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Three-dimensional quasi-steady-state problem of moving heat and diffusion sources in an infinite solid

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ABSTRACT

The three-dimensional quasi-steady-state temperature and moisture concentration induced by a constantly moving point heat source and a constantly moving point diffusion source in an infinite isotropic solid are derived. Here the thermodiffusion (Soret) and the diffusionthermo (Dufour) effects are taken into account in our modelling. It is observed that the obtained coupled set of partial differential equations can be decoupled into two independent differential equations for two newly introduced functions, whose solutions can be expediently derived in a moving coordinate system which moves together with the point source. The results show that two positive effective diffusivities are needed to describe the hygrothermal field. Numerical results are presented to illustrate the distributions of the hygrothermal Green's functions. This research can be considered as an extension of the well-known Jaeger-Rosenthal solution for a moving heat source to the more complex situation in which there exists coupling between heat and moisture. In the Appendix we also present the temperature and the moisture concentration induced by instantaneous heat source and diffusion source by using the known result of an instantaneous heat source and the decoupling methodology presented in this research. Some interesting physical interpretations are presented for the instantaneous heat source and instantaneous diffusion source.

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MECHANIC

1. Introduction

The temperature field induced by a moving point heat source is of great practical importance and theoretical interest (see for example, Jaeger, 1942; Rosenthal, 1946; Wang, 1989; Wang et al., 2007; Levin, 2008). In modern polymer–matrix composite, the influence of temperature and moisture changes on the behaviors of the material is critical and has been investigated extensively (see for example, Sih et al., 1986; Chang and Chao, 1993; Altay and Dokmeci, 2000; Aboudi and Williams, 2000; Hsieh and Hwu, 2006 and the references cited therein). Under hygrothermal environment, the thermodiffusion effect (Soret effect), which refers to the mass diffusion induced by thermal gradient, and the diffusionthermo effect (Dufour effect), which refers to the heat transfer induced by moisture concentration gradient, should be given special attention (Sih et al., 1986; Hyer, 1988; Altay and Dokmeci, 2000; Aboudi and Williams, 2000). It is added that the Soret effect and the Dufour effect are also present in binary liquid mixtures (Hort et al., 1992; Kim et al., 2007).

In this investigation we endeavor to address in detail the three-dimensional quasi-steady-state problem of constantly moving point heat source and point diffusion source in an infinite isotropic solid when both the Soret and the Duffet effects

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are taken into account. We also present in Appendix A the temperature and the moisture concentration induced by instantaneous heat source and diffusion source. In addition we discuss in Appendix B the uniqueness theorem for the uncoupled, quasi-static hygrothermoelastic theory and how to obtain the stress fields induced by constantly moving or instantaneous heat source and diffusion source.

2. The temperature and moisture concentration induced by constantly moving heat and diffusion sources

Here we consider a point heat source of strength P and a point diffusion source of strength M moving at a constant speed V along the positive direction of the x-axis. At the time t = 0, the heat source and the diffusion source are located at the origin of a fixed rectangular Cartesian coordinate system (x, y, z).

In the fixed rectangular Cartesian coordinate system (x, y, z), the modified Fourier's equation of heat conduction and Fick's equation of moisture diffusion can be written as (Sih et al., 1986; Aboudi and Williams, 2000)

$$q_{i,i} + \rho c_p \dot{\theta} - \Theta_0 d_t \dot{C} = P \delta(x - Vt) \delta(y) \delta(z), \tag{1}$$

$$f_{i,i} + \dot{C} = M \delta(x - Vt) \delta(y) \delta(z), \tag{2}$$

where q_i and f_i are the heat fluxes and moisture fluxes, respectively; ρ is the material density, c_p is the specific heat capacity for constant moisture concentration and pressure, d_t is a material constant, Θ_0 is the reference temperature (or the initial state of the temperature); θ and C are, respectively, the temperature change and the moisture concentration (the mass of moisture per unit volume contained at a point in the solid); $\delta($ is the Dirac delta function; and the dot notation is used to indicate derivative of a function with respect to time t. It is mentioned here that in the expression of Eq. (1) for the heat conduction equation, we have ignored the interconvertibility of thermal and mechanical energy. This kind of simplification is permissible in most practical situations (Boley and Weiner, 1960).

The heat and moisture fluxes are given by (Sih et al., 1986; Hyer, 1988; Aboudi and Williams, 2000)

$$q_{i} = -k\theta_{,i} - \zeta C_{,i},$$

$$f_{i} = -\zeta \theta_{,i} - \gamma C_{,i},$$
(3)

where k and γ are the thermal conductivity and the moisture constant, respectively; ξ and ζ are the Dufour and the Soret coefficients, respectively. In addition the four constants can be specifically expressed as (Sih et al., 1986; Aboudi and Williams, 2000)

$$k = L_{21}d_t + \frac{L_{22}}{\Theta_0}, \quad \xi = L_{21}d_c, \quad \zeta = L_{11}d_t + \frac{L_{12}}{\Theta_0}, \quad \gamma = L_{11}d_c,$$
(4)

where L_{ij} ($L_{12} = L_{21}$) and d_c are also material constants.

Substitution of Eq. (3) into Eqs. (1) and (2) yields the following:

$$k\nabla^2\theta + \xi\nabla^2\mathbf{C} - \rho c_p \dot{\theta} + \Theta_0 d_t \dot{\mathbf{C}} = -P\delta(\mathbf{x} - Vt)\delta(\mathbf{y})\delta(\mathbf{z}),\tag{5}$$

$$\zeta \nabla^2 \theta + \gamma \nabla^2 \mathbf{C} - \dot{\mathbf{C}} = -M\delta(\mathbf{x} - Vt)\delta(\mathbf{y})\delta(\mathbf{z}),\tag{6}$$

where $\nabla^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}$ is the 3D Laplace operator. Inserting the result of Eq. (6) into Eq. (5) and eliminating \dot{C} , then Eq. (5) can be equivalently expressed as

$$(k + \Theta_0 d_t \zeta) \nabla^2 \theta + (\xi + \Theta_0 d_t \gamma) \nabla^2 C - \rho c_p \dot{\theta} = -(P + \Theta_0 d_t M) \delta(x - Vt) \delta(y) \delta(z),$$

$$\tag{7}$$

For mathematical convenience, we can further write together Eqs. (6) and (7) into the following matrix form as:

$$\mathbf{A}\begin{bmatrix}\nabla^{2}\theta\\\nabla^{2}C\end{bmatrix} - \mathbf{B}\begin{bmatrix}\dot{\theta}\\\dot{C}\end{bmatrix} = -\mathbf{S}\delta(x - Vt)\delta(y)\delta(z),\tag{8}$$

where **A** and **B** are two real and symmetric 2×2 matrices given by

$$\mathbf{A} = \mathbf{A}^{\mathrm{T}} = \begin{bmatrix} k + \Theta_0 d_t \zeta & \zeta + \Theta_0 d_t \gamma \\ \zeta + \Theta_0 d_t \gamma & \frac{\gamma(\zeta + \Theta_0 d_t \gamma)}{\zeta} \end{bmatrix},$$

$$\mathbf{B} = \mathbf{B}^{\mathrm{T}} = \begin{bmatrix} \rho c_p & \mathbf{0} \\ \mathbf{0} & \frac{\zeta + \Theta_0 d_t \gamma}{\zeta} \end{bmatrix},$$

(9)

and **S** is a 2-D vector given by

$$\mathbf{S} = \begin{bmatrix} P + \Theta_0 d_t M \\ \frac{M(\xi + \Theta_0 d_t \gamma)}{\zeta} \end{bmatrix}.$$
(10)

In order to solve Eq. (8), we first consider the following eigenvalue problem:

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X. Wang, E. Pan/Mechanics Research Communications 35 (2008) 475-482

$$(\mathbf{A} - \lambda \mathbf{B})\mathbf{v} = \mathbf{0},$$

where λ and **v** are, respectively, the eigenvalue and the associated eigenvector. The two eigenvalues of Eq. (11) can be explicitly determined as

$$\lambda_{1} = \frac{A_{11}B_{22} + A_{22}B_{11} + \sqrt{(A_{11}B_{22} - A_{22}B_{11})^{2} + 4A_{12}^{2}B_{11}B_{22}}}{2B_{11}B_{22}} > 0,$$

$$\lambda_{2} = \frac{A_{11}B_{22} + A_{22}B_{11} - \sqrt{(A_{11}B_{22} - A_{22}B_{11})^{2} + 4A_{12}^{2}B_{11}B_{22}}}{2B_{11}B_{22}} > 0,$$
(12)

where A_{ij} and B_{ii} (i, j = 1, 2) are respectively the nonzero components of the matrices **A** and **B**. It is mentioned that the positiveness of the two eigenvalues λ_1 and λ_2 is a result of the fact that both **A** and **B** are positive definite to ensure the positive definiteness of the local entropy production. Meanwhile the two eigenvectors associated with the two eigenvalues are given by

$$\mathbf{v}_1 = \begin{bmatrix} A_{12} \\ \lambda_1 B_{11} - A_{11} \end{bmatrix}, \quad \mathbf{v}_2 = \begin{bmatrix} A_{12} \\ \lambda_2 B_{11} - A_{11} \end{bmatrix}.$$
(13)

Due to the fact that the two matrices **A** and **B** are real and symmetric, then the following orthogonal relationships with respect to the two matrices establish

$$\begin{bmatrix} \mathbf{v}_1^{\mathsf{T}} \\ \mathbf{v}_2^{\mathsf{T}} \end{bmatrix} \mathbf{A} \begin{bmatrix} \mathbf{v}_1 & \mathbf{v}_2 \end{bmatrix} = \begin{bmatrix} \delta_1 & \mathbf{0} \\ \mathbf{0} & \delta_2 \end{bmatrix},$$

$$\begin{bmatrix} \mathbf{v}_1^{\mathsf{T}} \\ \mathbf{v}_2^{\mathsf{T}} \end{bmatrix} \mathbf{B} \begin{bmatrix} \mathbf{v}_1 & \mathbf{v}_2 \end{bmatrix} = \begin{bmatrix} \lambda_1^{-1} \delta_1 & \mathbf{0} \\ \mathbf{0} & \lambda_2^{-1} \delta_2 \end{bmatrix},$$
(14)

where

$$\delta_{1} = \lambda_{1}^{3} B_{11}^{2} B_{22} - 2\lambda_{1}^{2} A_{11} B_{11} B_{22} + \lambda_{1} (A_{11}^{2} B_{22} + A_{12}^{2} B_{11}),$$

$$\delta_{2} = \lambda_{2}^{3} B_{11}^{2} B_{22} - 2\lambda_{2}^{2} A_{11} B_{11} B_{22} + \lambda_{2} (A_{11}^{2} B_{22} + A_{12}^{2} B_{11}).$$
(15)

Now we introduce two new functions *f* and *g*, which are related to θ and *C* through

$$\begin{bmatrix} \theta \\ C \end{bmatrix} = \begin{bmatrix} \mathbf{v}_1 & \mathbf{v}_2 \end{bmatrix} \begin{bmatrix} f \\ g \end{bmatrix}.$$
(16)

In view of Eqs. (8), (14) and (16), the two new functions f and g will satisfy the following two independent inhomogeneous partial differential equations:

$$\nabla^{2} f - \frac{1}{\lambda_{1}} \frac{\partial f}{\partial t} = -\frac{K_{1}}{\delta_{1}} \delta(\mathbf{x} - \mathbf{V}t) \delta(\mathbf{y}) \delta(\mathbf{z}),$$

$$\nabla^{2} g - \frac{1}{\lambda_{2}} \frac{\partial g}{\partial t} = -\frac{K_{2}}{\delta_{2}} \delta(\mathbf{x} - \mathbf{V}t) \delta(\mathbf{y}) \delta(\mathbf{z}),$$
(17)

where the two constants K_1 and K_2 are

$$K_{1} = A_{12}P + (\lambda_{1}B_{11}B_{22} - A_{11}B_{22} + A_{12}\Theta_{0}d_{t})M,$$

$$K_{2} = A_{12}P + (\lambda_{2}B_{11}B_{22} - A_{11}B_{22} + A_{12}\Theta_{0}d_{t})M.$$
(18)

It can be observed from Eq. (17) that the two eigenvalues λ_1 (>0) and λ_2 (>0) can be considered as two effective diffusivities for *f* and *g*, respectively. Next we introduce a new moving coordinate system ($\tilde{x}, \tilde{y}, \tilde{z}$) which is related to the fixed coordinate system (x, y, z) through

$$\tilde{\mathbf{x}} = \mathbf{x} - \mathbf{V}t, \quad \tilde{\mathbf{y}} = \mathbf{y}, \quad \tilde{\mathbf{z}} = \mathbf{z}.$$
 (19)

In view of the fact that the moving coordinate system moves at the same speed as the point source, then the temperature and moisture concentration (or equivalently the two newly introduced functions f and g) in the new moving coordinate system do not explicitly depend on the time t, and are in a quasi-steady-state (Wang et al., 2007; Levin, 2008). Consequently in the new coordinate system, Eq. (17) can be changed into the following two independent inhomogeneous perturbed Laplace equations:

$$\frac{\partial^2 f}{\partial \tilde{x}^2} + \frac{\partial^2 f}{\partial \tilde{y}^2} + \frac{\partial^2 f}{\partial \tilde{z}^2} + \frac{V}{\lambda_1} \frac{\partial f}{\partial \tilde{x}} = -\frac{K_1}{\delta_1} \delta(\tilde{x}) \delta(\tilde{y}) \delta(\tilde{z}),$$

$$\frac{\partial^2 g}{\partial \tilde{x}^2} + \frac{\partial^2 g}{\partial \tilde{y}^2} + \frac{\partial^2 g}{\partial \tilde{z}^2} + \frac{V}{\lambda_2} \frac{\partial g}{\partial \tilde{x}} = -\frac{K_2}{\delta_2} \delta(\tilde{x}) \delta(\tilde{y}) \delta(\tilde{z}).$$
(20)

477

(11)

We then introduce two new functions *F* and *G* which are related to *f* and *g* through (Wang et al., 2007)

$$f = \exp\left(-\frac{V\tilde{x}}{2\lambda_1}\right)F, \quad g = \exp\left(-\frac{V\tilde{x}}{2\lambda_2}\right)G.$$
(21)

As a result Eq. (20) can be equivalently rewritten into the following two 3D inhomogeneous Helmholtz equations:

$$\frac{\partial^2 F}{\partial \tilde{x}^2} + \frac{\partial^2 F}{\partial \tilde{y}^2} + \frac{\partial^2 F}{\partial \tilde{z}^2} - \frac{V^2}{4\lambda_1^2} F = -\frac{K_1}{\delta_1} \delta(\tilde{x}) \delta(\tilde{y}) \delta(\tilde{z}),$$

$$\frac{\partial^2 G}{\partial \tilde{x}^2} + \frac{\partial^2 G}{\partial \tilde{y}^2} + \frac{\partial^2 G}{\partial \tilde{z}^2} - \frac{V^2}{4\lambda_2^2} G = -\frac{K_2}{\delta_2} \delta(\tilde{x}) \delta(\tilde{y}) \delta(\tilde{z}),$$
(22)

whose solutions are expediently given by

$$F = \frac{K_1}{4\pi\delta_1} \frac{\exp\left(-\frac{V}{2\lambda_1}\sqrt{\tilde{x}^2 + \tilde{y}^2 + \tilde{z}^2}\right)}{\sqrt{\tilde{x}^2 + \tilde{y}^2 + \tilde{z}^2}}, \quad G = \frac{K_2}{4\pi\delta_2} \frac{\exp\left(-\frac{V}{2\lambda_2}\sqrt{\tilde{x}^2 + \tilde{y}^2 + \tilde{z}^2}\right)}{\sqrt{\tilde{x}^2 + \tilde{y}^2 + \tilde{z}^2}}.$$
(23)

It follows from Eqs. (21) and (23) that

$$f = \frac{K_1}{4\pi\delta_1} \frac{\exp\left(-\frac{v}{2\lambda_1} \left[\sqrt{\tilde{x}^2 + \tilde{y}^2 + \tilde{z}^2} + \tilde{x}\right]\right)}{\sqrt{\tilde{x}^2 + \tilde{y}^2 + \tilde{z}^2}},$$

$$g = \frac{K_2}{4\pi\delta_2} \frac{\exp\left(-\frac{v}{2\lambda_2} \left[\sqrt{\tilde{x}^2 + \tilde{y}^2 + \tilde{z}^2} + \tilde{x}\right]\right)}{\sqrt{\tilde{x}^2 + \tilde{y}^2 + \tilde{z}^2}}.$$
(24)

In view of Eqs. (16), (19) and (24), the temperature θ and the moisture concentration C induced by the moving point source can be finally expressed in the fixed rectangular Cartesian coordinates (x, y, z) as follows:

$$\theta = \frac{A_{12}K_{1}\exp\left\{-\frac{v}{2\lambda_{1}}\left[\sqrt{(x-Vt)^{2}+y^{2}+z^{2}}+(x-Vt)\right]\right\}}{4\pi\delta_{1}\sqrt{(x-Vt)^{2}+y^{2}+z^{2}}} + \frac{A_{12}K_{2}\exp\left\{-\frac{v}{2\lambda_{2}}\left[\sqrt{(x-Vt)^{2}+y^{2}+z^{2}}+(x-Vt)\right]\right\}}{4\pi\delta_{2}\sqrt{(x-Vt)^{2}+y^{2}+z^{2}}},$$

$$C = \frac{K_{1}(\lambda_{1}B_{11}-A_{11})\exp\left\{-\frac{v}{2\lambda_{1}}\left[\sqrt{(x-Vt)^{2}+y^{2}+z^{2}}+(x-Vt)\right]\right\}}{4\pi\delta_{1}\sqrt{(x-Vt)^{2}+y^{2}+z^{2}}} + \frac{K_{2}(\lambda_{2}B_{11}-A_{11})\exp\left\{-\frac{v}{2\lambda_{2}}\left[\sqrt{(x-Vt)^{2}+y^{2}+z^{2}}+(x-Vt)\right]\right\}}{4\pi\delta_{2}\sqrt{(x-Vt)^{2}+y^{2}+z^{2}}}.$$
(25)

Consequently we can write down the 3D quasi-steady-state hygrothermal Green's functions as follows:

$$G_{\theta P} = \frac{A_{12}^{2} \exp\left\{-\frac{V}{2\lambda_{1}} \left[\sqrt{(x-Vt)^{2}+y^{2}+z^{2}}+(x-Vt)\right]\right\}}{4\pi\delta_{1}\sqrt{(x-Vt)^{2}+y^{2}+z^{2}}} + \frac{A_{12}^{2} \exp\left\{-\frac{V}{2\lambda_{2}} \left[\sqrt{(x-Vt)^{2}+y^{2}+z^{2}}+(x-Vt)\right]\right\}}{4\pi\delta_{2}\sqrt{(x-Vt)^{2}+y^{2}+z^{2}}},$$
(26a)

$$G_{\theta M} = \frac{A_{12}(\lambda_1 B_{11} B_{22} - A_{11} B_{22} + A_{12} \Theta_0 d_t) \exp\left\{-\frac{V}{2\lambda_1} \left[\sqrt{(x - Vt)^2 + y^2 + z^2} + (x - Vt)\right]\right\}}{4\pi \delta_1 \sqrt{(x - Vt)^2 + y^2 + z^2}} + \frac{A_{12}(\lambda_2 B_{11} B_{22} - A_{11} B_{22} + A_{12} \Theta_0 d_t) \exp\left\{-\frac{V}{2\lambda_2} \left[\sqrt{(x - Vt)^2 + y^2 + z^2} + (x - Vt)\right]\right\}}{4\pi \delta_2 \sqrt{(x - Vt)^2 + y^2 + z^2}}, \quad (26b)$$

$$G_{CP} = \frac{A_{12}(\lambda_{1}B_{11} - A_{11})\exp\left\{-\frac{V}{2\lambda_{1}}\left[\sqrt{(x - Vt)^{2} + y^{2} + z^{2}} + (x - Vt)\right]\right\}}{4\pi\delta_{1}\sqrt{(x - Vt)^{2} + y^{2} + z^{2}}} + \frac{A_{12}(\lambda_{2}B_{11} - A_{11})\exp\left\{-\frac{V}{2\lambda_{2}}\left[\sqrt{(x - Vt)^{2} + y^{2} + z^{2}} + (x - Vt)\right]\right\}}{4\pi\delta_{2}\sqrt{(x - Vt)^{2} + y^{2} + z^{2}}},$$

$$G_{CM} = \frac{(\lambda_{1}B_{11} - A_{11})(\lambda_{1}B_{11}B_{22} - A_{11}B_{22} + A_{12}\Theta_{0}d_{t})\exp\left\{-\frac{V}{2\lambda_{1}}\left[\sqrt{(x - Vt)^{2} + y^{2} + z^{2}} + (x - Vt)\right]\right\}}{4\pi\delta_{1}\sqrt{(x - Vt)^{2} + y^{2} + z^{2}}} + \frac{(\lambda_{2}B_{11} - A_{11})(\lambda_{2}B_{11}B_{22} - A_{11}B_{22} + A_{12}\Theta_{0}d_{t})\exp\left\{-\frac{V}{2\lambda_{2}}\left[\sqrt{(x - Vt)^{2} + y^{2} + z^{2}} + (x - Vt)\right]\right\}}{4\pi\delta_{2}\sqrt{(x - Vt)^{2} + y^{2} + z^{2}}},$$
(26c)

(26d)

where the hygrothermal Green's functions are defined as: $G_{\theta P}$ is the temperature induced by a unit heat source (P = 1); $G_{\theta M}$ is the temperature induced by a unit diffusion source (M = 1); G_{CP} is the moisture concentration induced by a unit heat source (*P* = 1); and G_{CM} is the moisture concentration induced by a unit diffusion source (*M* = 1). Apparently $G_{\partial M} \neq G_{CP}$.

Now that the well-known Jaeger-Rosenthal solution for a moving heat source (Jaeger, 1942; Rosenthal, 1946; Wang, 1989) has been extended to the more complex situation in which there exists coupling between heat and moisture.

478

2

3. Numerical results

Here we consider T300/5208 graphite/epoxy quasi-isotropic laminate subjected to relative humidity change from 0 to 75% at $\Theta_0 = 21 \text{ °C}$. The pertinent material constants are presented in Table 1.

By using Eq. (12), the two effective diffusivities λ_1 and λ_2 are calculated as $\lambda_1 = 2.3385 \times 10^{-12} \text{ m}^2/\text{s}$ and $\lambda_2 = 5.3037 \times 10^{-15} \text{ m}^2/\text{s}$. If we ignore the coupling Dufour and Soret effects, i.e., $L_{12} = d_t = 0$, the thermal diffusivity is $2.5062 \times 10^{-12} \text{ m}^2/\text{s}$ while the moisture diffusivity is $6.5989 \times 10^{-15} \text{ m}^2/\text{s}$. It is observed that the existence of the Dufour and Soret effects will reduce the values of the two diffusivities for the decoupled case. We demonstrate in Fig. 1 the distributions of the four hygrothermal Green's functions $G_{\theta P}$, G_{CP} , $G_{\theta M}$, G_{CM} along the moving \tilde{x} axis ($\tilde{y} = \tilde{z} = 0$). It is observed from Fig. 1 that: (i) all the four Green's functions are infinite at $\tilde{x} = 0$, the location of the point source, and they decay fast as \tilde{x} is further away from $\tilde{x} = 0$; (ii) all the four Green's functions are not symmetric with respect to $\tilde{x} = 0$; (iii) G_{CP} and $G_{\theta M}$ are non-zero due to the coupling Dufour and Soret effects, in addition it is verified that $G_{\theta M} \neq G_{CP}$; (iv) $G_{\theta P}$ is always positive, $G_{\theta M}$ is always negative, while G_{CP} and G_{CM} can change their signs along the positive \tilde{x} axis. This change of sign phenomenon cannot be observed from the decoupled case when ignoring the Dufour and Soret effects.

4. Conclusions and discussion

We have derived the temperature and the moisture concentration induced by a constantly moving point heat source and a constantly moving point diffusion source. The results show that two positive effective diffusivities for *f* and *g* are introduced (see Eq. (17)). The positiveness of the two diffusivities λ_1 and λ_2 is a consequence of the positive definiteness of the local entropy production. As presented in Appendix A, it is not a difficult task to derive the temperature and the moisture

Table 1

Material constants of T300/5208 graphite/epoxy quasi-isotropic laminate subjected to change in moisture from 0 to 75% relative humidity at $\Theta_0 = 21^{\circ}$ C (Sih et al., 1986; Aboudi and Williams, 2000)

Property	Value
$ \begin{array}{l} \rho \ (\text{kg/m}^3) \\ c_p \ (\text{m}^2/\text{s}^2\text{K}) \\ L_{11} \ (\text{kg/m}^3) \\ L_{12} \ (\text{kg/ms}) \\ L_{22} \ (\text{kg/ms}) \\ L_{22} \ (\text{kgm/s}^3) \\ d_c \ (\text{m}^5/\text{kgs}^2) \\ d_t \ (\text{m}^2/\text{s}^2\text{K}) \end{array} $	$\begin{array}{c} 1590\\ 806.461\\ 3.544\times 10^{-18}\\ 9.153\times 10^{-12}\\ 9.453\times 10^{-5}\\ 1862\\ -1373\end{array}$



Fig. 1. Distributions of the four hygrothermal Green's functions $G_{\partial P}$, G_{CP} , $G_{\partial M}$, G_{CM} along the \tilde{x} axis.

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X. Wang, E. Pan/Mechanics Research Communications 35 (2008) 475-482

concentration induced by instantaneous heat source and diffusion source by using the known result of an instantaneous heat source (Boley and Weiner, 1960; Wang, 1989) and the decoupling methodology presented in this research. Furthermore due to the fact that the stress fields induced by a constantly moving heat source and by an instantaneous heat source have been derived (see Eqs. (6.162) and (6.178) in Wang, 1989), then it is simple to extend the previous result to calculate the stresses induced by constantly moving or instantaneous heat source and diffusion source through the introduction of two new functions f and g in view of Eqs. (16), (17), (A4) and (A5), and through application of the superposition principle for a linear system. The details for the last discussion are presented in Appendix B.

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Appendix A. The temperature and moisture concentration induced by instantaneous heat source and diffusion source

We consider an instantaneous point heat source of strength *Q* and an instantaneous point diffusion source of strength *N* released at the origin of the fixed rectangular Cartesian coordinate system (x, y, z) at t = 0. Then the initial conditions of temperature and moisture concentration can be given by (Wang, 1989)

$$\theta(\mathbf{x}, \mathbf{y}, \mathbf{z}, \mathbf{0}) = \mathbf{Q}\delta(\mathbf{x})\delta(\mathbf{y})\delta(\mathbf{z}), \quad \mathbf{C}(\mathbf{x}, \mathbf{y}, \mathbf{z}, \mathbf{0}) = \mathbf{N}\delta(\mathbf{x})\delta(\mathbf{y})\delta(\mathbf{z}). \tag{A1}$$

In the discussion of instantaneous heat source and diffusion source, the temperature and the moisture concentration should satisfy the following set of homogeneous partial differential equations

$$k\nabla^2\theta + \xi\nabla^2\mathbf{C} - \rho c_p \dot{\theta} + \Theta_0 d_t \dot{\mathbf{C}} = \mathbf{0},\tag{A2}$$

$$\zeta \nabla^2 \theta + \gamma \nabla^2 \mathbf{C} - \mathbf{C} = \mathbf{0}. \tag{A3}$$

In view of Eq. (16), the above initial conditions and the governing partial differential equations can be expressed in terms of the two new functions f and g as

$$f(x, y, z, 0) = \widetilde{Q}\delta(x)\delta(y)\delta(z), \quad g(x, y, z, 0) = \widetilde{N}\delta(x)\delta(y)\delta(z)$$
(A4)

and

$$\nabla^2 f - \frac{1}{\lambda_1} \frac{\partial f}{\partial t} = \mathbf{0}, \quad \nabla^2 g - \frac{1}{\lambda_2} \frac{\partial g}{\partial t} = \mathbf{0}, \tag{A5}$$

where

$$\begin{bmatrix} \widetilde{Q} \\ \widetilde{N} \end{bmatrix} = \begin{bmatrix} \delta_1^{-1} & \mathbf{0} \\ \mathbf{0} & \delta_2^{-1} \end{bmatrix} \begin{bmatrix} \mathbf{v}_1^{\mathsf{T}} \\ \mathbf{v}_2^{\mathsf{T}} \end{bmatrix} \mathbf{A} \begin{bmatrix} Q \\ N \end{bmatrix}.$$
(A6)

Now that the expressions of *f* and *g* can be easily obtained as (Boley and Weiner, 1960; Wang, 1989)

$$f(x, y, z, t) = \frac{Q}{(2\sqrt{\lambda_1 \pi t})^3} \exp\left(-\frac{x^2 + y^2 + z^2}{4\lambda_1 t}\right),$$

$$g(x, y, z, t) = \frac{\widetilde{N}}{(2\sqrt{\lambda_2 \pi t})^3} \exp\left(-\frac{x^2 + y^2 + z^2}{4\lambda_2 t}\right).$$
(A7)

Consequently we can obtain the expressions of temperature and moisture concentration as

$$\theta(x, y, z, t) = \frac{\widetilde{Q}A_{12}}{(2\sqrt{\lambda_1\pi t})^3} \exp\left(-\frac{x^2 + y^2 + z^2}{4\lambda_1 t}\right) + \frac{\widetilde{N}A_{12}}{(2\sqrt{\lambda_2\pi t})^3} \exp\left(-\frac{x^2 + y^2 + z^2}{4\lambda_2 t}\right),$$

$$C(x, y, z, t) = \frac{\widetilde{Q}(\lambda_1 B_{11} - A_{11})}{(2\sqrt{\lambda_1\pi t})^3} \exp\left(-\frac{x^2 + y^2 + z^2}{4\lambda_1 t}\right) + \frac{\widetilde{N}(\lambda_2 B_{11} - A_{11})}{(2\sqrt{\lambda_2\pi t})^3} \exp\left(-\frac{x^2 + y^2 + z^2}{4\lambda_2 t}\right).$$
(A8)

In fact the two constants Q and N possess important physical meanings. Following the procedure of Boley and Weiner (1960), the total energy due to temperature rise in the infinite region is (t > 0)

$$\int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \rho c_p \theta \, \mathrm{d}x \, \mathrm{d}y \, \mathrm{d}z = \frac{\rho c_p \widetilde{Q} A_{12}}{(2\sqrt{\lambda_1 \pi t})^3} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \exp\left(-\frac{x^2 + y^2 + z^2}{4\lambda_1 t}\right) \mathrm{d}x \, \mathrm{d}y \, \mathrm{d}z + \frac{\rho c_p \widetilde{N} A_{12}}{(2\sqrt{\lambda_2 \pi t})^3} \\ \times \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \exp\left(-\frac{x^2 + y^2 + z^2}{4\lambda_2 t}\right) \mathrm{d}x \, \mathrm{d}y \, \mathrm{d}z \\ = \rho c_p [A_{12} \quad A_{12}] \begin{bmatrix} \widetilde{Q} \\ \widetilde{N} \end{bmatrix} = \rho c_p [1 \quad 0] [\mathbf{v}_1 \quad \mathbf{v}_2] \begin{bmatrix} \delta_1^{-1} & 0 \\ 0 \quad \delta_2^{-1} \end{bmatrix} \begin{bmatrix} \mathbf{v}_1^T \\ \mathbf{v}_2^T \end{bmatrix} \mathbf{A} \begin{bmatrix} Q \\ N \end{bmatrix} = \rho c_p Q, \tag{A9}$$

and the total mass of moisture in the infinite region is (t > 0)

$$\int_{-\infty}^{\infty} \int_{-\infty}^{\infty} C \, dx \, dy \, dz = \frac{\widetilde{Q} \left(\lambda_1 B_{11} - A_{11}\right)}{\left(2\sqrt{\lambda_1 \pi t}\right)^3} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \exp\left(-\frac{x^2 + y^2 + z^2}{4\lambda_1 t}\right) dx \, dy \, dz + \frac{\widetilde{N} (\lambda_2 B_{11} - A_{11})}{\left(2\sqrt{\lambda_2 \pi t}\right)^3} \\ \times \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \exp\left(-\frac{x^2 + y^2 + z^2}{4\lambda_2 t}\right) dx \, dy \, dz \\ = \left[\lambda_1 B_{11} - A_{11} - \lambda_2 B_{11} - A_{11}\right] \left[\begin{array}{c} \widetilde{Q} \\ \widetilde{N} \end{array} \right] = \left[0 - 1 \right] \left[\mathbf{v}_1 - \mathbf{v}_2 \right] \left[\begin{array}{c} \delta_1^{-1} & 0 \\ 0 - \delta_2^{-1} \end{array} \right] \left[\begin{array}{c} \mathbf{v}_1^T \\ \mathbf{v}_2^T \end{array} \right] \mathbf{A} \left[\begin{array}{c} Q \\ N \end{array} \right] = N.$$
 (A10)

During the above derivations, we have utilized the orthogonal relationships in Eq. (14). Thus it is observed from Eqs. (A9) and (A10) that the total energy and the total mass of moisture in the solid are constant at all times. Carrying out integrations to Eq. (A8) will yield the temperature and moisture concentration induced by instantaneous line and plane sources (Boley and Weiner, 1960).

Appendix B. The determination of the stress field induced by constantly moving or instantaneous heat and diffusion sources

Following an identical theoretical development presented by Boley and Weiner (1960), we can obtain the following uniqueness theorem for the uncoupled, quasi-static hygrothermoelastic theory:

Uniqueness theorem. Given a regular region of space D + B with boundary B and functions $f_i(P)$, $\theta(P)$, C(P) defined in D + B; then there exists at most one set of single-valued functions $\sigma_{ij}(P)$, $\varepsilon_{ij}(P)$ and $u_i(P)$, all in class $C^{(1)}$ in D + B, which satisfy the following field equations:

$$\sigma_{ij,j} + f_i = 0 \quad \text{in } D,$$

$$\sigma_{ij} = \delta_{ij}\lambda\varepsilon_{kk} + 2\mu\varepsilon_{ij} - \delta_{ij}(3\lambda + 2\mu)\alpha\theta - \delta_{ij}(3\lambda + 2\mu)\etaC \quad \text{in } B + D,$$

$$\varepsilon_{ij} = \frac{1}{2}(u_{i,j} + u_{j,i}) \quad \text{in } B + D$$
(B1)

and which lead (through $S_i = \sigma_{ij}n_j$ where n_j is the surface normal) to specified values on *B* for three quantities with distinct subscripts from the six quantities $[S_I, S_{II}, S_{III}]$ and $[u_I, u_{II}, u_{III}]$, where I, II and III denote orthogonal directions. In Eq. (B1) σ_{ij} and ε_{ij} are the stress and strain tensor components, S_i is traction component, u_i is the displacement vector component, δ_{ij} is the Kronecker delta, λ and μ are Lamé constants of the material and α and η are the linear thermal expansion and linear moisture expansion coefficients. (B1)₁ is the equilibrium equations in the presence of body forces f_i ; (B1)₂ is the hygrothermoelastic constitutive law for an isotropic material with small strains (Aboudi and Williams, 2000); while (B1)₃ is the strain–displacement relations.

If we introduce the two new functions f and g defined in Eq. (16), then Eq. (B1)₂ for the hygrothermoelastic constitutive law can be equivalently written into

$$\sigma_{ij} = \delta_{ij} \lambda \varepsilon_{kk} + 2\mu \varepsilon_{ij} - \delta_{ij} (3\lambda + 2\mu) \tilde{\alpha} f - \delta_{ij} (3\lambda + 2\mu) \tilde{\eta} g, \tag{B2}$$

where

$$\tilde{\chi} = \alpha A_{12} + \eta (\lambda_1 B_{11} - A_{11}), \\ \tilde{\eta} = \alpha A_{12} + \eta (\lambda_2 B_{11} - A_{11}).$$
(B3)

In addition *f* and *g* satisfy the two decoupled equations in Eq. (17) for a constantly moving heat source and diffusion source, and they satisfy Eqs. (A4) and (A5) for an instantaneous heat source and an instantaneous diffusion source. Based on Eq. (B2) and the known results of stress fields induced by a constantly moving point heat source and by an instantaneous point heat source (see Eqs. (6.162) and (6.178) in Wang, 1989), we can easily obtain the unique stress field induced by constantly moving or instantaneous point heat source and point diffusion source by using the superposition principle for a linear system and the uniqueness theorem we just presented.

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