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# On the screw dislocation in a functionally graded piezoelectric plane and half-plane

X. Wang\*, E. Pan

Department of Civil Engineering and Department of Applied Mathematics, University of Akron, Akron, OH 44325-3905, United States

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# Abstract

In this paper closed-form expressions of the electroelastic field induced by a piezoelectric screw dislocation in a functionally graded piezoelectric plane and half-plane are derived. The material properties are assumed to vary exponentially along the x and y-directions. The solution for a screw dislocation in a functionally graded piezoelectric plane is obtained through introduction of two generalized stress functions. The solution for a screw dislocation in a functionally graded piezoelectric half-plane is derived by using the method of image. It is also found that the interaction between a piezoelectric screw dislocation and a circular insulating hole in the functionally graded piezoelectric material can be solved by using series expansion method.

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Keywords: Functionally graded piezoelectric material; Screw dislocation; Stress function; Method of image; Half-plane; Circular hole

## 1. Introduction

Functionally graded piezoelectric materials (FGPMs) usually refer to the pieozoelectric materials in which the material properties vary smoothly in a given direction. FGPMs can be utilized to provide the desirable thermal-mechanical and piezoelectric properties. Applications of FGPMs can be found in tribology, electronic and biomechanics. Recent analysis (Almajid and Taya, 2001; Wu et al., 2002; Taya et al., 2003) and experimental tests (Zhu et al., 2000; Almajid and Taya, 2001; Taya et al., 2003) have clearly shown the advantage of FGPM actuator or sensor over the traditional ones. Due to their potential application in intelligent/smart structures, FGPMs have attracted wide attention in recent years (see for example Jin and Zhong, 2002; Li and Weng, 2002; Pan and Han, 2005; Collet et al., 2006). A review of the principal developments in functionally graded materials (FGMs) since 2000 can be found in the most recent work by Birman and Byrd (2007).

Most recently by means of the stress function technique, Lazar (2007) addressed a screw dislocation in an elastic medium exponentially graded in the *y*-direction within the framework of the classical elasticity theory.

\* Corresponding author. Tel.: +1 330 972 2457; fax: +1 330 972 6020.

E-mail addresses: xuwang@uakron.edu (X. Wang), pan2@uakron.edu (E. Pan).

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Motivated by Lazar (2007), we investigate in this paper a piezoelectric screw dislocation in an FGPM plane and half-plane within the framework of linear theory of piezoelectricity. Also discussed is a piezoelectric screw dislocation near a circular insulating hole in an FGPM plane.

# 2. A piezoelectric screw dislocation in an FGPM plane

We first consider a static screw dislocation located at origin in an FGPM plane. The FGPM plane is transversely isotropic with the poling direction parallel to the z-axis. The screw dislocation is assumed to be straight and infinitely long in the z-direction, suffering a displacement jump b and an electric potential jump  $\Delta\phi$  across the slip plane.

The engineering shear strains  $\gamma_{zx}$ ,  $\gamma_{zy}$  and the electric fields  $E_x$ ,  $E_y$  should satisfy the following incompatibility conditions (Lazar, 2007):

$$\frac{\partial \gamma_{zy}}{\partial x} - \frac{\partial \gamma_{zx}}{\partial y} = b\delta(x)\delta(y), \tag{1}$$

$$\frac{\partial E_y}{\partial x} - \frac{\partial E_x}{\partial y} = -\Delta\phi\delta(x)\delta(y),\tag{2}$$

where  $\delta()$  is the Dirac delta function.

In addition the strains and electric fields can be expressed in terms of stresses  $\sigma_{zx}$ ,  $\sigma_{zy}$  and electric displacements  $D_x$ ,  $D_y$  as

$$\begin{bmatrix} \gamma_{zy} \\ E_y \end{bmatrix} = \frac{1}{\tilde{c}_{44}(x,y)\epsilon_{11}(x,y)} \begin{bmatrix} \epsilon_{11}(x,y) & e_{15}(x,y) \\ -e_{15}(x,y) & c_{44}(x,y) \end{bmatrix} \begin{bmatrix} \sigma_{zy} \\ D_y \end{bmatrix},$$
(3a)

$$\begin{bmatrix} \gamma_{zx} \\ E_x \end{bmatrix} = \frac{1}{\tilde{c}_{44}(x,y)\epsilon_{11}(x,y)} \begin{bmatrix} \epsilon_{11}(x,y) & e_{15}(x,y) \\ -e_{15}(x,y) & c_{44}(x,y) \end{bmatrix} \begin{bmatrix} \sigma_{zx} \\ D_x \end{bmatrix},$$
(3b)

where  $c_{44}$ ,  $e_{15}$  and  $\epsilon_{11}$  are, respectively, the elastic modulus, the piezoelectric coefficient, and the dielectric permittivity;  $\tilde{c}_{44} = c_{44} + e_{15}^2/\epsilon_{11} \ge c_{44}$  is the piezoelectrically stiffened elastic coefficient. We remark that different from the homogeneous material case,  $c_{44}$ ,  $e_{15}$  and  $\epsilon_{11}$  (and as a result  $\tilde{c}_{44}$ ) in this paper are all functions of the coordinates x and y. In particular, we assume that

$$c_{44} = \exp(2\beta_1 x + 2\beta_2 y)c_{44}^0, \quad e_{15} = \exp(2\beta_1 x + 2\beta_2 y)e_{15}^0, \quad \epsilon_{11} = \exp(2\beta_1 x + 2\beta_2 y)\epsilon_{11}^0, \tag{4}$$

where  $c_{44}^0$ ,  $e_{15}^0$ ,  $\epsilon_{11}^0$ ,  $\beta_1$  and  $\beta_2$  are material constants. It is noticed that the above assumption of exponential variation for the material properties has been adopted by many authors (see for example, Jin and Zhong, 2002; Wang, 2003; Kwon, 2003; Collet et al., 2006) to simplify the analysis involved. Here it should be pointed out that there exist explicit solutions for FGPMs with variations other than the exponential variations assumed in this research (see for example, Collet et al., 2006).

Due to the fact that the stresses and electric displacements satisfy the following equilibrium equations:

$$\frac{\partial \sigma_{zx}}{\partial x} + \frac{\partial \sigma_{zy}}{\partial y} = 0,$$
(5)
$$\frac{\partial D_x}{\partial x} + \frac{\partial D_y}{\partial y} = 0$$
(6)

$$\frac{1}{\partial x} + \frac{y}{\partial y} = 0, \tag{6}$$

then it's convenient to introduce the following generalized stress functions f and g which are related to the stresses and electric displacements through

$$\sigma_{zy} = \frac{\partial f}{\partial x}, \quad \sigma_{zx} = -\frac{\partial f}{\partial y},$$
  

$$D_y = \frac{\partial g}{\partial x}, \quad D_x = -\frac{\partial g}{\partial y}.$$
(7)

As a result the strains and electric fields can be expressed in terms of the two introduced generalized stress functions f and g as

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$$\begin{bmatrix} \gamma_{zy} \\ E_{y} \end{bmatrix} = \frac{1}{\tilde{c}_{44}(x,y)\epsilon_{11}(x,y)} \begin{bmatrix} \epsilon_{11}(x,y) & e_{15}(x,y) \\ -e_{15}(x,y) & c_{44}(x,y) \end{bmatrix} \begin{bmatrix} f_{,x} \\ g_{,x} \end{bmatrix},$$
(8a)

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$$\begin{bmatrix} \gamma_{zx} \\ E_x \end{bmatrix} = -\frac{1}{\tilde{c}_{44}(x,y)\epsilon_{11}(x,y)} \begin{bmatrix} \epsilon_{11}(x,y) & e_{15}(x,y) \\ -e_{15}(x,y) & c_{44}(x,y) \end{bmatrix} \begin{bmatrix} f_{,y} \\ g_{,y} \end{bmatrix}.$$
(8b)

Inserting (8a) and (8b) into Eqs. (1) and (2) yields

$$\nabla^{2} f - 2\beta_{1} f_{,x} - 2\beta_{2} f_{,y} = (c_{44}^{0} b + e_{15}^{0} \Delta \phi) \delta(x) \delta(y),$$
  

$$\nabla^{2} g - 2\beta_{1} g_{,x} - 2\beta_{2} g_{,y} = (e_{15}^{0} b - \epsilon_{11}^{0} \Delta \phi) \delta(x) \delta(y),$$
(9)

We now introduce two new functions  $\varphi$  and  $\psi$  which are related to f and g through

$$f = \exp(\beta_1 x + \beta_2 y)\varphi, \quad g = \exp(\beta_1 x + \beta_2 y)\psi, \tag{10}$$

As a result Eq. (9) can be rewritten in terms of  $\varphi$  and  $\psi$  as

$$\nabla^2 \varphi - \beta^2 \varphi = (c_{44}^0 b + e_{15}^0 \Delta \phi) \delta(x) \delta(y),$$
  

$$\nabla^2 \psi - \beta^2 \psi = (e_{15}^0 b - \epsilon_{11}^0 \Delta \phi) \delta(x) \delta(y),$$
(11)

where  $\beta = \sqrt{\beta_1^2 + \beta_2^2}$ . The solutions to the two inhomogeneous Helmholtz equations in Eq. (11) can be expediently given by (Lazar, 2007)

$$\varphi = -\frac{c_{44}^{0}b + e_{15}^{0}\Delta\phi}{2\pi}K_{0}(\beta r), 
\psi = \frac{\epsilon_{11}^{0}\Delta\phi - e_{15}^{0}b}{2\pi}K_{0}(\beta r),$$
(12)

where  $r = \sqrt{x^2 + y^2}$  and  $K_n$  is the *n*th order modified Bessel function of the second kind.

In view of Eqs. (10) and (12), the explicit expressions of f and g are given by

$$f = -\frac{c_{44}^{0}b + e_{15}^{0}\Delta\phi}{2\pi}e^{\beta_{1}x + \beta_{2}y}K_{0}(\beta r),$$

$$g = \frac{\epsilon_{11}^{0}\Delta\phi - e_{15}^{0}b}{2\pi}e^{\beta_{1}x + \beta_{2}y}K_{0}(\beta r).$$
(13)

Substituting Eq. (13) into Eq. (7), we arrive at the expressions of stresses and electric displacements as

$$\sigma_{zx} = \frac{c_{44}^{0}b + e_{15}^{0}\Delta\phi}{2\pi} e^{\beta_{1}x + \beta_{2}y} \Big[ \beta_{2}K_{0}(\beta r) - \beta \frac{y}{r}K_{1}(\beta r) \Big],$$
  

$$\sigma_{zy} = \frac{c_{44}^{0}b + e_{15}^{0}\Delta\phi}{2\pi} e^{\beta_{1}x + \beta_{2}y} \Big[ \beta \frac{x}{r}K_{1}(\beta r) - \beta_{1}K_{0}(\beta r) \Big],$$
(14)

$$D_{x} = \frac{e_{15}^{0}b - \epsilon_{11}^{0}\Delta\phi}{2\pi} e^{\beta_{1}x + \beta_{2}y} \Big[ \beta_{2}K_{0}(\beta r) - \beta\frac{y}{r}K_{1}(\beta r) \Big],$$
  
$$D_{y} = \frac{e_{15}^{0}b - \epsilon_{11}^{0}\Delta\phi}{2\pi} e^{\beta_{1}x + \beta_{2}y} \Big[ \beta\frac{x}{r}K_{1}(\beta r) - \beta_{1}K_{0}(\beta r) \Big].$$
(15)

Using Eqs. (3a) and (3b), the strains and electric fields can be derived to be

$$\gamma_{zx} = \frac{b}{2\pi} e^{-\beta_1 x - \beta_2 y} \Big[ \beta_2 K_0(\beta r) - \beta \frac{y}{r} K_1(\beta r) \Big],$$
  

$$\gamma_{zy} = \frac{b}{2\pi} e^{-\beta_1 x - \beta_2 y} \Big[ \beta \frac{x}{r} K_1(\beta r) - \beta_1 K_0(\beta r) \Big],$$
(16)

$$E_{x} = \frac{\Delta\phi}{2\pi} e^{-\beta_{1}x-\beta_{2}y} \Big[\beta \frac{y}{r} K_{1}(\beta r) - \beta_{2} K_{0}(\beta r)\Big],$$
  

$$E_{y} = \frac{\Delta\phi}{2\pi} e^{-\beta_{1}x-\beta_{2}y} \Big[\beta_{1} K_{0}(\beta r) - \beta \frac{x}{r} K_{1}(\beta r)\Big].$$
(17)

It is observed from Eqs. (16) and (17) that the displacement jump b can only induce the mechanical strains  $\gamma_{zx}, \gamma_{zy}$ ; whilst the electric potential jump  $\Delta \phi$  can only induce the electric fields  $E_x, E_y$ . In addition the strains and electric fields are independent of the specific values of the three material constants  $c_{44}^0, e_{15}^0$  and  $\epsilon_{11}^0$ .

Furthermore the stresses and electric displacements can be expressed in the polar coordinate system as

$$\sigma_{z\theta} = \frac{\partial f}{\partial r} = \frac{c_{44}^0 b + e_{15}^0 \Delta \phi}{2\pi} e^{r(\beta_1 \cos \theta + \beta_2 \sin \theta)} [\beta K_1(\beta r) - (\beta_1 \cos \theta + \beta_2 \sin \theta) K_0(\beta r)],$$

$$\sigma_{zr} = -\frac{\partial f}{r\partial \theta} = \frac{c_{44}^0 b + e_{15}^0 \Delta \phi}{2\pi} (\beta_2 \cos \theta - \beta_1 \sin \theta) e^{r(\beta_1 \cos \theta + \beta_2 \sin \theta)} K_0(\beta r),$$

$$D_{\theta} = \frac{\partial g}{\partial r} = \frac{e_{15}^0 b - e_{11}^0 \Delta \phi}{2\pi} e^{r(\beta_1 \cos \theta + \beta_2 \sin \theta)} [\beta K_1(\beta r) - (\beta_1 \cos \theta + \beta_2 \sin \theta) K_0(\beta r)],$$

$$D_r = -\frac{\partial g}{r\partial \theta} = \frac{e_{15}^0 b - e_{11}^0 \Delta \phi}{2\pi} (\beta_2 \cos \theta - \beta_1 \sin \theta) e^{r(\beta_1 \cos \theta + \beta_2 \sin \theta)} K_0(\beta r).$$
(18)

It can be easily checked that when we ignore the piezoelectric effect, i.e.,  $e_{15} = 0$  and let the material be graded only in the *y*-direction, i.e.,  $\beta_1 = 0$ , Eqs. (14), (16) and (19) reduce to those derived by Lazar (2007). On the other hand if we ignore the material gradation, i.e.,  $\beta_1 = \beta_2 = 0$ , Eqs. (14)–(19) will reduce to those derived by Pak (1990) (also see Lee et al., 2000) by observing the following asymptotic behaviors for  $K_0(x)$  and  $K_1(x)$ 

$$K_0(x) \to -\ln(x/2) - 0.57721, K_1(x) \to x^{-1}, \text{ when } x \to 0^+$$
 (20)

We point out that with the derived full-field expressions for the stresses and electric displacements, the generalized Peach-Koehler force acting on the piezoelectric screw dislocation can be obtained by using the Peach-Koehler formulation (Lee et al., 2000). As expected the Peach-Koehler force is no longer a central force due to the existence of the tangential component of the force in the polar coordinates (Lazar, 2007).

# 3. A piezoelectric screw dislocation in an FGPM half-plane

# 3.1. An FGPM half-plane with a traction-free and charge-free surface

We now consider the case where the piezoelectric screw dislocation is located at (0, d),  $(d \ge 0)$  in an FGPM half-plane  $y \ge 0$ . The material properties of the FGPM half-plane are also assumed to vary exponentially as described by Eq. (4). The boundary conditions on the surface of the FGPM half-plane is assumed to be traction-free and charge-free, i.e.,  $\sigma_{zy} = D_y = 0$  at y = 0. By using the image method (Pak, 1990), the two generalized stress functions f and g for a half-plane with a traction-free and charge-free surface are found to be

$$f = -\frac{c_{44}^{0}b + e_{15}^{0}\Delta\phi}{2\pi}e^{\beta_{1}x + \beta_{2}(y+d)}\left\{K_{0}[\beta\sqrt{x^{2} + (y-d)^{2}}] - K_{0}[\beta\sqrt{x^{2} + (y+d)^{2}}]\right\},$$

$$g = \frac{\epsilon_{11}^{0}\Delta\phi - e_{15}^{0}b}{2\pi}e^{\beta_{1}x + \beta_{2}(y+d)}\left\{K_{0}[\beta\sqrt{x^{2} + (y-d)^{2}}] - K_{0}[\beta\sqrt{x^{2} + (y+d)^{2}}]\right\}.$$
(21)

It can be easily checked that Eq. (21) satisfies the traction-free and charge-free boundary conditions  $\sigma_{zy} = D_y = 0$  (or equivalently f = g = 0) on the surface y=0. Eq. (21) can be physically interpreted as the summation of the electroelastic field caused by a piezoelectric screw dislocation with displacement jump  $\Delta \phi$  located at (0, d) and another one by an *image* piezoelectric screw dislocation with displacement jump  $-e^{2\beta_2 d}b$  and electric potential jump  $-e^{2\beta_2 d}\Delta\phi$  located at the image point (0, -d). It is noticed that the magnitudes of the displacement and electric potential jumps for the image screw dislocation at (0, -d) are influenced by the gradient parameter  $\beta_2 d$ . Therefore with the derived generalized stress functions f and g, we can easily find all the electroelastic quantities. One special case is for the one when the material is graded only in one coordinate. For example, if we ignore the gradation in the x-direction, i.e.,  $\beta_1 = 0$ , the stress component  $\sigma_{zy}$  and the electric displacement  $D_y$  can be expressed as

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$$\sigma_{zy} = \frac{c_{44}^{0}b + e_{15}^{0}\Delta\phi}{2\pi d}\Omega(x, y, \beta_{2}),$$

$$D_{y} = \frac{e_{15}^{0}b - \epsilon_{11}^{0}\Delta\phi}{2\pi d}\Omega(x, y, \beta_{2}),$$
(22)

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where the function  $\Omega(x, y, \beta_2)$  is defined by

$$\Omega(x, y, \beta_2) = d|\beta_2| e^{\beta_2(y+d)} x \left\{ \frac{K_1[|\beta_2|\sqrt{x^2 + (y-d)^2}]}{\sqrt{x^2 + (y-d)^2}} - \frac{K_1[|\beta_2|\sqrt{x^2 + (y+d)^2}]}{\sqrt{x^2 + (y+d)^2}} \right\}.$$
(23)

It can also be easily checked that the results derived in this section can be reduced to those derived by Pak (1990) for a screw dislocation interacting with a traction-free and charge-free surface if we ignore the material gradation, i.e.,  $\beta_1 = \beta_2 = 0$ . We present in Fig. 1 the contour plots of the function  $\Omega(x, y, \beta_2)$  in Eq. (23) for  $\beta_2 d = -3$ , -1, 1, 3. It is observed that the gradient parameter  $\beta_2$  can significantly influence the values of the function  $\Omega(x, y, \beta_2)$ , and consequently influence the stress component  $\sigma_{zy}$  and the electric displacement  $D_y$ .



Fig. 1. Contours of the function  $\Omega(x, y, \beta_2)$  in Eq. (23) for  $\beta_2 d = -3, -1, 1, 3$ .

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# 3.2. An FGPM half-plane bonded by a rigid conductor

It shall be mentioned that in general the image method cannot be easily applied to address the problem of a piezoelectric screw dislocation located at (0, d), (d > 0) in an FGPM half-plane  $y \ge 0$  bonded by a rigid conductor, i.e.,  $\gamma_{zx} = E_x = 0$  on y = 0. However when we ignore the gradation in the y-direction, i.e.,  $\beta_2 = 0$  in Eq. (4), the two generalized stress functions f and g for a half-plane bonded by a rigid conductor can still be easily obtained by using the image method as follows

$$f = -\frac{c_{44}^{0}b + e_{15}^{0}\Delta\phi}{2\pi}e^{\beta_{1}x}\left\{K_{0}[|\beta_{1}|\sqrt{x^{2} + (y-d)^{2}}] + K_{0}\left[|\beta_{1}|\sqrt{x^{2} + (y+d)^{2}}\right]\right\},$$

$$g = \frac{\epsilon_{11}^{0}\Delta\phi - e_{15}^{0}b}{2\pi}e^{\beta_{1}x}\left\{K_{0}\left[|\beta_{1}|\sqrt{x^{2} + (y-d)^{2}}\right] + K_{0}\left[|\beta_{1}|\sqrt{x^{2} + (y+d)^{2}}\right]\right\},$$
(24)

which can be interpreted as the summation of the electroelastic field caused by a piezoelectric screw dislocation with displacement jump b and electric potential jump  $\Delta\phi$  located at (0, d) and another one by an *image* piezoelectric screw dislocation with displacement jump b and electric potential jump  $\Delta\phi$  located at the image point (0, -d). In this case the full field expressions of  $\gamma_{zx}$  and  $E_x$  are given by

$$\gamma_{zx} = -\frac{|\beta_{1}|b}{2\pi} e^{-\beta_{1}x} \left\{ \frac{(y-d)K_{1}[|\beta_{1}|\sqrt{x^{2}+(y-d)^{2}}]}{\sqrt{x^{2}+(y-d)^{2}}} + \frac{(y+d)K_{1}\left[|\beta_{1}|\sqrt{x^{2}+(y+d)^{2}}\right]}{\sqrt{x^{2}+(y+d)^{2}}} \right\},$$

$$E_{x} = \frac{|\beta_{1}|\Delta\phi}{2\pi} e^{-\beta_{1}x} \left\{ \frac{(y-d)K_{1}\left[|\beta_{1}|\sqrt{x^{2}+(y-d)^{2}}\right]}{\sqrt{x^{2}+(y-d)^{2}}} + \frac{(y+d)K_{1}\left[|\beta_{1}|\sqrt{x^{2}+(y+d)^{2}}\right]}{\sqrt{x^{2}+(y+d)^{2}}} \right\},$$
(25)

which satisfy the boundary conditions  $\gamma_{zx} = E_x = 0$  on y = 0.

# 4. Conclusion and discussion

In this paper we derived closed-form solutions for a straight piezoelectric screw dislocation with a displacement jump b and an electric potential jump  $\Delta \phi$  in an FGPM plane by using the stress function technique and in an FGPM half-plane by using the stress the method of image. The solutions were verified by comparison with existing solutions (Pak, 1990; Lazar, 2007). It's expected that the derived solutions for a screw dislocation can be further applied to investigate crack problems in FGPM. In fact by using the general solution to the Helmholtz equations in Eq. (11) in polar coordinates, we can discuss more complex interaction problems. For example, as shown in Fig. 2, we consider the case in which there is a circular insulating (or charge-free) hole of radius R in an FGPM plane described by Eq. (4). The center of the circular hole is at the origin of the coordinate system, and the piezoelectric screw dislocation is located at ( $\delta$ , 0), ( $\delta > R$ ) on the positive real axis in the matrix. The two generalized stress functions f and g for this case can be expressed in infinite series form as

$$f = -\frac{c_{44}^{0}b + e_{15}^{0}\Delta\phi}{2\pi}e^{\beta_{1}(x+\delta) + \beta_{2}y} \left[ K_{0}(\beta\sqrt{r^{2} + \delta^{2} - 2\delta r\cos\theta}) - A_{0}K_{0}(\beta r) - 2\sum_{n=1}^{+\infty}A_{n}K_{n}(\beta r)\cos(n\theta) \right],$$

$$g = \frac{\epsilon_{11}^{0}\Delta\phi - e_{15}^{0}b}{2\pi}e^{\beta_{1}(x+\delta) + \beta_{2}y} \left[ K_{0}(\beta\sqrt{r^{2} + \delta^{2} - 2\delta r\cos\theta}) - A_{0}K_{0}(\beta r) - 2\sum_{n=1}^{+\infty}A_{n}K_{n}(\beta r)\cos(n\theta) \right], \quad (r \ge R)$$
(26)

where  $A_0, A_1, A_2, \ldots$  are unknown constants to be determined.

By enforcing the traction-free and insulating boundary conditions  $\sigma_{zr} = D_r = 0$  (or equivalently f = g = 0) at r = R, and observing the following Graf's addition theorem (Chew, 1995)

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Fig. 2. Interaction between a piezoelectric screw dislocation and a circular insulating hole of radius R in an FGPM plane described by Eq. (4).

$$K_0 \left[ \beta \sqrt{r^2 + \delta^2 - 2\delta r \cos \theta} \right] = K_0(\beta \delta) I_0(\beta r) + 2 \sum_{n=1}^{+\infty} K_n(\beta \delta) I_n(\beta r) \cos(n\theta), \quad \text{for } r < \delta$$
(27)

where  $I_n$  is the *n*th order modified Bessel function of the first kind, we can finally arrive at all the unknowns

$$A_0 = \frac{K_0(\beta\delta)I_0(\beta R)}{K_0(\beta R)}, \quad A_n = \frac{K_n(\beta\delta)I_n(\beta R)}{K_n(\beta R)}, \quad n = 1, 2, 3, \dots, +\infty,$$
(28)

In practice the series in Eq. (26) is truncated at a large integer n = N to get sufficiently accurate result.

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