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Stoneley (interfacial) waves between two magneto-electro-elastic half planes

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We investigated the propagation properties of Stoneley waves between two magneto-electro-elastic half planes. Magneto-electro-elastic materials are assumed to possess hexagonal (6 mm) symmetry. Twenty-five sets of magneto-electrical interface conditions were adopted and generalized frequency equations were derived and solved numerically. It was found that, for each set of interface conditions, existing Stoneley waves are always non-dispersive. Numerical results further show that material properties have a significant effect on both the number and velocity of Stoneley waves, and that, although different magneto-electrical interface conditions could influence the existence of Stoneley waves, they have no effect on wave velocities.

Keywords: Stoneley wave; phase velocity; magneto-electron-elastic materials; dispersion

1. Introduction

In recent years, piezoelectric and piezomagnetic materials have increasingly found applications in various engineering structures, especially in smart/intelligent systems as intelligent sensors, damage detectors, etc. Composite materials consisting of piezoelectric and piezomagnetic phases exhibit a magnetoelectric effect that is absent in either constituent. The magnetoelectric effects of piezoelectric-piezomagnetic composites were first reported in 1972 [1]. Since then, numerous investigations have been devoted to the static deformation, free vibration and fracture behaviour of magneto-electro-elastic materials. Although advances have been made in piezoelectric ceramics [2-12], research on wave propagation problems in magneto-electro-elastic materials are, to date, still limited. Liu et al. [13] studied piezoelectric-piezomagnetic multilayers with simultaneously negative permeability and permittivity. The scattering properties of an arc-shaped interface crack between piezoelectric and piezomagnetic materials have also been evaluated [14,15]. Soh and Liu [16] analyzed interfacial shear horizontal (SH) waves in a piezoelectric-piezomagnetic bi-material and SH surface waves in transversely isotropic magneto-electro-elastic materials have also been investigated [17]. Wave propagation in magneto-electro-elastic multilayered plates was studied using the propagator matrix and state-vector approaches [18]. Chen and Shen [19] studied the propagation of axial shear

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magneto-electro-elastic waves in piezoelectric-piezomagnetic composites with randomly distributed cylindrical inhomogeneities. Melkumyan [20] discussed 12 shear surface waves guided by clamped/free boundaries in magneto-electro-elastic materials. All these studies concentrated on anti-plane problems. Abbudi and Barnett [21] carried out a detailed study on the interfacial wave in the corresponding piezoelectric bimaterial. Nevertheless, to the best of our knowledge, there are no reports on two-dimensional (2D) wave propagation in magneto-electro-elastic bimaterials.

This study investigates the propagation properties of Stoneley waves between two transversely isotropic and magneto-electro-elastic half planes. Twenty five sets of magneto-electrical interface conditions are considered. By assuming the possible form of the solution, generalized frequency equations are derived. Numerical examples are presented to show the effects of magneto-electrical interface conditions on the velocity of Stoneley waves. It is expected that our results will have applications in surface acoustic wave devices.

2. General equations and interface conditions

Consider two transversely isotropic magneto-electro-elastic half planes with their poling directions all parallel to the *y*-axis, as schematically shown in Figure 1. The two half planes are perfectly bounded mechanically. The constitutive equations within the framework of the theory of linear magneto-electro-elastic medium take the form:

$$\begin{cases} \sigma_{xx}^{(l)} \\ \sigma_{yy}^{(l)} \\ \sigma_{xy}^{(l)} \end{cases} = \begin{bmatrix} c_{11}^{(l)} & c_{13}^{(l)} & 0 \\ c_{13}^{(l)} & c_{33}^{(l)} & 0 \\ 0 & 0 & c_{44}^{(l)} \end{bmatrix} \begin{cases} u_{,x}^{(l)} \\ v_{,y}^{(l)} \\ u_{,y}^{(l)} + v_{,x}^{(l)} \end{cases} + \begin{bmatrix} 0 & e_{31}^{(l)} \\ 0 & e_{33}^{(l)} \\ e_{15}^{(l)} & 0 \end{bmatrix} \begin{cases} \phi_{,x}^{(l)} \\ \phi_{,y}^{(l)} \end{cases} \\ + \begin{bmatrix} 0 & f_{31}^{(l)} \\ 0 & f_{33}^{(l)} \\ f_{15}^{(l)} & 0 \end{bmatrix} \begin{cases} \psi_{,x}^{(l)} \\ \psi_{,y}^{(l)} \end{cases} , \quad l = 1,2 \end{cases}$$

$$\begin{bmatrix} D_{x}^{(l)} \\ 0 \end{bmatrix} \begin{bmatrix} 0 & 0 & e_{15}^{(l)} \end{bmatrix} \begin{bmatrix} u_{,x}^{(l)} \\ \psi_{,y}^{(l)} \end{bmatrix} \quad \begin{bmatrix} \varepsilon_{11}^{(l)} & 0 \end{bmatrix} \begin{bmatrix} \phi_{,x}^{(l)} \\ \phi_{,x}^{(l)} \end{bmatrix}$$

$$\begin{cases} D_{y}^{(l)} \\ D_{y}^{(l)} \end{cases} = \begin{bmatrix} e_{31}^{(l)} & e_{33}^{(l)} & 0 \end{bmatrix} \begin{cases} \psi_{y}^{(l)} \\ u_{y}^{(l)} + v_{x}^{(l)} \\ \psi_{x}^{(l)} + v_{x}^{(l)} \end{cases} - \begin{bmatrix} g_{11}^{(l)} & 0 \\ 0 & g_{33}^{(l)} \end{bmatrix} \begin{cases} \psi_{y}^{(l)} \\ \psi_{y}^{(l)} \\ \psi_{y}^{(l)} \end{cases} , \quad l = 1, 2$$
 (1b)

$$\begin{cases} B_{x}^{(l)} \\ B_{y}^{(l)} \end{cases} = \begin{bmatrix} 0 & 0 & f_{15}^{(l)} \\ f_{31}^{(l)} & f_{33}^{(l)} & 0 \end{bmatrix} \begin{cases} u_{,x}^{(l)} \\ v_{,y}^{(l)} \\ u_{,y}^{(l)} + v_{,x}^{(l)} \end{cases} - \begin{bmatrix} g_{11}^{(l)} & 0 \\ 0 & g_{33}^{(l)} \end{bmatrix} \begin{cases} \phi_{,x}^{(l)} \\ \phi_{,y}^{(l)} \end{cases}$$

$$- \begin{bmatrix} \mu_{11}^{(l)} & 0 \\ 0 & \mu_{33}^{(l)} \end{bmatrix} \begin{cases} \psi_{,x}^{(l)} \\ \psi_{,y}^{(l)} \end{cases}, \quad l = 1, 2$$

$$(1c)$$

where the superscript (l)(l=1,2) represents the upper and lower half planes, respectively; $u^{(l)}(x, y, t)$ and $v^{(l)}(x, y, t)$ are the displacement components in the x- and y-directions,



Figure 1. Bonded magneto-electro-elastic bimaterial plane possessing hexagonal symmetry with their poling directions along the *y*-axis.

respectively; $\phi^{(l)}(x, y, t)$ and $\psi^{(l)}(x, y, t)$ are the electric and magnetic potentials, respectively; $\sigma_{T\Gamma}^{(l)}(x, y, t)$, $D_T^{(l)}(x, y, t)$ and $B_T^{(l)}(x, y, t)$ (T, $\Gamma = x, y$) are the stress, electric displacement and magnetic induction, respectively; $c_{ij}^{(l)}$, $e_{ij}^{(l)}$, $f_{ij}^{(l)}$ and $g_{ij}^{(l)}$ are the elastic, piezoelectric, piezomagnetic and magnetoelectric constants, respectively; $\varepsilon_{ij}^{(l)}$ and $\mu_{ij}^{(l)}$ are the dielectric permittivities and magnetic permeabilities, respectively.

In the absence of body forces, electric charges and magnetic charges, the governing equations for the elastic displacements $u^{(l)}$ and $v^{(l)}$, electric potential $\phi^{(l)}$, and magnetic potential $\psi^{(l)}$ can be written as follows

$$c_{11}^{(l)}u_{,xx}^{(l)} + c_{44}^{(l)}u_{,yy}^{(l)} + \left(c_{13}^{(l)} + c_{44}^{(l)}\right)v_{,xy}^{(l)} + \left(e_{31}^{(l)} + e_{15}^{(l)}\right)\phi_{,xy}^{(l)} + \left(f_{31}^{(l)} + f_{15}^{(l)}\right)\psi_{,xy}^{(l)} = \rho^{(l)}u_{,tt}^{(l)}$$
(2a)

$$\left(c_{13}^{(l)} + c_{44}^{(l)}\right)u_{,xy}^{(l)} + c_{44}^{(l)}v_{,xx}^{(l)} + c_{33}^{(l)}v_{,yy}^{(l)} + e_{15}^{(l)}\phi_{,xx}^{(l)} + e_{33}^{(l)}\phi_{,yy}^{(l)} + f_{15}^{(l)}\psi_{,xx}^{(l)} + f_{33}^{(l)}\psi_{,yy}^{(l)} = \rho^{(l)}v_{,tt}^{(l)}$$
(2b)

$$\left(e_{31}^{(l)} + e_{15}^{(l)}\right)u_{,xy}^{(l)} + e_{15}^{(l)}v_{,xx}^{(l)} + e_{33}^{(l)}v_{,yy}^{(l)} - \varepsilon_{11}^{(l)}\phi_{,xx}^{(l)} - \varepsilon_{33}^{(l)}\phi_{,yy}^{(l)} - g_{11}^{(l)}\psi_{,xx}^{(l)} - g_{33}^{(l)}\psi_{,yy}^{(l)} = 0$$
(2c)

$$\left(f_{31}^{(l)} + f_{15}^{(l)}\right)u_{,xy}^{(l)} + f_{15}^{(l)}v_{,xx}^{(l)} + f_{33}^{(l)}v_{,yy}^{(l)} - g_{11}^{(l)}\phi_{,xx}^{(l)} - g_{33}^{(l)}\phi_{,yy}^{(l)} - \mu_{11}^{(l)}\psi_{,xx}^{(l)} - \mu_{33}^{(l)}\psi_{,yy}^{(l)} = 0$$
(2d)

where $\rho^{(l)}$ is the material density.

Mechanical bonding conditions along the interface require that:

$$u^{(1)}(x,0,t) = u^{(2)}(x,0,t)$$
(3a)

$$v^{(1)}(x,0,t) = v^{(2)}(x,0,t)$$
(3b)

$$\sigma_{xy}^{(1)}(x,0,t) = \sigma_{xy}^{(2)}(x,0,t)$$
(3c)

$$\sigma_{yy}^{(1)}(x,0,t) = \sigma_{yy}^{(2)}(x,0,t)$$
(3d)

The mechanical contact (or interface) conditions (3) together with the following ideal 25 sets of magneto-electrical contact (interface) conditions on the interface y=0 are of interest to us:

Case 1:
$$\phi^{(1)} = \phi^{(2)}, D_y^{(1)} = D_y^{(2)}, \psi^{(1)} = \psi^{(2)}, B_y^{(1)} = B_y^{(2)}$$
 (4a-1)

Case 2:
$$\phi^{(1)} = \phi^{(2)}, D_y^{(1)} = D_y^{(2)}, \psi^{(1)} = \psi^{(2)} = 0$$
 (4a-2)

Case 3:
$$\phi^{(1)} = \phi^{(2)}, D_y^{(1)} = D_y^{(1)}, B_y^{(1)} = B_y^{(2)} = 0$$
 (4a-3)

Case 4:
$$\phi^{(1)} = \phi^{(2)}, \ D_y^{(1)} = D_y^{(2)}, \ \psi^{(1)} = 0, \ B_y^{(2)} = 0$$
 (4a-4)

Case 5:
$$\phi^{(1)} = \phi^{(2)}, D_y^{(1)} = D_y^{(2)}, B_y^{(1)} = 0, \psi^{(2)} = 0$$
 (4a-5)

Case 6:
$$\phi^{(1)} = \phi^{(2)} = 0, \ \psi^{(1)} = \psi^{(2)}, \ B_y^{(1)} = B_y^{(2)}$$
 (4b-1)

Case 7:
$$\phi^{(1)} = \phi^{(2)} = 0, \ \psi^{(1)} = \psi^{(2)} = 0$$
 (4b-2)

Case 8:
$$\phi^{(1)} = \phi^{(2)} = 0, B_y^{(1)} = B_y^{(2)} = 0$$
 (4b-3)

Case 9:
$$\phi^{(1)} = \phi^{(2)} = 0, \ \psi^{(1)} = 0, \ B_y^{(2)} = 0$$
 (4b-4)

Case 10:
$$\phi^{(1)} = \phi^{(2)} = 0, \ B_y^{(1)} = 0, \ \psi^{(2)} = 0$$
 (4b-5)

Case 11:
$$D_y^{(1)} = D_y^{(2)} = 0, \ \psi^{(1)} = \psi^{(2)}, \ B_y^{(1)} = B_y^{(2)}$$
 (4c-1)

Case 12:
$$D_y^{(1)} = D_y^{(2)} = 0, \ \psi^{(2)} = \psi^{(2)} = 0$$
 (4c-2)
Case 13: $D_y^{(1)} = D_y^{(2)} = 0, \ B_y^{(1)} = B_y^{(2)} = 0$ (4c-3)

Case 13:
$$D_y = D_y = 0, B_y = 0$$
 (4c-3)
Case 14: $D_v^{(1)} = D_v^{(2)} = 0, \psi^{(1)} = 0, B_v^{(2)} = 0$ (4c-4)

Case 15:
$$D_y^{(1)} = D_y^{(2)} = 0, \ B_y^{(1)} = 0, \ \psi^{(2)} = 0$$
 (4c-5)

Case 16:
$$\phi^{(1)} = 0, \ D_y^{(2)} = 0, \ \psi^{(1)} = \psi^{(2)}, \ B_y^{(1)} = B_y^{(2)}$$
 (4d-1)

Case 17:
$$\phi^{(1)} = 0, D_y^{(2)} = 0, \psi^{(1)} = \psi^{(2)} = 0$$
 (4d-2)
Case 18: $\phi^{(1)} = 0, D_y^{(2)} = 0, B_y^{(1)} = B_y^{(2)} = 0$ (4d-3)

Case 18:
$$\phi^{(1)} = 0, D_y^{(2)} = 0, B_y^{(1)} = B_y^{(2)} = 0$$
 (4d-3)

Case 19:
$$\phi^{(1)} = 0, D_y^{(2)} = 0, \psi^{(1)} = 0, B_y^{(2)} = 0$$
 (4d-4)

Case 20:
$$\phi^{(1)} = 0, D_y^{(2)} = 0, B_y^{(1)} = 0, \psi^{(2)} = 0$$
 (4d-5)

Case 21:
$$D_y^{(1)} = 0, \ \phi^{(2)} = 0, \ \psi^{(1)} = \psi^{(2)}, \ B_y^{(1)} = B_y^{(2)}$$
 (4e-1)

Case 22:
$$D_y^{(1)} = 0, \ \phi^{(2)} = 0, \ \psi^{(1)} = \psi^{(2)} = 0$$
 (4e-2)

Case 23:
$$D_y^{(1)} = 0, \ \phi^{(2)} = 0, \ B_y^{(1)} = B_y^{(2)} = 0$$
 (4e-3)

Case 24:
$$D_y^{(1)} = 0, \ \phi^{(2)} = 0, \ \psi^{(1)} = 0, \ B_y^{(2)} = 0$$
 (4e-4)

Case 25:
$$D_y^{(1)} = 0, \ \phi^{(2)} = 0, \ B_y^{(1)} = 0, \ \psi^{(2)} = 0$$
 (4e-5)

Equations (4) imply that the magneto-electrical interface conditions are classified into five groups, each of which simultaneously includes five different subsets of interface conditions.

3. Derivation of frequency equation

We consider the possible solution to Equation (2) in the form of the Stoneley waves:

$$\mathbf{u}^{(l)} = \mathbf{\Xi}^{(l)}(y, k, \omega) \exp[ikx - i\omega t], \quad l = 1, 2$$
(5)

where $\mathbf{u}^{(l)} = \{ u^{(l)} \ v^{(l)} \ \phi^{(l)} \ \psi^{(l)} \}$ is called the extended displacement; k and ω are wave number and frequency, respectively. For a 2D in-plane problem, the extended displacements are generally exponential functions of y. Therefore, we can express Equation (2) as:

$$\mathbf{u}^{(l)}(x, y, t) = F^{(l)}(k) \left\{ \begin{array}{cc} A_u^{(l)} & A_{\psi}^{(l)} & A_{\psi}^{(l)} \end{array} \right\}^{\mathrm{T}} \exp(\lambda^{(l)} ky) \exp(ikx) \exp(-ikct), \quad l = 1, 2$$
(6)

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in which, $kc = \omega$ and c is the phase velocity of Stoneley waves; $F^{(l)}$ is an unknown function; $A_u^{(l)}, A_{\psi}^{(l)}, A_{\psi}^{(l)}, A_{\psi}^{(l)}$ are related to the material constants. By inserting extended displacement $\mathbf{u}^{(l)}$ of Equation (6) into Equations (2), one obtains the following secular equation (or eigenvalue equation):

$$|\mathbf{M}^{(l)}| = 0, \quad l = 1, 2 \tag{7}$$

with

$$\mathbf{M}^{(l)} = \begin{bmatrix} \rho^{(l)}c^{2} + c_{44}^{(l)}\lambda^{(l)2} - c_{11}^{(l)} & i\left(c_{13}^{(l)} + c_{44}^{(l)}\right)\lambda^{(l)} & i\left(e_{31}^{(l)} + e_{15}^{(l)}\right)\lambda^{(l)} & i\left(f_{31}^{(l)} + f_{15}^{(l)}\right)\lambda^{(l)} \\ i\left(c_{13}^{(l)} + c_{44}^{(l)}\right)\lambda^{(l)} & \rho^{(l)}c^{2} + c_{33}^{(l)}\lambda^{(l)2} - c_{44}^{(l)} & e_{33}^{(l)}\lambda^{(l)2} - e_{15}^{(l)} & f_{33}^{(l)}\lambda^{(l)2} - f_{15}^{(l)} \\ i\left(e_{31}^{(l)} + e_{15}^{(l)}\right)\lambda^{(l)} & e_{33}^{(l)}\lambda^{(l)2} - e_{15}^{(l)} & \varepsilon_{11}^{(l)} - \varepsilon_{33}^{(l)}\lambda^{(l)2} & g_{11}^{(l)} - g_{33}^{(l)}\lambda^{(l)2} \\ i\left(f_{31}^{(l)} + f_{15}^{(l)}\right)\lambda^{(l)} & f_{33}^{(l)}\lambda^{(l)2} - f_{15}^{(l)} & g_{11}^{(l)} - g_{33}^{(l)}\lambda^{(l)2} & \mu_{11}^{(l)} - \mu_{33}^{(l)}\lambda^{(l)2} \end{bmatrix}$$

$$\tag{8}$$

It can be shown from Equation (8) that there are, in total, eight roots from Equation (7) and, if $\tilde{\lambda}(c)$ is a root of Equation (7), so is $-\tilde{\lambda}(c)$. Thus, only four are independent. Without loss of generality, we can denote $\lambda_m^{(l)}$ (l = 1, 2, m = 1, 2, 3, 4) to be the roots of Equation (7), which satisfy $\operatorname{Re}(\lambda_m^{(1)}) < 0$ and $\operatorname{Re}(\lambda_m^{(2)}) > 0$. In addition, since all the field quantities must vanish as |y| approaches infinity, we can further express the extended displacements as:

$$\mathbf{u}^{(l)}(x,y,t) = \sum_{m=1}^{4} F_m^{(l)}(k) \left\{ a_{um}^{(l)} \quad a_{vm}^{(l)} \quad a_{\phi m}^{(l)} \quad a_{\psi m}^{(l)} \right\}^{\mathrm{T}} \exp(\lambda_m^{(l)} ky) \exp(ikx) \exp(-ikct), \quad l = 1,2$$
(9)

where $a_{um}^{(l)}(\lambda_m^{(l)}, c)$, $a_{vm}^{(l)}(\lambda_m^{(l)}, c)$, $a_{\phi m}^{(l)}(\lambda_m^{(l)}, c)$ and $a_{\psi m}^{(l)}(\lambda_m^{(l)}, c)$ (l=1,2) are known material constants satisfying the following relations:

$$\mathbf{M}^{(l)} \left\{ a_{um}^{(l)} \quad a_{\psi m}^{(l)} \quad a_{\phi m}^{(l)} \quad a_{\psi m}^{(l)} \right\}^{\mathrm{T}} = \mathbf{0}, \quad l = 1, 2$$
(10)

It should be noted that for given material parameters, $a_{um}^{(l)}$, $a_{\nu m}^{(l)}$, $a_{\phi m}^{(l)}$ and $a_{\psi m}^{(l)}$ in Equation (10) depend only on the wave velocity c, since $\lambda_m^{(l)}$ is actually the function of c.

Substituting Equation (9) into the constitutive equations, i.e. Equations (1), the expressions for stresses $\sigma_{xy}^{(l)}$ and $\sigma_{yy}^{(l)}$, electric displacement $D_y^{(l)}$ and magnetic induction $B_y^{(l)}$ (i.e. extended traction $\tau^{(l)}(x, y, t)$) are obtained as follows:

$$\tau^{(l)}(x,y,t) = \sum_{m=1}^{4} F_m^{(l)}(k) \left\{ G_{1m}^{(l)} \quad G_{2m}^{(l)} \quad G_{3m}^{(l)} \quad G_{4m}^{(l)} \right\}^{\mathrm{T}} \exp(\lambda_m^{(l)} ky) \exp(ikx) \exp(-ikct), \quad l = 1,2$$
(11)

in which

$$\boldsymbol{\tau}^{(l)}(x, y, t) = \left\{ \begin{array}{cc} \sigma_{xy}^{(l)} & \sigma_{yy}^{(l)} & D_{y}^{(l)} & B_{y}^{(l)} \end{array} \right\}, \quad l = 1, 2$$
(12)

$$G_{1m}^{(l)}(k,c,\lambda_m^{(l)}) = c_{44}^{(l)}\lambda_m^{(l)}ka_{um}^{(l)} + ik\left(c_{44}^{(l)}a_{vm}^{(l)} + e_{15}^{(l)}a_{\phi m}^{(l)} + f_{15}^{(l)}a_{\psi m}^{(l)}\right), \quad l = 1,2$$
(13a)

$$G_{2m}^{(l)}(k,c,\lambda_m^{(l)}) = c_{13}^{(l)}ika_{um}^{(l)} + \lambda_m^{(l)}k\Big(c_{33}^{(l)}a_{vm}^{(l)} + e_{33}^{(l)}a_{\phi m}^{(l)} + f_{33}^{(l)}a_{\psi m}^{(l)}\Big), \quad l = 1,2$$
(13b)

$$G_{3m}^{(l)}(k,c,\lambda_m^{(l)}) = e_{31}^{(l)}ika_{um}^{(l)} + \lambda_m^{(l)}k\left(e_{33}^{(l)}a_{\nu m}^{(l)} - \varepsilon_{33}^{(l)}a_{\phi m}^{(l)} - g_{33}^{(l)}a_{\psi m}^{(l)}\right), \quad l = 1,2$$
(13c)

$$G_{4m}^{(l)}(k,c,\lambda_m^{(l)}) = f_{31}^{(l)}ika_{um}^{(l)} + \lambda_m^{(l)}k\Big(f_{33}^{(l)}a_{\nu m}^{(l)} - g_{33}^{(l)}a_{\phi m}^{(l)} - \mu_{33}^{(l)}a_{\psi m}^{(l)}\Big), \quad l = 1,2$$
(13d)

It is important to point out that $G_{1m}^{(l)}$, $G_{2m}^{(l)}$, $G_{3m}^{(l)}$ and $G_{4m}^{(l)}$ in Equation (13) are all linear functions of the wave number k.

Substituting Equations (11) and (9) into Equations (3) and (4) yields:

$$\Omega \mathbf{F} = \mathbf{0} \tag{14}$$

where

$$\mathbf{F} = \left\{ F_1^{(1)} \quad F_2^{(1)} \quad F_3^{(1)} \quad F_4^{(1)} \quad F_1^{(2)} \quad F_2^{(2)} \quad F_3^{(2)} \quad F_4^{(2)} \right\}$$
(15)

and Ω is a 8 × 8 matrix. The elements of the matrix Ω are:

$$\Omega_{1m} = a_{um}^{(1)}, \ \Omega_{2m} = a_{vm}^{(1)}, \ \Omega_{3m} = G_{1m}^{(1)}, \ \Omega_{4m} = G_{2m}^{(1)}, \quad m = 1, 2, \dots, 4$$
(16a)

$$\Omega_{1(m+4)} = -a_{um}^{(2)}, \ \Omega_{2(m+4)} = -a_{vm}^{(2)}, \ \Omega_{3(m+4)}$$
$$= -G_{1m}^{(2)}, \ \Omega_{4(m+4)} = -G_{2m}^{(2)}, \quad m = 1, 2, \dots, 4$$
(16b)

and Ω_{km} , $\Omega_{k(m+4)}$ (k = 5, 6, 7, 8, m = 1, 2, 3, 4) corresponding to the 25 sets of interface conditions can also be obtained. For example, for Case 1 interface condition:

$$\Omega_{5m} = a_{\phi m}^{(1)}, \ \Omega_{6m} = G_{3m}^{(1)}, \ \Omega_{7m} = a_{\psi m}^{(1)}, \ \Omega_{8m} = G_{4m}^{(1)}, \quad m = 1, 2, \dots, 4$$
(17a)

$$\Omega_{5(m+4)} = -a_{\phi m}^{(2)}, \ \Omega_{6(m+4)} = -G_{3m}^{(2)}, \ \Omega_{7(m+4)}$$
$$= -a_{\psi m}^{(2)}, \ \Omega_{8(m+4)} = -G_{4m}^{(2)}, \quad m = 1, 2, \dots, 4$$
(17b)

For a nontrivial solution of \mathbf{F} , the determinant of its coefficient in Equation (14) must be equal to zero, which leads to:

$$|\mathbf{\Omega}| = 0 \tag{18}$$

Equation (18) is the derived frequency equation, which reveals the frequency dispersion characters of the Stoneley wave in magneto-electro-elastic bimaterials.

For a fixed row or column in the matrix \mathbf{Q} , its elements are either composed of one of $a_{um}^{(l)}, a_{vm}^{(l)}, a_{\phi m}^{(l)}$ and $a_{\psi m}^{(l)}$ or composed of one of $G_{1m}^{(l)}, G_{2m}^{(l)}, G_{3m}^{(l)}$ and $G_{4m}^{(l)}$, it is easily concluded from Equations (16) and (17) together with Equations (13) and (10) that the wave velocity c satisfying Equation (18) is independent of k, which implies that the Stoneley wave in a mechanically bonded magneto-electro-elastic bimaterial is non-dispersive. This statement is also consistent with the result by Abbudi and Barnett [21] for the corresponding piezoelectric bimaterial plane and can be further validated by the numerical results discussed below.

4. Numerical results and discussion

Equation (18) is very complicated and, thus, it is impossible for us to solve analytically the Stoneley wave velocities. In this section, numerical results are presented and discussed for magneto-electro-elastic materials composed of different volumetric ratios of BaTiO₃-CoFe₂O₄. Material properties of the magneto-electro-elastic materials as volume percentage (or volume fraction v_f) of BaTiO₃–CoFe₂O₄ are listed in Table 1 [22–27].

We also define $c_0 = \min\{c/c_s^{(1)}, c/c_s^{(2)}\}$, where $c_s^{(l)} = \sqrt{\tilde{c}_{44}^{(l)}/\rho^{(l)}}(l=1,2)$ and

$$\tilde{c}_{44}^{(l)} = c_{44}^{(l)} + \frac{\varepsilon_{11}^{(l)} f_{15}^{(l)2} - 2 e_{15}^{(l)} f_{15}^{(l)} g_{11}^{(l)} + \mu_{11}^{(l)} e_{15}^{(l)2}}{\mu_{11}^{(l)} \varepsilon_{11}^{(l)} - g_{11}^{(l)2}}$$

Actually, $c_s^{(1)}$ and $c_s^{(2)}$ denote the shear wave velocities corresponding to the two kinds of the magneto-electro-elastic materials, respectively. For purely elastic materials, the corresponding Stoneley waves are less than one, thus our numerical studies in this paper are limited within the range $0 < c_0 < 1$.

The piezomagnetic/piezoelectric composite structure (i.e. CoFe₂O₄-BaTiO₃) is widely used in engineering. Thus, the Stoneley wave velocities in this bimaterial infinite plane are firstly examined (CoFe₂O₄ (in y > 0 half plane) and BaTiO₃ (in y < 0 half plane) correspond to $v_f = 0.0$ and $v_f = 1.0$, respectively). As shown in Table 2, there are two Stoneley waves in this bimaterial CoFe₂O₄-BaTiO₃ infinite plane. It is also interesting that the magnetoelectrical interface conditions have no influence on the wave velocities.

To further understand the properties of the existing Stoneley waves in bimaterial $CoFe_2O_4$ -BaTiO₃, we assume $f_{31}^{(1)} = f_{33}^{(1)} = f_{15}^{(1)} = e_{31}^{(2)} = e_{33}^{(2)} = e_{15}^{(2)} = 0$ with the other

Table 1. Material properties of BaTiO₃-CoFe₂O₄ composites as a percentage (volume fraction v_f) (c_{ij} in 10^9 N/m², e_{ij} in C/m², ε_{ij} in 10^{-9} C/Vm, f_{ij} in N/Am, μ_{ij} in 10^{-4} Ns²/C², g_{ij} in 10^{-12} Ns/VC, ρ in kg/m³) $v_f = 0.0$ corresponding to CoFe₂O₄ and $v_f = 1.0$ to BaTiO₃.

	$v_{\rm f} = 0.0$	$v_{\rm f} = 0.2$	$v_{\rm f} = 0.4$	$v_{\rm f} = 0.6$	$v_{\rm f} = 0.8$	$v_{\rm f} = 1.0$
c ₁₁	286	250	225	200	175	166
C ₁₂	173	146	125	110	100	77
c ₁₃	170	145	125	110	100	78
C33	269.5	240	220	190	170	162
C ₄₄	45.3	45	45	45	50	43
e ₁₅	0	0	0	0	0	11.6
e ₃₁	0	-2	-3	-3.5	-4	-4.4
e ₃₃	0	4	7	11	14	18.6
e ₁₁	0.08	0.33	0.8	0.9	1.0	11.2
e ₃₃	0.093	2.5	5.0	7.5	10	12.6
μ_{11}	5.9	3.9	2.5	1.5	0.8	0.05
μ_{33}	1.57	1.33	1.0	0.75	0.5	0.1
f_{15}	560	340	220	180	80	0
f_{31}	580	410	300	200	100	0
f ₃₃	700	550	380	260	120	0
g ₁₁	0	2.8	4.8	6.0	6.8	0
g ₃₃	0	2000	2750	2500	1500	0
ρ	5300	5400	5500	5600	5700	5800

Table 2. Stoneley wave velocity *c* and corresponding normalized value c_0 in CoFe₂O₄–BaTiO₃ bimaterial; material properties of CoFe₂O₄ (in *y*>0) and BaTiO₃ (in *y*<0) are the same as those given in Table 1 when $v_f = 0.0$ and $v_f = 1.0$, respectively.

	Cases 1–25
$c_1 ({\rm ms}^{-1})$	0.3081×10^{3}
(c_{01})	(0.1006)
$c_2({\rm ms}^{-1})$	2.7089×10^{3}
(c ₀₂)	(0.9320)

Table 3. Stoneley wave velocity *c* and corresponding normalized value c_0 for the purely elastic bimaterial reduced from CoFe₂O₄-BaTiO₃ by letting both piezomagnetic and piezoelectric constants equal to zero in these materials. Material properties of CoFe₂O₄ and BaTiO₃ are the same as those given in Table 1 when v_f =0.0 and v_f =1.0, respectively, and their piezomagnetic and piezoelectric constants are set to zero.

	Cases 1–25
$c_1 (ms^{-1})$ (c_{01})	$\begin{array}{c} 0.4822 \times 10^{3} \\ (0.1771) \end{array}$

material properties being the same as those for $CoFe_2O_4$ and $BaTiO_3$, respectively. The Stoneley wave velocities of this uncoupled bimaterial infinite plane are, therefore, examined, as shown in Table 3. Obviously, in this case, only one Stoneley wave velocity exists, which is also independent of the magnetoelectrical interface conditions. Thus, Tables 2 and 3 further indicate that, for the bimaterial $CoFe_2O_4$ – $BaTiO_3$ infinite plane, the first Stoneley wave with velocity c_1 is closely associated with the elastic properties of both the piezomagnetic and piezoelectric materials; while the second Stoneley wave with velocity c_2 is associated with both the piezomagnetic properties of $CoFe_2O_4$ and the piezoelectric properties of $BaTiO_3$.

As discussed in [20], for 2D anti-plane problems of the homogeneous magnetoelectro-elastic medium, different magnetoelectrical interface conditions would generally induce waves along the interface. For the present 2D in-plane problems, we assume 25 sets of interface conditions as Case 1 to Case 25 *via* Equation (4), but with the same homogeneous material in the infinite plane. Numerical calculations are carried out for various percentages of $CoFe_2O_4$ –BaTiO₃ to show the effects of magnetoelectrical interface conditions on the Stoneley waves.

As shown in Table 4, for an infinite homogeneous magneto-electro-elastic medium containing an interface (*xoz* plane), both the material properties and magneto-electrical contact conditions along the interface have important effects on the Stoneley waves. For example, for $v_f = 0.2$ and $v_f = 0.4$, there are two Stoneley wave velocities for each case. However, for material volume ratio $v_f = 0.6$, only one Stoneley wave velocity exists. Furthermore, if $v_f = 0.8$, there is no Stoneley wave for the magnetically impermeable

Table 4. Stoneley wave velocity c and corresponding normalized value c_0 in an infinite homogeneous magneto-electro-elastic plane under the 25 sets of magneto-electro-elastic interface conditions for different volumetric ratios of BaTiO₃–CoFe₂O₄.

	$v_{\rm f} = 0.2$	$v_{\rm f} = 0.4$	$v_{\rm f} = 0.6$	$v_{\rm f} = 0.8$ Case 1-25 except for
	Case 1–25	Case 1–25	Case 1–25	Cases 3, 8, 13, 18, 23
$c_1(\text{ms}^{-1})$	2.0734×10^{3}	1.6702×10^{3}	2.0129×10^{3}	2.6001×10^{3}
$c_2(\text{ms}^{-1})$	(0.7139) 2.7861 × 10 ³	(0.3034) 0.7659×10^{3}	(0.7084)	(0.8772)
(c_{02})	(0.9620)	(0.9649)	—	_

interface conditions (i.e. $B_y^{(1)} = B_y^{(2)} = 0$). Table 4 also implies that magnetoelectrical interface conditions have no influence on the existing Stoneley wave velocities. In addition, it should also be noted that in this case, Case 4 and Case 5, Case 9 and 10, Case 14 and Case 15, Case 19 and case 20 are, respectively, equivalent; and Case 21 to Case 25 are equivalent to Case 16 to Case 20, respectively. Therefore, there are actually only 16 sets of independent magnetoelectrical interface conditions.

5. Conclusions

- For a mechanically bonded magneto-electro-elastic infinite bimaterial plane, the existing Stoneley wave is a non-dispersive guided wave under the 25 sets of magneto-electrical interface conditions.
- For the bimaterial plane made of $CoFe_2O_4$ in the half plane y>0 and $BaTiO_3$ in the half plane y<0, there are two Stoneley wave velocities. One is closely associated with the combined elastic properties, and the other with the piezomagnetic properties of $CoFe_2O_4$ and piezoelectric properties of $BaTiO_3$.
- For an infinite homogeneous magneto-electro-elastic medium under the 25 sets of interface conditions, both the material properties and the magnetoelectrical interface conditions can have a significant effect on the existence of the Stoneley waves.
- Although the magnetoelectrical interface conditions could affect the existence of the Stoneley wave, they have no influence on the existing Stoneley wave velocities.

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