13} + c_{55})u_{1,13} + c_{55}u_{3,11} + c_{33}u_{3,33} + e_{15}\phi_{,11} + e_{33}\phi_{,33} + q_{15}\psi_{,11} + q_{33}\psi_{,33} = \rho \ddot{u}_3 \\
&(e_{15} + e_{31})u_{1,13} + e_{15}u_{3,11} + e_{33}u_{3,33} - \varepsilon_{11}\phi_{,11} - \varepsilon_{33}\phi_{,33} - d_{11}\psi_{,11} - d_{33}\psi_{,33} = 0 \\
&(q_{15} + q_{31})u_{1,13} + q_{15}u_{3,11} + q_{33}u_{3,33} - d_{11}\phi_{,11} - d_{33}\phi_{,33} - \mu_{11}\psi_{,11} - \mu_{33}\psi_{,33} = 0
\end{aligned} \tag{6}$$

Since the phase angle difference between the elastic displacement along the x_1 axis and that along the x_3 axis is 90 deg, and the phase angle of the electric and magnetic potentials is the same as u_3 [7], we can write the general displacements (elastic displacements and electric and magnetic potentials) in the vector form as

$$\begin{bmatrix} u_1 \\ u_3 \\ \phi \\ \psi \end{bmatrix} = \begin{bmatrix} A \\ iB \\ iC \\ iD \end{bmatrix} e^{bkx_3} e^{ik(x_1 - ct)} \tag{7}$$

where $i = \sqrt{-1}$; A, B, C, and D are the amplitudes of the wave to be determined; k is the wave number along the x_1 direction; and c is the phase velocity of the wave. Substituting Eq. (7) into Eq. (6) yields [6]

$$\mathbf{\Gamma S} = 0 \tag{8}$$

where

$$\mathbf{\Gamma} = \begin{bmatrix} c_{11} - c_{55}b^2 - \rho c^2 & (c_{13} + c_{55})b & (e_{31} + e_{15})b & (q_{31} + q_{15})b \\ & c_{33}b^2 - c_{55} + \rho c^2 & e_{33}b^2 - e_{15} & q_{33}b^2 - q_{15} \\ & & \varepsilon_{11} - \varepsilon_{33}b^2 & d_{11} - d_{33}b^2 \\ & \text{sym} & & \mu_{11} - \mu_{33}b^2 \end{bmatrix}$$

$$\mathbf{S} = [A \quad B \quad C \quad D]^T$$

Equation (8) is called the general Christoffel equation. It is apparent that a nontrivial solution for A, B, C, and D requires that

$$|\mathbf{\Gamma}| = 0 \tag{9}$$

For a given phase velocity c, there are eight eigenvalues for b, each corresponding to a wave propagating in the magnetoelectroelastic layer and yielding a partial solution to the magnetoelectroelastic layer. These roots can be divided into two categories with each having four eigenvalues representing the quasilongitudinal wave and quasitransverse wave and those associated with the electric and magnetic potentials. The following rules are utilized in order to identify the wave mode for a given eigenvalue: (1) because the electric and magnetic potentials must satisfy the Laplace equation (in the uncoupled case), the eigenvalues corresponding to them are in general real, and (2) the eigenvalue corresponding to the longitudinal or transverse wave can be real or complex. A real eigenvalue represents an attenuate wave, while a complex one represents a general harmonic wave.

We further remark that, for the two types of the eigenvalues discussed above, one describes the wave propagating along the positive x_3 direction, and the other along the negative direction. The eigenvalues describing the wave along the positive x_3 direction are either negative real (representing an attenuate wave) or complex with positive image part (representing a harmonic wave). For the wave propagating along the opposite direction, the properties of the corresponding eigenvalues are just opposite. Thus, we can assume that b_m ($m = 1, 2, 3, 4$) represent the waves associated with the two potentials and the longitudinal and transverse waves propagating along the positive x_3 direction, and b_m ($m = 5, 6, 7, 8$) represent those along the opposite direction.

For the semi-infinite elastic base, the general Christoffel equation (8) is reduced to the following simple eigenvalue problems:

$$\mathbf{\Gamma}'\mathbf{S}' = 0 \tag{10}$$

where

$$\mathbf{\Gamma}' = \begin{bmatrix} c'_{11} - c'_{55}b'^2 - \rho' c^2 & (c'_{13} + c'_{55})b' \\ (c'_{13} + c'_{55})b' & c'_{33}b'^2 - c'_{55} + \rho' c^2 \end{bmatrix}$$

$$\mathbf{S}' = [A' \quad B']^T$$

In general, there are four eigenvalues from Eq. (10). However, since in the semi-infinite base wave propagates only along the positive x_3 direction, only two roots are reserved. These two roots have either a negative real part or a positive image part.

3 Reflection and Transmission Coefficients

Having stated the basic rules in solving the eigensystem, we can now derive the reflection and transmission coefficients in the first magnetoelectroelastic layer (Layer 1) and the Nth elastic layer of the system. As for other layered systems, the propagator matrix method can be employed [7].

First, for any layer with the exception of the Nth layer (the semi-infinite elastic base), the general displacements can be expressed, with the subwave method, as

$$\begin{bmatrix} u_1 \\ u_3 \\ \phi \\ \psi \end{bmatrix} = \sum_{m=1}^{8} \xi_m \begin{bmatrix} A_m \\ iB_m \\ iC_m \\ iD_m \end{bmatrix} e^{b_m k x_3} e^{ik(x_1-ct)} \tag{11}$$

where ξ_m $(m=1-8)$ are unknown coefficients to be determined. Substituting Eq. (11) into the general constitutive relation for the magnetoelectroelastic material, we then find the stress, electric displacement, and magnetic induction as

$$\begin{bmatrix} \sigma_{33} \\ \sigma_{13} \\ \Phi_3 \\ \Psi_3 \end{bmatrix} = \sum_{m=1}^{8} \xi_m \begin{bmatrix} ik(c_{13}A_m + c_{33}B_m b_m + e_{33}C_m b_m + q_{33}D_m b_m) \\ k(c_{55}A_m b_m - c_{55}B_m - e_{15}C_m - q_{15}D_m) \\ ik(e_{31}A_m + e_{33}B_m b_m - \varepsilon_{33}C_m b_m - d_{33}D_m b_m) \\ ik(q_{31}A_m + q_{33}B_m b_m - d_{33}C_m b_m - \mu_{33}D_m b_m) \end{bmatrix} \times e^{b_m k x_3} e^{ik(x_1-ct)} \tag{12}$$

where σ_{33} and σ_{13} are the normal and shear stresses, and $\Phi_3(\equiv D_3)$ and $\Psi_3(\equiv B_3)$ are, respectively, the electric displacement and magnetic induction in the x_3 direction.

Second, for the Nth layer (e.g., the semi-infinite elastic base), since the ultrasonic wave propagates only along the positive x_3 direction, the displacements and stresses are given by

$$\begin{bmatrix} u_1' \\ u_3' \end{bmatrix} = \sum_{m=1}^{2} \xi_m' \begin{bmatrix} A_m' \\ iB_m' \end{bmatrix} e^{b_m' k x_3} e^{ik(x_1-ct)} \tag{13}$$

$$\begin{bmatrix} \sigma_{33}' \\ \sigma_{13}' \end{bmatrix} = \sum_{m=1}^{2} \xi_m' \begin{bmatrix} ik(c_{13}'A_m' + c_{33}'B_m' b_m') \\ k(c_{55}'A_m' b_m' - c_{55}'B')_m \end{bmatrix} e^{b_m' k x_3} e^{ik(x_1-ct)} \tag{14}$$

where ξ_m' $(m=1,2)$ are new unknown coefficients to be determined.

Finally, in the layered structure, we assume that all the magnetoelectroelastic layers ($j=1$ to $N-1$) are well bonded to each other. Therefore, the out-of-plane variables are continuous along these interfaces. In other words, at all interfaces, these quantities satisfy

$$\begin{bmatrix} u_1 \\ u_3 \\ \phi \\ \psi \end{bmatrix}_{x_3=z_j}^{(j)} = \begin{bmatrix} u_1 \\ u_3 \\ \phi \\ \psi \end{bmatrix}_{x_3=z_j}^{(j+1)}$$

$$\begin{bmatrix} \sigma_{33} \\ \sigma_{13} \\ \Phi_3 \\ \Psi_3 \end{bmatrix}_{x_3=z_j}^{(j)} = \begin{bmatrix} \sigma_{33} \\ \sigma_{13} \\ \Phi_3 \\ \Psi_3 \end{bmatrix}_{x_3=z_j}^{(j+1)} \quad j=1 \quad 2,\ldots,N-2 \tag{15}$$

As for the interface between Layer $N-1$ and Layer N (the semi-infinite elastic base), both the open and short circuit conditions for the electric and magnetic fields are assumed. In other words, for the open circuit interface, the elastic displacements and tractions, and the electrical displacement and magnetic induction should satisfy the following continuity conditions:

$$\begin{bmatrix} u_1 \\ u_3 \end{bmatrix}_{x_3=z_{N-1}}^{(N-1)} = \begin{bmatrix} u_1' \\ u_3' \end{bmatrix}_{x_3=z_{N-1}}^{(N)} \quad \begin{bmatrix} \sigma_{33} \\ \sigma_{13} \end{bmatrix}_{x_3=z_{N-1}}^{(N-1)} = \begin{bmatrix} \sigma_{33}' \\ \sigma_{13}' \end{bmatrix}_{x_3=z_{N-1}}^{(N)}$$

$$[\Phi_3]_{x_3=z_{N-1}}^{(N-1)} = 0 \quad [\Psi_3]_{x_3=z_{N-1}}^{(N-1)} = 0 \tag{16}$$

For the short circuit interface, the above continuity conditions are replaced by

$$\begin{bmatrix} u_1 \\ u_3 \end{bmatrix}_{x_3=z_{N-1}}^{(N-1)} = \begin{bmatrix} u_1' \\ u_3' \end{bmatrix}_{x_3=z_{N-1}}^{(N)} \quad \begin{bmatrix} \sigma_{33} \\ \sigma_{13} \end{bmatrix}_{x_3=z_{N-1}}^{(N-1)} = \begin{bmatrix} \sigma_{33}' \\ \sigma_{13}' \end{bmatrix}_{x_3=z_{N-1}}^{(N)}$$

$$[\phi]_{x_3=z_{N-1}}^{(N-1)} = 0 \quad [\psi]_{x_3=z_{N-1}}^{(N-1)} = 0 \tag{17}$$

In summary, for a given phase velocity, we find that the total number of unknowns is $5+8(N-2)+2$ (from the above formulation). On the other hand, the total number of equations including the continuity conditions and boundary conditions is $4+8(N-2)+2$, one less than the total number of unknowns. Therefore, it is clear that the reflection and transmission coefficients can be solved from these equations (i.e., the relative amplitude of the induced wave over the incident wave).

Using Eq. (11), the amplitude of the incident wave in the first layer is given by

$$u_{\text{in}} = \xi_4 \sqrt{A_4\bar{A}_4 + B_4\bar{B}_4} \tag{18}$$

where an overbar represents the conjugate of a complex variable. Similarly, the amplitudes of the reflected longitudinal and transverse waves in the first layer are

$$u_7 = \xi_7 \sqrt{A_7\bar{A}_7 + B_7\bar{B}_7} \quad u_8 = \xi_8 \sqrt{A_8\bar{A}_8 + B_8\bar{B}_8} \tag{19}$$

Making use of these expressions for the wave amplitudes, the reflection coefficients of the longitudinal and transverse waves in the first magnetoelectroelastic layer are found to be

$$R_L = \left|\frac{u_7}{u_{\text{in}}}\right| = \left|\frac{\xi_7 \sqrt{A_7\bar{A}_7 + B_7\bar{B}_7}}{\xi_4 \sqrt{A_4\bar{A}_4 + B_4\bar{B}_4}}\right|$$

$$R_T = \left|\frac{u_8}{u_{\text{in}}}\right| = \left|\frac{\xi_8 \sqrt{A_8\bar{A}_8 + B_8\bar{B}_8}}{\xi_4 \sqrt{A_4\bar{A}_4 + B_4\bar{B}_4}}\right| \tag{20}$$

Similarly, the transmission coefficients of the longitudinal and transverse wave in the semi-infinite elastic base are given by

$$T_L = \left|\frac{u_1'}{u_{\text{in}}}\right| = \left|\frac{\xi_1' \sqrt{A_1'\bar{A}_1' + B_1'\bar{B}_1'}}{\xi_4 \sqrt{A_4\bar{A}_4 + B_4\bar{B}_4}}\right|$$

$$T_T = \left|\frac{u_2'}{u_{\text{in}}}\right| = \left|\frac{\xi_2' \sqrt{A_2'\bar{A}_2' + B_2'\bar{B}_2'}}{\xi_4 \sqrt{A_4\bar{A}_4 + B_4\bar{B}_4}}\right| \tag{21}$$

Furthermore, the general displacements and stresses at any other interfaces can also be solved by the state space method [7].

Table 1 Material constants of aluminum and organic glass

Material	Constants				
	c_{11} ($\mathrm{N/m^2}$)	c_{13} ($\mathrm{N/m^2}$)	c_{33} ($\mathrm{N/m^2}$)	c_{55} ($\mathrm{N/m^2}$)	ρ ($\mathrm{kg/m^3}$)
Aluminum	108×10^9	51×10^9	108×10^9	28.5×10^9	2700
Organic glass	8.41×10^9	5.05×10^9	8.41×10^9	1.48×10^9	1180

It can be easily shown that the state vector of the second layer at $z=z_1$ is related to that of Layer $N-1$ at $z=z_{N-1}$ by the following propagating relation [7]:

$$\mathbf{X}^{(N-1)}(z_{N-1}) = \mathbf{T}\mathbf{X}^{(2)}(z_1) \tag{22}$$

where $\mathbf{X}=[u_1\ i\Phi_3\ i\Psi_3\ i\sigma_{33}\ \sigma_{13}\ i\phi\ i\psi\ iu_3]^T$ is the state vector [7], and $\mathbf{T}$ is the global propagator matrix given as

$$\mathbf{T} = \mathbf{P}_{N-1}(h_{N-1})\mathbf{P}_{N-2}(h_{N-2}), \ldots, \mathbf{P}_2(h_2)$$

with $\mathbf{P}_j(h_j)$ and h_j being, respectively, the propagator matrix and thickness of Layer j.

Noticing the interfaces between Layers $N-1$ and N, and between the first and second layers, the state vectors satisfy

$$\mathbf{X}^{(N-1)}(z_{N-1}) = \mathbf{X}^{(N)}(z_{N-1}) \quad \mathbf{X}^{(2)}(z_1) = \mathbf{X}^{(1)}(z_1) \tag{23}$$

The state vectors at these two interfaces can then be written as

$$\mathbf{X}^{(N)}(z_{N-1}) = \begin{bmatrix} a_{11} & a_{12} \\ 0 & 0 \\ 0 & 0 \\ a_{41} & a_{42} \\ a_{51} & a_{52} \\ \cdots & \cdots \\ \cdots & \cdots \\ a_{81} & a_{82} \end{bmatrix} \begin{bmatrix} \xi_1' \\ \xi_2' \end{bmatrix}$$

$$\mathbf{X}^{(1)}(z_1) = \mathbf{B}\begin{bmatrix} \xi_4 \\ \xi_5 \\ \xi_6 \\ \xi_7 \\ \xi_8 \end{bmatrix} = \begin{bmatrix} b_{14} & \cdots & b_{18} \\ b_{24} & \cdots & b_{28} \\ b_{34} & \cdots & b_{38} \\ b_{44} & \cdots & b_{48} \\ b_{54} & \cdots & b_{58} \\ b_{64} & \cdots & b_{68} \\ b_{74} & \cdots & b_{78} \\ b_{84} & \cdots & b_{88} \end{bmatrix}_{8\times5} \begin{bmatrix} \xi_4 \\ \xi_5 \\ \xi_6 \\ \xi_7 \\ \xi_8 \end{bmatrix} \tag{24}$$

where

$$a_{1m} = A_m' e^{b_m' k z_{N-1}} \quad a_{4m} = -k(c_{13}'A_m' + c_{33}'B_m'b_m')e^{b_m' k z_{N-1}}$$

$$a_{5m} = k(c_{55}'A_m'b_m' - c_{55}'B_m')e^{b_m' k z_{N-1}} \quad a_{8m} = -B_m' e^{b_m' k z_{N-1}} \quad (m=1,2)$$

$$b_{1m} = A_m \quad b_{2m} = -k(e_{31}A_m + e_{33}B_m b_m - \varepsilon_{33}C_m b_m - d_{33}D_m b_m)$$

$$b_{3m} = -k(q_{31}A_m + q_{33}B_m b_m - d_{33}C_m b_m - \mu_{33}D_m b_m)$$

$$b_{4m} = -k(c_{13}A_m + c_{33}B_m b_m + e_{33}C_m b_m + q_{33}D_m b_m)$$

$$b_{5m} = k(c_{55}A_m b_m - c_{55}B_m - e_{15}C_m - q_{15}D_m) \quad b_{6m} = -C_m$$

$$b_{7m} = -D_m \quad b_{8m} = -B_m \quad (m=4,5,\ldots,8)$$

Substituting Eq. (23) into Eq. (24), and then into Eq. (22) leads to

$$\begin{bmatrix} a_{11} & a_{12} \\ 0 & 0 \\ 0 & 0 \\ a_{41} & a_{42} \\ a_{51} & a_{52} \\ \cdots & \cdots \\ \cdots & \cdots \\ a_{81} & a_{82} \end{bmatrix} \begin{bmatrix} \xi_1' \\ \xi_2' \end{bmatrix} = \begin{bmatrix} c_{11} & \cdots & c_{15} \\ c_{21} & \cdots & c_{25} \\ c_{31} & \cdots & c_{35} \\ c_{41} & \cdots & c_{45} \\ c_{51} & \cdots & c_{55} \\ c_{61} & \cdots & c_{65} \\ c_{71} & \cdots & c_{75} \\ c_{81} & \cdots & c_{85} \end{bmatrix}_{8\times5} \begin{bmatrix} \xi_4 \\ \xi_5 \\ \xi_6 \\ \xi_7 \\ \xi_8 \end{bmatrix} = \mathbf{TB}\begin{bmatrix} \xi_4 \\ \xi_5 \\ \xi_6 \\ \xi_7 \\ \xi_8 \end{bmatrix} \tag{25}$$

Applying the open circuit condition and the interface continuity condition described by Eq. (16), we can rearrange Eq. (25) into the following form:

$$\begin{bmatrix} a_{11} & a_{12} & -c_{12} & -c_{13} & -c_{14} & -c_{15} \\ 0 & 0 & -c_{22} & -c_{23} & -c_{24} & -c_{25} \\ 0 & 0 & -c_{32} & -c_{33} & -c_{34} & -c_{35} \\ a_{41} & a_{42} & -c_{42} & -c_{43} & -c_{44} & -c_{45} \\ a_{51} & a_{52} & -c_{52} & -c_{53} & -c_{54} & -c_{55} \\ a_{81} & a_{82} & -c_{82} & -c_{83} & -c_{84} & -c_{85} \end{bmatrix} \begin{bmatrix} \xi_1' \\ \xi_2' \\ \xi_5 \\ \xi_6 \\ \xi_7 \\ \xi_8 \end{bmatrix} = \begin{bmatrix} c_{11} \\ c_{21} \\ c_{31} \\ c_{41} \\ c_{51} \\ c_{81} \end{bmatrix} \xi_4 \tag{26}$$

Similar expression can be found for the short circuit condition (Eq. (17)). Therefore, for a fixed coefficient of incident wave ξ_4, this equation can be solved for the unknown coefficients on the left-hand side. Particularly, we can assume that ξ_4 is equal to 1, which gives us the reflection and transmission coefficients of the multilayered structure from Eqs. (20) and (21).

4 Numerical Example

In this section, the above formulation is applied to analyze the reflection and transmission coefficients of the multilayered plate. The plate is composed of piezoelectric, magnetic, and purely elastic materials. While $BaTiO_3$ is selected for the piezoelectric layer, $CoFe_2O_4$ is selected for the magnetostrictive layer. Before studying the magnetoelectroelastic system, a purely elastic multilayered structure composed of aluminum and organic glass is employed to verify our formulation.

4.1 Purely Elastic Layered Structure. Our purely elastic structure is similar to that of Rose [14]. As shown in Fig. 1, it is assumed that the structure has four layers: three layers of aluminum with thickness of 0.001 m each, plus the organic glass base. The elastic constants c_{ij} and density ρ of the materials are listed in Table 1.

Figure 2 shows the variation of the reflection coefficients at the interface of the first and second layers and the transmission coefficients at the interface of the third aluminum layer and organic glass base due to both transverse and longitudinal incident waves. It is observed that the variation of these coefficients with the incident angle is the same as that of Rose [14].

We also observed from our numerical calculation that, among these four coefficients, only the reflection coefficient of longitudinal wave varies with the frequency of the incident wave. This is shown in Fig. 3, where it is noticed that the reflection coefficient

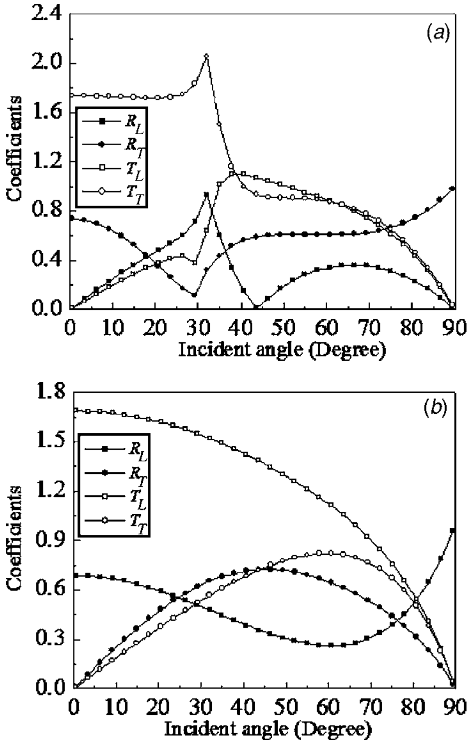

Fig. 2 Variation of reflection and transmission coefficients of the elastic wave in Al-glass structure: (*a*) transverse incident wave and (*b*) longitudinal incident wave

decreases with increasing frequency when the incident angle is larger than the first critical angle (about 30 deg). Furthermore, instead of the reflected longitudinal wave, interfacial wave appears at the interface for this case, with a high incident frequency corresponding to a wave with fast attenuate amplitude.

4.2 Magnetoelectroelastic Layered Structure. After validating our formulation for the purely elastic layered structure, we now apply it to a multilayered magnetoelectroelastic (MEE) structure, which composed of four layers. The first and third layers are made of piezoelectric material $BaTiO_3$, and the second layer of piezomagnetic material $CoFe_2O_4$, and the semi-infinite elastic base of organic glass. It is also assumed that the thicknesses of piezoelectric and piezomagnetic layers are the same and equal to 0.001 m. The material properties are listed in Table 2 [7]. Following Refs. [24,25], positive magnetic permeability is chosen in the calculation. Again, the reflection coefficients are at the interface of

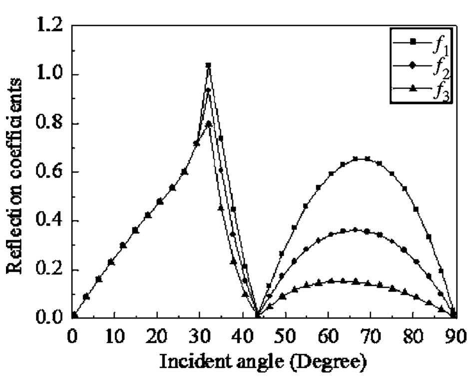

Fig. 3 Variation of reflection coefficient of longitudinal wave with incident angle and at different frequencies (transverse incident wave, $f_1 < f_2 < f_3$)

Table 2 Material properties (c_{ij} in N/m^2, e_{ij} in C/m^2, q_{ij} in N/A m, ε_{ij} in C^2/(N m^2), μ_{ij} in N s^2/C^2, and ρ in kg/m^3)

Properties	$BaTiO_3$	$CoFe_2O_4$	Properties	$BaTiO_3$	$CoFe_2O_4$
c_{11}	166×10^9	286×10^9	q_{31}	0	580.3
c_{13}	78×10^9	170.5×10^9	q_{33}	0	699.7
c_{33}	162×10^9	269.5×10^9	q_{15}	0	550
c_{55}	43×10^9	45.3×10^9	ε_{11}	11.2×10^{-9}	0.08×10^{-9}
e_{31}	−4.4	0	ε_{33}	12.6×10^{-9}	0.093×10^{-9}
e_{33}	18.6	0	μ_{11}	5×10^{-6}	590×10^{-6}
e_{15}	11.6	0	μ_{33}	10×10^{-6}	157×10^{-6}
ρ	5800	5300			

the first and second layers, and the transmission coefficients are at the interface of the $N-1$th layer and the semi-infinite elastic base (refer to Fig. 1).

Figure 4 presents the three-dimensional plot for the four coefficients as functions of the incident angle (varying from 0 deg to 90 deg) and the dimensionless frequency $\omega H\sqrt{\rho_{\max}/c_{\max}}$ (herein $\omega=kc$, $\rho_{\max}$ and $c_{\max}$ are the maximum density and elastic constant of the system, H is the total thickness of magnetoelectroelastic layered plate, and the dimensionless frequency varies from 0 to 2). The incident wave is transverse. From Fig. 4, we observed that near the critical incident angle, the coefficients are very large at low frequencies except for the reflection coefficient of the transverse wave (Fig. 4(*b*)). Furthermore, these coefficients decrease rapidly with increasing frequency. On the other hand, if the incident angle is much smaller or much larger than the critical incident angle, these coefficients are, in general, insensitive to the frequency of the incident wave. When the incident angle is equal to 90 deg (corresponding to a sweeping incident wave), the reflection coefficient of transverse wave is equal to 1, while other coefficients are equal to zero. In this case, all energy is reflected in the form of transverse polarization.

Figure 5 shows the three-dimensional plot for the four coefficients as the function of the incident angle and the dimensionless frequency when the incident wave is longitudinal. Similar to the transverse incident wave, the variation of the reflection and transmission coefficients due to longitudinal incident wave is also sensitive to the frequency of incident wave. However, different from Fig. 4, we observed from Figs. 5(*b*) and 5(*d*) that the reflection and transmission coefficients of transverse wave increase with increasing frequency. We further remark that when the incident angle is equal to 90 deg, the reflection coefficient of the longitudinal wave is equal to 1, which is independent of the incident frequency. This phenomenon is also consistent with that observed in the purely elastic structure.

In order to investigate the effects of the electric circuit condition on the reflection and transmission coefficients, the four coefficients due to a longitudinal incident wave under open and short circuits are presented in Fig. 6. It is clear that the circuit boundary condition has only slight effect on the reflection and transmission coefficients.

5 Conclusions

The reflection and transmission coefficients for plane waves at oblique incidence on a multilayered system of piezomagnetic and/or piezoelectric materials are analyzed. The proposed procedure and formulation is simple and universal (e.g., as compared to the potential function method [26]). Typical numerical examples are presented for both purely elastic and magnetoelectroelastic layered structures. For the purely elastic structure, the reflection and transmission coefficients obtained by the present formulation are exactly the same as previously published results. For the magnetoelectroelastic structure, our three-dimensional plots of the reflection and transmission coefficients clearly demonstrate the dependence of these coefficients on the incident angle and

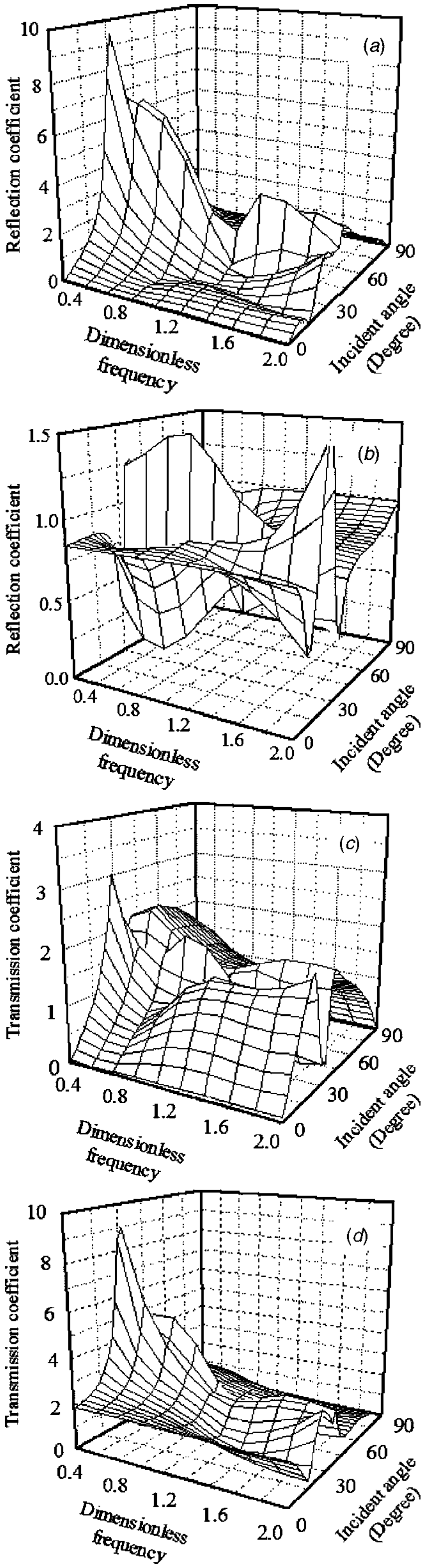

Fig. 4 Variation of reflection and transmission coefficients with incident angle and frequency for a transverse incident wave in MEE system: (*a*) reflection coefficient of longitudinal wave, (*b*) reflection coefficient of transverse wave, (*c*) transmission coefficient of longitudinal wave, and (*d*) transmission coefficient of transverse wave

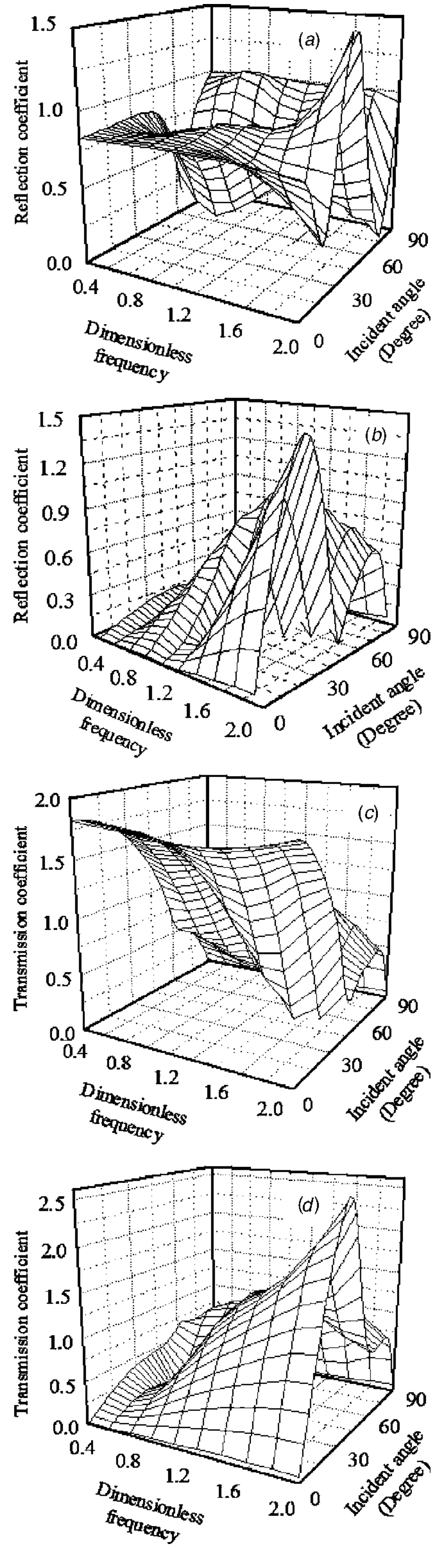

Fig. 5 Variation of reflection and transmission coefficients with incident angle and frequency for a longitudinal incident wave in MEE system: (*a*) reflection coefficient of longitudinal wave, (*b*) reflection coefficient of transverse wave, (*c*) transmission coefficient of longitudinal wave, and (*d*) transmission coefficient of transverse wave

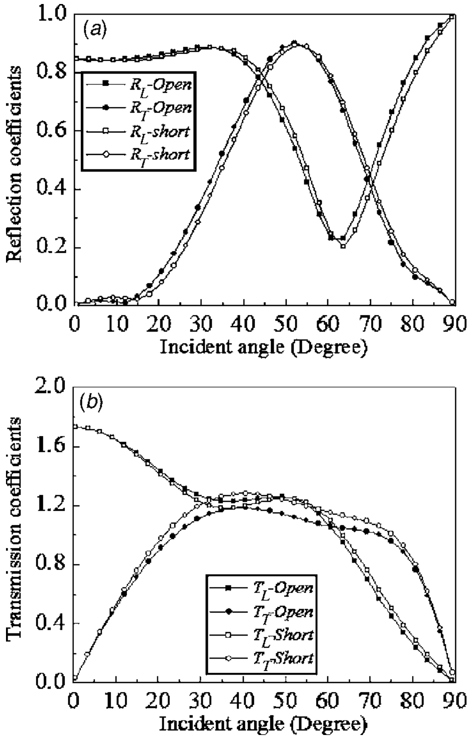

Fig. 6 Variation of reflection (*a*) and transmission (*b*) coefficients with incident angle under both open and short circuit conditions (longitudinal incident wave)

frequencies of the incident wave. Furthermore, we also observe that the electric circuit condition (open or short circuit) has only slight effect on these wave coefficients. These basic features are important to wave studies in real structures where the magneto-electroelastic material serves as one of their members, e.g., for energy conversion among mechanical, electric, and magnetic fields.

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