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On a semi-infinite crack penetrating a piezoelectric circular inhomogeneity with a viscous interface

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ABSTRACT

We investigate a semi-infinite crack penetrating a piezoelectric circular inhomogeneity bonded to an infinite piezoelectric matrix through a linear viscous interface. The tip of the crack is at the center of the circular inhomogeneity. By means of the complex variable and conformal mapping methods, exact closed-form solutions in terms of elementary functions are derived for the following three loading cases: (i) nominal Mode-III stress and electric displacement intensity factors at infinity; (ii) a piezoelectric screw dislocation located in the unbounded matrix; and (iii) a piezoelectric screw dislocation located in the inhomogeneity. The time-dependent electroelastic field in the cracked composite system is obtained. Particularly the time-dependent stress and electric displacement intensity factors at the crack tip, jumps in the displacement and electric potential across the crack surfaces, displacement jump across the viscous interface, and image force acting on the piezoelectric screw dislocation are all derived. It is found that the value of the relaxation (or characteristic) time for this cracked composite system is just twice as that for the same fibrous composite system without crack. Finally, we extend the methods to the more general scenario where a semi-infinite wedge crack is within the inhomogeneity/matrix composite system with a viscous interface.

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1. Introduction

Study of a semi-infinite crack half-way penetrating a circular inhomogeneity (Steif, 1987; Hutchinson, 1987) is fundamental to the understanding of failure mechanism in fiber reinforced composites (Erdogan and Gupta, 1975; Steif, 1987) and plays a special role in quantifying the crack-tip shielding effect by micro-cracking at the tip of a macroscopic crack (Hutchinson, 1987). The stress level at the tip of a macroscopic crack (represented by a semi-infinite crack or wedge crack) can also be reduced or shielded by dislocations that are generated in the vicinity of the crack tip (Majumdar and Burns, 1981; Ohr et al., 1985). At elevated temperatures mass diffusion becomes important along the interface with periodically distributed microscopic steps (Raj and Ashby, 1971; Ashby, 1972). The microscopic mass diffusion-controlled mechanism along the interface can be macroscopically described by the linear rheologic law for a viscous interface (raj and Ashby, 1971; Ashby, 1972; He and Lim, 2001). Consequently it would be more realistic if the viscous interface (or time-dependent sliding interface) and the nearby dislocations can be simultaneously incorporated into the benchmark problem of a semi-infinite crack penetrating a circular inhomogeneity.

Even though various defect problems, such as cracks, dislocations, inhomogeneities (or inclusions), and interfaces in piezoelectric materials which possess the intrinsic electromechanical coupling phenomenon, have been thoroughly investigated (see, for example, Deeg, 1980; Pak, 1990a,b, 1992; Suo et al., 1992; Meguid and Deng, 1998; Deng and Meguid, 1999; Liu

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et al., 1999; Lee et al., 2000; Ru, 2001; Chen et al., 2002a,b; He and Lim, 2003; Wang and Pan, 2008; Wang et al., 2008a), there are no analytical studies of the interaction among all these different kinds of defects in piezoelectric solids within a unified framework.

Therefore, in this work we analytically investigate in detail a semi-infinite insulating crack (or wedge crack) half-way penetrating a piezoelectric circular inhomogeneity bonded to an infinite piezoelectric matrix through a linear viscous interface in the presence of a screw dislocation by means of the complex variable and conformal mapping techniques. Here, the screw dislocation can be located either in the inhomogeneity or in the matrix. Due to the influence of the time-dependent linear viscous interface, the analytic function vectors characterizing the electroelastic field are not only functions of the complex variable *z*, but also functions of the real time *t* (Wang and Pan, 2008; Wang et al., 2008a). It should be noted that a solution to this problem is unavailable, even in the simple framework of pure elasticity. The original problem can be more conveniently discussed in the mapped ζ -plane. The reason why we can obtain closed-form solutions to this problem is due to the fact that: (i) the number of the static and moving image singularities is *finite* in the ζ -plane even though we have two boundaries (one straight, the other one circular) to address (Ting, 2005; Palaniappan, 2005; Wang et al., 2008a). Here, we are particularly interested in the absence of the semi-infinite crack or wedge crack (Wang et al., 2008a). Here, we are particularly interested in the fracture parameters, such as stress and electric displacement intensity factors at the crack tip, jumps in displacement and electric potential across the crack surfaces; the displacement jump across the viscous interface and image force acting on the screw dislocation. All these results are time-dependent due to the influence of the viscous interface.

This paper is structured as follows. In Section 2, we present the basic equations which are essential for the ensuing analysis of a semi-infinite crack half-way penetrating a circular inhomogeneity with a viscous interface. Sections 3–5 are devoted to the study of a semi-infinite insulating crack penetrating the inhomogeneity with a viscous interface under the action of remote nominal stress and electric displacement intensity factors (Section 3), a screw dislocation in the matrix (Section 4) and a screw dislocation in the inhomogeneity (Section 5). In Section 6, we discuss the more general scenario where a semiinfinite wedge crack half-way penetrates a circular inhomogeneity with a viscous interface.

2. Basic equations

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We consider an inhomogeneity/matrix composite plane containing a semi-infinite crack, as shown in Fig. 1. Cartesian and polar coordinate systems are established with their origins at the crack tip such that the crack lies on the negative *x*-axis. Both the circular inhomogeneity of radius *R* and the surrounding unbounded matrix are hexagonal piezoelectric materials with their poling directions parallel to the fiber axis. In addition the circular inhomogeneity is bonded to the surrounding matrix through a linear viscous interface *L* which will be described in more detail below. Throughout this paper, the subscripts 1 and 2 (or the superscripts (1) and (2)) are adopted to identify the quantities in the inhomogeneity and matrix, respectively. In this research we assume that the two-phase composite system is in a state of anti-plane deformation (Pak, 1990a,b; Lee et al., 2000), and the inertia effect in both the inhomogeneity and matrix is ignored. Consequently the non-trivial basic equations expressed in the Cartesian coordinate system (*x*, *y*) are listed below

$$\sigma_{zx,x} + \sigma_{zy,y} = 0, \quad D_{x,x} + D_{y,y} = 0, \tag{1}$$

$$\begin{bmatrix} \sigma_{zy} \\ D_y \end{bmatrix} = \begin{bmatrix} c_{44} & -e_{15} \\ e_{15} & \epsilon_{11} \end{bmatrix} \begin{bmatrix} w_{,y} \\ E_y \end{bmatrix}, \quad \begin{bmatrix} \sigma_{zx} \\ D_x \end{bmatrix} = \begin{bmatrix} c_{44} & -e_{15} \\ e_{15} & \epsilon_{11} \end{bmatrix} \begin{bmatrix} w_{,x} \\ E_x \end{bmatrix},$$
(2)

where a comma followed by x (or y) denotes partial derivatives with respect to x (or y); σ_{zx} , σ_{zy} are the shear stresses; D_x , D_y are the electric displacements; E_x , E_y are the electric fields; w is the out-of-plane displacement; ϕ is the electric potential; c_{44} , e_{15} , and ϵ_{11} are, respectively, the elastic modulus, piezoelectric constant, and dielectric permittivity. In addition we define $\tilde{c}_{44} = c_{44} + e_{15}^2/\epsilon_{11}$ as the piezoelectrically stiffened elastic constant which will also be used in the following analysis.

The electroelastic field can be expressed in terms of a two-dimensional (2D) analytic function vector $\mathbf{f}(z,t) = [f_1(z,t) \ f_2(z,t)]^{\mathrm{T}}$ with z = x + iy being the complex variable and t being the time as follows:

$$\begin{bmatrix} w\\ \phi \end{bmatrix} = \operatorname{Im}\{\mathbf{f}(z,t)\},\tag{3}$$

$$\begin{bmatrix} \sigma_{zy} + i\sigma_{zx} \\ D_y + iD_x \end{bmatrix} = \mathbf{C}\mathbf{f}'(z,t), \quad \begin{bmatrix} \sigma_{z\theta} + i\sigma_{zr} \\ D_{\theta} + iD_r \end{bmatrix} = \frac{z}{|z|}\mathbf{C}\mathbf{f}'(z,t), \tag{4}$$

where the material matrix **C** is defined as $\mathbf{C} = \begin{bmatrix} c_{44} & e_{15} \\ e_{15} & -\epsilon_{11} \end{bmatrix}$. Also in Eq. (4), the prime denotes differentiation with respect to the complex variable *z*, and $\sigma_{z\theta}$, σ_{zr} , D_{θ} , D_r are the stresses and electric displacements in the polar coordinate system (*r*, θ). The appearance of the real time *t* in the analytic function vector **f** is due solely to the influence of the viscous interface (Wang and Pan, 2008; Wang et al., 2008a).

If we further introduce a conformal mapping function $z = m(\zeta)$, then

$$\begin{bmatrix} \sigma_{zt} + i\sigma_{zn} \\ D_t + iD_n \end{bmatrix} = \frac{\zeta \mathbf{C} \mathbf{f}'(\zeta, t)}{|\zeta m'(\zeta)|},\tag{5}$$

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X. Wang et al./International Journal of Solids and Structures 46 (2009) 203-216



Fig. 1. Schematic of a semi-infinite crack penetrating a piezoelectric circular inhomogeneity bonded to the surrounding matrix through a linear viscous interface.

where $\mathbf{f}(z,t) = \mathbf{f}[m(\zeta),t] = \mathbf{f}(\zeta,t)$ has been adopted for the convenience of analysis; σ_{zt} , D_t , σ_{zn} , D_n are the tangential and normal stresses and electric displacements in the curvilinear coordinate system expressed by $m(\zeta)$.

The boundary conditions on the viscous interface between the circular inhomogeneity and the matrix can be expressed as (He and Lim, 2003; Wang et al., 2008a)

$$\begin{aligned}
\sigma_{zr}^{(1)} &= \sigma_{zr}^{(2)}, \quad D_r^{(1)} = D_r^{(2)}, \\
\phi^{(1)} &= \phi^{(2)}, \quad r = R \text{ and } t > 0 \\
\sigma_{zr}^{(1)} &= \eta(\dot{w}^{(2)} - \dot{w}^{(1)}),
\end{aligned} \tag{6}$$

where a dot over the quantity denotes differentiation with respect to time t, and η is the nonnegative interface slip coefficient which can be measured through properly designed experiment (He and Lim, 2003). At the initial time t = 0 the interface L is a perfect one due to the fact that at t = 0 the displacement across the interface has no time to experience any jump (Fan and Wang, 2003).

The boundary conditions on the upper and lower surfaces of the semi-infinite crack are traction-free and charge-free (Pak, 1990b; Lee et al., 2000)

$$\sigma_{zy} = 0, \quad D_y = 0 \quad \text{at } x < 0 \quad \text{and} \quad y = 0 \tag{7}$$

The original boundary value problem can be more conveniently discussed by introducing the following conformal mapping function:

$$z = m(\zeta) = \zeta^2, \tag{8}$$

which maps the cracked *z*-plane onto the right half-plane in the ζ -plane (Re{ ζ } \geq 0), as shown in Fig. 2. More specifically the cracked circular inhomogeneity is mapped onto the half-circular region | ζ | < \sqrt{R} and Re{ ζ } \geq 0 in the ζ -plane; the cracked



Fig. 2. The mapped ζ -plane by using $z = \zeta^2$ for the semi-infinite crack problem in *z*-plane.

matrix is mapped onto $|\zeta| > \sqrt{R}$ and $\operatorname{Re}{\zeta} \ge 0$ in the ζ -plane; the inhomogeneity–matrix interface |z| = R is mapped onto the half-circle $|\zeta| = \sqrt{R}$ and $\operatorname{Re}{\zeta} \ge 0$ in the ζ -plane. In the ζ -plane we have two boundaries to address: one is the straight boundary $\operatorname{Re}{\zeta} = 0$, the other is the half-circular interface $|\zeta| = \sqrt{R}$ and $\operatorname{Re}{\zeta} \ge 0$. One reason why we can obtain closed-form solutions to this problem is that the number of image singularities is *finite* in the ζ -plane (Ting, 2005; Palaniappan, 2005; Wang et al., 2008b). During the analysis we can first satisfy the boundary conditions on the straight surface $|\zeta| = \sqrt{R}$.

In the following we discuss in detail three loading cases:

- (i) Far from the crack tip, the electroelastic field approaches the singular field specified by the Mode-III stress and electric displacement intensity factors K^{σ} and K^{D} .
- (ii) A piezoelectric screw dislocation located in the unbounded matrix. Here the screw dislocation is assumed to be straight and infinitely long along the fiber axis, experiencing a displacement jump *b* and an electric potential jump $\Delta \phi$ across the slip plane. The dislocation can also have a line force *p* and line charge *q* along its core (Lee et al., 2000).
- (iii) A piezoelectric screw dislocation located in the inhomogeneity. In this loading case it is assumed that p = q = 0.

3. Nominal field intensity factors at infinity

3.1. The complex potentials

When the matrix is subjected to nominal Mode-III stress and electric displacement intensity factors K^{σ} and K^{D} at infinity, the asymptotic behavior of $\mathbf{f}_{2}(z,t)$ defined in the matrix at infinity can be found as (Lee et al., 2000)

$$\mathbf{f}_2(z,t) \to \sqrt{\frac{2z}{\pi}} \mathbf{C}_2^{-1} \mathbf{K}, \text{ as } z \to \infty$$
(9)

where $\mathbf{K} = \begin{bmatrix} K^{\sigma} & K^{D} \end{bmatrix}^{T}$. Consequently in the ζ -plane, we obtain the following asymptotic behavior for $\mathbf{f}_{2}(\zeta, t)$

$$\mathbf{f}_2(\zeta, t) \to \sqrt{\frac{2}{\pi}} \zeta \mathbf{C}_2^{-1} \mathbf{K}, \text{ as } \zeta \to \infty$$
 (10)

In view of the basic equations presented in Section 2, the boundary conditions on the viscous interface $|\zeta| = \sqrt{R}$ can be concisely expressed in terms of $\mathbf{f}_1(\zeta, t)$ defined in the inhomogeneity and $\mathbf{f}_2(\zeta, t)$ defined in the matrix as (here we have implicitly

extended the original half-circular interface $|\zeta| = \sqrt{R}$ and $\operatorname{Re}{\zeta} \ge 0$ to an imaginary total circular interface $|\zeta| = \sqrt{R}$ in view of the remote loading in Eq. (10). Consequently the corresponding definition region for $\mathbf{f}_1(\zeta, t)$ is now $|\zeta| < \sqrt{R}$, whilst that for $\mathbf{f}_2(\zeta, t)$ is now $|\zeta| > \sqrt{R}$)

$$\mathbf{C}_{1}\mathbf{f}_{1}^{+}(\zeta,t) + \mathbf{C}_{1}\overline{\mathbf{f}}_{1}^{-}\left(\frac{R}{\zeta},t\right) = \mathbf{C}_{2}\mathbf{f}_{2}^{-}(\zeta,t) + \mathbf{C}_{2}\overline{\mathbf{f}}_{2}^{+}\left(\frac{R}{\zeta},t\right),$$

$$\dot{\mathbf{f}}_{2}^{-}(\zeta,t) - \dot{\overline{\mathbf{f}}}_{2}^{+}\left(\frac{R}{\zeta},t\right) - \dot{\mathbf{f}}_{1}^{+}(\zeta,t) + \dot{\overline{\mathbf{f}}}_{1}^{-}\left(\frac{R}{\zeta},t\right) = \Lambda\mathbf{C}_{1}\left[\zeta\mathbf{f}_{1}^{\prime+}(\zeta,t) - \frac{R}{\zeta}\overline{\mathbf{f}}_{1}^{\prime-}\left(\frac{R}{\zeta},t\right)\right], (|\zeta| = \sqrt{R})$$

$$(11)$$

where the superscripts "+" and "-" denote the limit values from the inner and outer sides of the circle $|\zeta| = \sqrt{R}$, and

$$\Lambda = \frac{1}{2\eta R} \operatorname{diag}[1 \quad 0]. \tag{12}$$

It is of interest to notice that expression (11) in the ζ -plane is very similar in structure to those for a circular inhomogeneity with a viscous interface in the absence of the semi-infinite crack (Wang et al., 2008a). This is another reason why we can arrive at an analytical solution to this problem. It follows from Eq. (11)₁ that

$$\mathbf{f}_{2}(\zeta,t) = \mathbf{C}_{2}^{-1}\mathbf{C}_{1}\overline{\mathbf{f}}_{1}\left(\frac{R}{\zeta},t\right) + \mathbf{f}_{0}(\zeta) - \overline{\mathbf{f}}_{0}\left(\frac{R}{\zeta}\right),$$

$$\overline{\mathbf{f}}_{2}\left(\frac{R}{\zeta},t\right) = \mathbf{C}_{2}^{-1}\mathbf{C}_{1}\mathbf{f}_{1}(\zeta,t) - \mathbf{f}_{0}(\zeta) + \overline{\mathbf{f}}_{0}\left(\frac{R}{\zeta}\right).$$
(13)

where $\mathbf{f}_0(\zeta) = \sqrt{\frac{2}{\pi}} \mathbf{C}_2^{-1} \mathbf{K} \zeta$ denotes the singular asymptotic behavior of $\mathbf{f}_2(\zeta, t)$ at infinity. Substituting the above results into Eq. (12)₂ and eliminating $\mathbf{f}_2(\zeta, t)$ and $\hat{\mathbf{f}}_2(\frac{R}{\zeta}, t)$, we obtain

$$\Lambda \mathbf{C}_{1} \frac{R}{\zeta} \bar{\mathbf{f}}_{1}^{\prime-} \left(\frac{R}{\zeta}, t \right) + \mathbf{H} \mathbf{C}_{1} \dot{\bar{\mathbf{f}}}_{1}^{-} \left(\frac{R}{\zeta}, t \right) = \Lambda \mathbf{C}_{1} \zeta \mathbf{f}_{1}^{\prime+} (\zeta, t) + \mathbf{H} \mathbf{C}_{1} \dot{\mathbf{f}}_{1}^{+} (\zeta, t), \quad (|\zeta| = \sqrt{R})$$

$$\tag{14}$$

where $\mathbf{H} = \begin{bmatrix} H_{11} & H_{12} \\ H_{12} & -H_{22} \end{bmatrix} = \mathbf{C}_1^{-1} + \mathbf{C}_2^{-1}$ is real and symmetric. In addition the components $H_{11} > 0$, $H_{22} > 0$ and H_{12} are explicitly given by

$$H_{11} = \frac{1}{\tilde{c}_{44}^{(1)}} + \frac{1}{\tilde{c}_{44}^{(2)}}, \quad H_{22} = \frac{c_{44}^{(1)}}{\tilde{c}_{44}^{(1)} \in (1)} + \frac{c_{42}^{(2)}}{\tilde{c}_{44}^{(2)} \in (1)}, \quad H_{12} = \frac{e_{15}^{(1)}}{\tilde{c}_{44}^{(1)} \in (1)} + \frac{e_{15}^{(2)}}{\tilde{c}_{44}^{(2)} \in (1)}.$$
(15)

Apparently the right-hand side of Eq. (14) is analytic within the circle |z| = R, while the left-hand side of Eq. (14) is analytic outside the circle including the point at infinity. By employing the Liouville's theorem, the left- and right-hand sides should be identically zero. Consequently we obtain the following set of homogeneous first-order partial differential equation for $\mathbf{f}_1(\zeta, t)$

$$\mathbf{\Lambda C}_{1}\zeta\mathbf{f}_{1}'(\zeta,t) + \mathbf{HC}_{1}\dot{\mathbf{f}}_{1}(\zeta,t) = \mathbf{0}, \quad (|\zeta| < \sqrt{R})$$
(16)

In order to solve Eq. (16), we first consider the following eigenvalue problem:

$$(\mathbf{\Lambda} - \lambda \mathbf{H})\mathbf{v} = \mathbf{0},\tag{17}$$

The two eigenvalues λ_1 and λ_2 of the above eigenvalue problem can be explicitly determined as

$$\lambda_1 = \frac{H_{22}}{2\eta R(H_{11}H_{22} + H_{12}^2)} > 0, \quad \lambda_2 = 0.$$
⁽¹⁸⁾

The eigenvectors associated with the two eigenvalues are

$$\mathbf{v}_1 = \begin{bmatrix} H_{22} \\ H_{12} \end{bmatrix}, \quad \mathbf{v}_2 = \begin{bmatrix} \mathbf{0} \\ \mathbf{1} \end{bmatrix}. \tag{19}$$

It can be proved that the following orthogonal relationships with respect to the two real and symmetric matrices H and Λ hold

$$\boldsymbol{\Phi}^{\mathrm{T}} \mathbf{H} \boldsymbol{\Phi} = \mathrm{diag}[\delta_{1} \quad \delta_{2}], \quad \boldsymbol{\Phi}^{\mathrm{T}} \boldsymbol{\Lambda} \boldsymbol{\Phi} = \lambda_{1} \delta_{1} \mathrm{diag}[1 \quad 0], \tag{20}$$

where

$$\boldsymbol{\Phi} = \begin{bmatrix} \mathbf{v}_1 & \mathbf{v}_2 \end{bmatrix}, \tag{21a}$$

and

$$\delta_1 = \mathbf{v}_1^T \mathbf{H} \mathbf{v}_1 = \lambda_1^{-1} \mathbf{v}_1^T \Lambda \mathbf{v}_1 = H_{22} (H_{11} H_{22} + H_{12}^2) > \mathbf{0},$$

$$\delta_2 = \mathbf{v}_2^T \mathbf{H} \mathbf{v}_2 = -H_{22} < \mathbf{0},$$
(21b)

We now introduce a new function vector $\mathbf{\Omega}(\zeta, t) = \begin{bmatrix} \Omega_1(\zeta, t) & \Omega_2(\zeta, t) \end{bmatrix}^T$ defined by

$$\mathbf{C}_{1}\mathbf{f}_{1}(\zeta,t)=\mathbf{\Phi}\mathbf{\Omega}(\zeta,t),$$

In view of Eqs. (20) and (22), the original coupled set of differential Eq. (16) can be decoupled as follows:

$$\begin{array}{ll}
\Omega_1(\zeta,t) + \lambda_1 \zeta \Omega_1'(\zeta,t) = \mathbf{0}, \\
\dot{\Omega}_2(\zeta,t) = \mathbf{0},
\end{array} \quad (|\zeta| < \sqrt{R})$$
(23)

(22)

whose general solutions can be expediently given by

$$\Omega_1(\zeta, t) = \Omega_1(\exp(-\lambda_1 t)\zeta, \mathbf{0}), \qquad (|\zeta| < \sqrt{R})$$

$$\Omega_2(\zeta, t) = \Omega_2(\zeta, \mathbf{0}), \qquad (|\zeta| < \sqrt{R})$$
(24)

The above expression indicates that it is simple to arrive at the solutions of $\Omega_1(\zeta, t)$ and $\Omega_2(\zeta, t)$ once their initial values are known. It is of interest to observe that the component function $\Omega_2(\zeta, t)$ is in fact *time-independent*. Due to the fact at time t = 0 the interface is perfect, we arrive at the following initial state of $\Omega(\zeta, t)$

$$\Omega(\zeta, 0) = \Phi^{-1} \mathbf{C}_1 \mathbf{f}_1(\zeta, 0) = 2 \operatorname{diag}[\frac{1}{\delta_1} \quad \frac{1}{\delta_2}] \Phi^{\mathsf{T}} \mathbf{f}_0(\zeta).$$
⁽²⁵⁾

During the above derivation we have utilized the first orthogonal relationship in Eq. (20) and the following expression for $\mathbf{f}_1(\zeta, 0)$

$$\mathbf{f}_{1}(\zeta, \mathbf{0}) = 2\mathbf{C}_{1}^{-1}\mathbf{H}^{-1}\mathbf{f}_{0}(\zeta).$$
(26)

Finally, we arrive at

$$\mathbf{f}_{1}(z,t) = 2\sqrt{\frac{2z}{\pi}} \mathbf{C}_{1}^{-1} [\mathbf{H}^{-1} \exp(-\lambda_{1}t) + \mathbf{M}[1 - \exp(-\lambda_{1}t)]] \mathbf{C}_{2}^{-1} \mathbf{K}, \quad (|z| < R)$$
(27)

$$\mathbf{f}_{2}(z,t) = \sqrt{\frac{2z}{\pi}} \mathbf{C}_{2}^{-1} \mathbf{K} - R \sqrt{\frac{2}{\pi z}} \mathbf{C}_{2}^{-1} [\mathbf{I} - 2[\mathbf{H}^{-1} \exp(-\lambda_{1}t) + \mathbf{M}[1 - \exp(-\lambda_{1}t)]] \mathbf{C}_{2}^{-1}] \mathbf{K}, \quad (|z| > R)$$
(28)

where

$$\mathbf{M} = -\frac{1}{H_{22}} \begin{bmatrix} 0 & 0\\ 0 & 1 \end{bmatrix}.$$
(29)

The electroelastic field in the inhomogeneity and matrix can be conveniently obtained by using the derived complex potentials $\mathbf{f}_1(z,t)$ and $\mathbf{f}_2(z,t)$. Due to the fact that the relaxation (or characteristic) time t_0 is the inverse of λ_1 , it is observed that the value of the relaxation time for the cracked composite system is twice as that for the same composite system without crack (Wang et al., 2008a). It should also be noted that in the above discussion we adopt a different method than that presented by Wang et al. (2008a). It is observed that the method presented here is mathematically more elegant than that in Wang et al. (2008a).

3.2. The electroelastic field

The displacement and electric potential within the inhomogeneity and matrix are

$$\begin{bmatrix} \mathbf{w}^{(1)} \\ \phi^{(1)} \end{bmatrix} = 2\sqrt{\frac{2r}{\pi}}\sin\frac{\theta}{2}\mathbf{C}_{1}^{-1}[\mathbf{H}^{-1}\exp(-\lambda_{1}t) + \mathbf{M}[1 - \exp(-\lambda_{1}t)]]\mathbf{C}_{2}^{-1}\mathbf{K}, (r < R)$$
(30a)

$$\begin{bmatrix} \mathbf{w}^{(2)} \\ \phi^{(2)} \end{bmatrix} = \sqrt{\frac{2r}{\pi}} \mathbf{C}_2^{-1} \mathbf{K} \sin \frac{\theta}{2} + \sqrt{\frac{2}{\pi r}} R \sin \frac{\theta}{2} \mathbf{C}_2^{-1} [\mathbf{I} - 2[\mathbf{H}^{-1} \exp(-\lambda_1 t) + \mathbf{M}[1 - \exp(-\lambda_1 t)]] \mathbf{C}_2^{-1}] \mathbf{K}, (r > R)$$
(30b)

The corresponding stresses and electric displacements are

$$\begin{bmatrix} \sigma_{zy}^{(1)} \\ D_{y}^{(1)} \end{bmatrix} = \sqrt{\frac{2}{\pi r}} \cos \frac{\theta}{2} [\mathbf{H}^{-1} \exp(-\lambda_{1}t) + \mathbf{M}[1 - \exp(-\lambda_{1}t)]] \mathbf{C}_{2}^{-1} \mathbf{K},$$

$$\begin{bmatrix} \sigma_{x}^{(1)} \\ D_{x}^{(1)} \end{bmatrix} = -\sqrt{\frac{2}{\pi r}} \sin \frac{\theta}{2} [\mathbf{H}^{-1} \exp(-\lambda_{1}t) + \mathbf{M}[1 - \exp(-\lambda_{1}t)]] \mathbf{C}_{2}^{-1} \mathbf{K},$$

$$\begin{bmatrix} \sigma_{zy}^{(2)} \\ D_{y}^{(2)} \end{bmatrix} = \frac{\mathbf{K}}{\sqrt{2\pi r}} \cos \frac{\theta}{2} + \frac{R}{\sqrt{2\pi r^{3}}} \cos \frac{3\theta}{2} [\mathbf{I} - 2[\mathbf{H}^{-1} \exp(-\lambda_{1}t) + \mathbf{M}[1 - \exp(-\lambda_{1}t)]] \mathbf{C}_{2}^{-1}] \mathbf{K},$$

$$\begin{bmatrix} \sigma_{zx}^{(2)} \\ D_{x}^{(2)} \end{bmatrix} = -\frac{\mathbf{K}}{\sqrt{2\pi r}} \sin \frac{\theta}{2} - \frac{R}{\sqrt{2\pi r^{3}}} \sin \frac{3\theta}{2} [\mathbf{I} - 2[\mathbf{H}^{-1} \exp(-\lambda_{1}t) + \mathbf{M}[1 - \exp(-\lambda_{1}t)]] \mathbf{C}_{2}^{-1}] \mathbf{K},$$

$$(7 > R)$$

$$(31b)$$

208

Jumps in the displacement and electric potential across the crack surfaces are

$$\begin{bmatrix} \Delta w^{(1)} \\ \Delta \phi^{(1)} \end{bmatrix} = 4\sqrt{\frac{2r}{\pi}} \mathbf{C}_{1}^{-1} [\mathbf{H}^{-1} \exp(-\lambda_{1}t) + \mathbf{M}[1 - \exp(-\lambda_{1}t)]] \mathbf{C}_{2}^{-1} \mathbf{K}, (-R \leq x \leq 0, \ y = 0)$$

$$\begin{bmatrix} \Delta w^{(2)} \\ \Delta \phi^{(2)} \end{bmatrix} = 2\sqrt{\frac{2r}{\pi}} \mathbf{C}_{2}^{-1} \mathbf{K} + 2\sqrt{\frac{2}{\pi r}} \mathbf{R} \mathbf{C}_{2}^{-1} [\mathbf{I} - 2[\mathbf{H}^{-1} \exp(-\lambda_{1}t) + \mathbf{M}[1 - \exp(-\lambda_{1}t)]] \mathbf{C}_{2}^{-1}] \mathbf{K}, \quad (x \leq -R, \ y = 0)$$

$$(32a)$$

where Δ means the value at the upper crack surface minuses that at the lower crack surface.

It is observed from Eq. (31) the stresses and electric displacements are regular at the point where the crack intersects the inhomogeneity–matrix interface. This observation is in agreement with the result of Ting (2005). It should be noted that the regular condition is only valid for a Mode-III crack discussed here, the stresses for a Mode-I crack are singular at the intersection point (Erdogan and Gupta, 1975; Wang and Ballarini, 2003). Here we would like to consider Eq. (32) in more detail. Figs. 3 and 4 illustrate the jumps in the displacement and electric potential across the crack surfaces at five different times $\tilde{t} = \lambda_1 t = 0, 0.5, 1.0, 2.0, \infty$ for a piezoelectric BaTiO₃ fiber reinforced in a piezoelectric PZT-5 matrix under the remote intensity factors $K^{\sigma} = 1N\sqrt{m}/m^2$ and $K^{D} = 0$. The pertinent material properties of BaTiO₃ are $c_{44} = 4.4 \times 10^{10}$ N/m², $e_{15} = 11.4$ C/m², $\epsilon_{11} = 9.8722 \times 10^{-9}$ C²/Nm², whilst those of PZT-5 are given by $c_{44} = 2.11 \times 10^{10}$ N/m², $e_{15} = 12.3$ C/m², $\epsilon_{11} = 8.1103 \times 10^{-9}$ C²/Nm². It is clearly observed from Figs. 3 and 4 that: (i) Δw is discontinuous at x = -R when t > 0, while $\Delta \phi$ is always continuous along the negative *x*-axis at any time; (ii) Δw at the inhomogeneity portion of the crack is positive when $\tilde{t} < 1.0$, whereas it is negative when $\tilde{t} > 1.0$; (iii) Δw at the matrix portion of the crack and $\Delta \phi$ at any position of the crack are always positive at any time.

• The displacement jump along the inhomogeneity–matrix interface *r* = *R* can be obtained as

$$w^{(2)} - w^{(1)} = 2\sin\frac{\theta}{2} \left[1 - \exp(-\lambda_1 t)\right] \sqrt{\frac{2R}{\pi}} \frac{(H_{22} \in {}^{(2)}_{11} + H_{12} e^{(2)}_{15})K^{\sigma} + (H_{22} e^{(2)}_{15} - H_{12} c^{(2)}_{44})K^{D}}{H_{22} \tilde{c}^{(2)}_{44} \in {}^{(2)}_{11}},$$
(33)

which indicates that at a certain fixed time the magnitude of the displacement jump across the viscous interface attains its maximum when the interface intersects the semi-infinite crack at $\theta = \pm \pi$. If the inhomogeneity and matrix have the same material property with the same poling direction, i.e., $c_{44}^{(1)} = c_{44}^{(2)} = c_{44}$, $e_{15}^{(1)} = e_{15}^{(2)} = e_{15}$, $\epsilon_{11}^{(1)} = \epsilon_{11}^{(2)} = \epsilon_{11}$, then Eq. (33) reduces to

$$w^{(2)} - w^{(1)} = \sqrt{\frac{2R}{\pi}} \frac{2K^{\sigma}}{c_{44}} \sin\frac{\theta}{2} [1 - \exp(-\lambda_1 t)], \tag{34}$$

where $\lambda_1 = c_{44}/(4\eta R)$. It is of interest to notice that in this case the displacement jump along the interface r = R is in fact independent of the piezoelectric and dielectric properties of the inhomogeneity or matrix, and is also independent of the nominal electric displacement intensity factor at infinity. On the other hand, if the inhomogeneity and matrix have the same material property but are poled in opposite directions, i.e., $c_{44}^{(1)} = c_{44}^{(2)} = c_{44}$, $e_{15}^{(2)} = -e_{15}^{(1)} = e_{15}^{(2)} = e_{11}$, then Eq. (33) reduces to



Fig. 3. Displacement jump across the crack surfaces at five different times $\tilde{t} = \lambda_1 t = 0, 0.5, 1.0, 2.0, \infty$ under the remote intensity factors $K^{\sigma} = 1 N \sqrt{m}/m^2$ and $K^{D} = 0$.

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X. Wang et al. / International Journal of Solids and Structures 46 (2009) 203-216



Fig. 4. Electric potential jump across the crack surfaces at five different times $\tilde{t} = \lambda_1 t = 0, 0.5, 1.0, 2.0, \infty$ under the remote intensity factors $K^{\sigma} = 1N\sqrt{m}/m^2$ and $K^{D} = 0$.

$$w^{(2)} - w^{(1)} = 2\sqrt{\frac{2R}{\pi}} \frac{\epsilon_{11}K^{\sigma} + e_{15}K^{D}}{\tilde{c}_{44}\epsilon_{11}} \sin\frac{\theta}{2} [1 - \exp(-\lambda_{1}t)],$$
(35)

where $\lambda_1 = \tilde{c}_{44}/(4\eta R)$.

• The stress and electric displacement intensity factors at the crack tip are

$$\mathbf{K}_{\text{tip}}(t) = \begin{bmatrix} K_{\text{tip}}^{\sigma}(t) \\ K_{\text{tip}}^{D}(t) \end{bmatrix} = \begin{bmatrix} \lim_{z \to 0} \left(\sqrt{2\pi |z|} \sigma_{zy} \right) \\ \lim_{z \to 0} \left(\sqrt{2\pi |z|} D_{y} \right) \end{bmatrix} = 2[\mathbf{H}^{-1} \exp(-\lambda_{1} t) + \mathbf{M}[1 - \exp(-\lambda_{1} t)]]\mathbf{C}_{2}^{-1}\mathbf{K},$$
(36)

or explicitly

$$K_{\rm tip}^{\sigma}(t) = \frac{2\exp(-\lambda_{1}t)}{\tilde{c}_{44}^{(2)} \in_{11}^{(2)} \left(H_{11}H_{22} + H_{12}^{2}\right)} \left[\left(H_{22} \in_{11}^{(2)} + H_{12}e_{15}^{(2)}\right) K^{\sigma} + \left(H_{22}e_{15}^{(2)} - H_{12}c_{44}^{(2)}\right) K^{\rm D} \right],$$

$$K_{\rm tip}^{\rm D}(t) = \frac{H_{12}}{H_{22}} K_{\rm tip}^{\sigma}(t) - \frac{2\left(e_{15}^{(2)}K^{\sigma} - c_{44}^{(2)}K^{\rm D}\right)}{\tilde{c}_{44}^{(2)} \in_{11}^{(2)}H_{22}}.$$
(37)

It is observed from the above expression that $K_{tip}^{\sigma}(\infty) = 0$ due to the fact that when $t \to \infty$ the viscous interface becomes freesliding and does not sustain shear loads.

When ignoring the piezoelectric effect by letting $e_{15}^{(1)} = e_{15}^{(2)} = 0$, then we obtain

$$\frac{K_{\rm tip}^{\sigma}(0)}{K^{\sigma}} = \frac{2c_{44}^{(1)}}{c_{44}^{(1)} + c_{44}^{(2)}},\tag{38}$$

which is the result obtained by Steif (1987).

If the inhomogeneity and matrix have the same material property with the same poling direction, i.e., $c_{44}^{(1)} = c_{44}^{(2)} = c_{44}$, $e_{15}^{(1)} = e_{15}^{(2)} = e_{15}$, $\epsilon_{11}^{(1)} = \epsilon_{11}^{(2)} = \epsilon_{11}$, then Eq. (37) reduces to

$$K_{\rm tip}^{\sigma}(t) = \exp(-\lambda_1 t)K^{\sigma}, \quad K_{\rm tip}^{\rm D}(t) = K^{\rm D} - \frac{e_{15}[1 - \exp(-\lambda_1 t)]K^{\sigma}}{c_{44}}, \tag{39}$$

which indicates that the stress intensity factor at the crack tip is independent of the piezoelectric and dielectric properties of the inhomogeneity or matrix, and is also independent of the nominal electric displacement intensity factor at infinity.

On the other hand, if the inhomogeneity and matrix have the same material property but are poled in opposite directions, i.e., $c_{44}^{(1)} = c_{44}^{(2)} = c_{44}$, $e_{15}^{(2)} = -e_{15}^{(1)} = e_{15}$, $\epsilon_{11}^{(1)} = \epsilon_{11}^{(2)} = \epsilon_{11}$, then Eq. (37) reduces to

$$K_{\rm tip}^{\sigma}(t) = \exp(-\lambda_1 t) \left(K^{\sigma} + \frac{e_{15}}{\epsilon_{11}} K^{\rm D} \right), \quad K_{\rm tip}^{\rm D}(t) = K^{\rm D} - \frac{e_{15}}{c_{44}} K^{\sigma}, \tag{40}$$

which indicates that the electric displacement intensity factor at the crack tip is in fact time-independent.

It should be pointed out that the method presented in this section can also be adopted conveniently to address more complex interaction problems such as a screw dislocation located in the matrix or in the inhomogeneity as discussed in the next two sections.

4. A piezoelectric screw dislocation in the matrix

In this section, we consider the loading case of a piezoelectric screw dislocation located in the unbounded matrix. The original boundary value problem in the ζ -plane shown in Fig. 2 can be equivalently considered as the original extended dislocation $\hat{\mathbf{b}} = \begin{bmatrix} b & \Delta \phi \end{bmatrix}^T$ and extended force $\hat{\mathbf{f}} = \begin{bmatrix} p & -q \end{bmatrix}^T$ located at $\zeta = \zeta_0$, and the image dislocation $-\hat{\mathbf{b}}$ and image force $\hat{\mathbf{f}}$ located at $\zeta = -\overline{\zeta}_0$ interacting with an *intact* circular inhomogeneity as shown in Fig. 5. Here, we ignore the intermediate steps, which are similar to but somewhat more complicated than those presented previously for the remote load case. The two analytic function vectors $-\mathbf{f}_1(\zeta, t)$ defined in the inhomogeneity and $\mathbf{f}_2(\zeta, t)$ defined in the matrix - due to the action of a piezoelectric screw dislocation located at $z = z_0$ (or $\zeta = \zeta_0$) in the matrix can be finally obtained as

$$\begin{aligned} \mathbf{f}_{1}(\zeta,t) &= \mathbf{C}_{1}^{-1} \left[\mathbf{H}^{-1} \ln[\zeta - \exp(\lambda_{1}t)\zeta_{0}] + \mathbf{M} \ln \frac{\zeta - \zeta_{0}}{\zeta - \exp(\lambda_{1}t)\zeta_{0}} \right] \frac{\mathbf{\dot{b}} - \mathbf{i}\mathbf{C}_{2}^{-1}\mathbf{f}}{\pi} \\ &- \mathbf{C}_{1}^{-1} [\mathbf{H}^{-1} \ln \left[\zeta + \exp(\lambda_{1}t)\overline{\zeta}_{0} \right] + \mathbf{M} \ln \frac{\zeta + \overline{\zeta}_{0}}{\zeta + \exp(\lambda_{1}t)\overline{\zeta}_{0}} \right] \frac{\mathbf{\dot{b}} + \mathbf{i}\mathbf{C}_{2}^{-1}\mathbf{\hat{f}}}{\pi}, \end{aligned}$$
(41)
$$\mathbf{f}_{2}(\zeta,t) &= \mathbf{C}_{2}^{-1} \left[\mathbf{H}^{-1} \ln \frac{\zeta - \exp(-\lambda_{1}t)R\overline{\zeta}_{0}^{-1}}{\zeta} + \mathbf{M} \ln \frac{\zeta - R\overline{\zeta}_{0}^{-1}}{\zeta - \exp(-\lambda_{1}t)R\overline{\zeta}_{0}^{-1}} \right] \frac{\mathbf{\dot{b}} + \mathbf{i}\mathbf{C}_{2}^{-1}\mathbf{\hat{f}}}{\pi} \\ &- \mathbf{C}_{2}^{-1} \left[\mathbf{H}^{-1} \ln \frac{\zeta + \exp(-\lambda_{1}t)R\overline{\zeta}_{0}^{-1}}{\zeta} + \mathbf{M} \ln \frac{\zeta + R\zeta_{0}^{-1}}{\zeta + \exp(-\lambda_{1}t)R\overline{\zeta}_{0}^{-1}} \right] \frac{\mathbf{\dot{b}} - \mathbf{i}\mathbf{C}_{2}^{-1}\mathbf{\hat{f}}}{\pi} \\ &+ \frac{\mathbf{\dot{b}} - \mathbf{i}\mathbf{C}_{2}^{-1}\mathbf{\hat{f}}}{2\pi} \ln(\zeta - \zeta_{0}) - \frac{\mathbf{\dot{b}} + \mathbf{i}\mathbf{C}_{2}^{-1}\mathbf{\hat{f}}}{2\pi} \ln(\zeta + \overline{\zeta}_{0}) - \frac{\mathbf{\dot{b}} + \mathbf{i}\mathbf{C}_{2}^{-1}\mathbf{\hat{f}}}{2\pi} \ln \frac{\zeta - R\overline{\zeta}_{0}^{-1}}{\zeta} + \frac{\mathbf{\dot{b}} - \mathbf{i}\mathbf{C}_{2}^{-1}\mathbf{\hat{f}}}{2\pi} \ln \frac{\zeta + R\zeta_{0}^{-1}}{\zeta}, \end{aligned}$$

where $\zeta_0 = \sqrt{z_0}, z_0 = r_0 \exp(i\theta_0)$.

Eq. (41) implies that the solution in the inhomogeneity in the ζ -plane can be considered as the superposition of the following two static and two moving singularities in a homogeneous infinite piezoelectric plane with material property C_1 : (i) an extended dislocation $2C_1^{-1}M\hat{b}$ and an extended force $2MC_2^{-1}\hat{f}$ located at the static singular point $\zeta = \zeta_0$; (ii) an extended dislocation $2C_1^{-1}M\hat{b}$ and an extended force $2MC_2^{-1}\hat{f}$ located at the static singular point $\zeta = -\overline{\zeta}_0$; (iii) an extended dislocation $2C_1^{-1}(H^{-1} - M)\hat{b}$ and an extended force $2(H^{-1} - M)C_2^{-1}\hat{f}$ located at the moving singular point $\zeta = \exp(\lambda_1 t)\zeta_0$; (iv) an extended dislocation $-2C_1^{-1}(H^{-1} - M)\hat{b}$ and an extended force $2(H^{-1} - M)C_2^{-1}\hat{f}$ located at the moving singular point $\zeta = -\exp(\lambda_1 t)\zeta_0$; (iv) an extended dislocation $-2C_1^{-1}(H^{-1} - M)\hat{b}$ and an extended force $2(H^{-1} - M)C_2^{-1}\hat{f}$ located at the moving singular point $\zeta = -\exp(\lambda_1 t)\zeta_0$. Thus the sum of the extended forces applied at the above four singularities is $4H^{-1}C_2^{-1}\hat{f}$ while the sum of the extended dislocation locations vanishes.



Fig. 5. Illustration of the image singularity.

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X. Wang et al./International Journal of Solids and Structures 46 (2009) 203-216

Similarly, Eq. (42) implies that the solution in the matrix in the ζ -plane can be considered as the superposition of the following five static and two moving singularities in a homogeneous infinite piezoelectric plane with material property C_2 : (i) an extended dislocation $\hat{\mathbf{b}}$ and an extended force $\hat{\mathbf{f}}$ located at the original static singular point $\zeta = \zeta_0$; (ii) an extended dislocation $(2C_2^{-1}\mathbf{M} - \mathbf{I})\hat{\mathbf{b}}$ and an extended force $\hat{\mathbf{f}}$ located at the static singular point $\zeta = -\overline{\zeta}_0$; (iii) an extended dislocation $(\mathbf{I} - 2\mathbf{M}\mathbf{C}_2^{-1})\hat{\mathbf{f}}$ located at the static singular point $\zeta = -\overline{\zeta}_0^{-1}$; (iv) an extended dislocation $(\mathbf{I} - 2\mathbf{C}_2^{-1}\mathbf{M})\hat{\mathbf{b}}$ and an extended force $(\mathbf{I} - 2\mathbf{M}\mathbf{C}_2^{-1})\hat{\mathbf{f}}$ located at the static singular point $\zeta = -R\zeta_0^{-1}$; (v) an extended dislocation $(\mathbf{I} - 2\mathbf{C}_2^{-1}\mathbf{M})\hat{\mathbf{b}}$ and an extended force $(\mathbf{I} - 2\mathbf{M}\mathbf{C}_2^{-1})\hat{\mathbf{f}}$ located at the static singular point $\zeta = -R\zeta_0^{-1}$; (v) an extended force $2(2\mathbf{H}^{-1}\mathbf{C}_2^{-1} - \mathbf{I})\hat{\mathbf{f}}$ located at the static singular point $\zeta = -R\zeta_0^{-1}$; (v) an extended force $-2(\mathbf{H}^{-1} - \mathbf{M})\mathbf{C}_2^{-1}\hat{\mathbf{f}}$ located at the moving singular point $\zeta = \exp(-\lambda_1 t)R\zeta_0^{-1}$; (vi) an extended dislocation $-2\mathbf{C}_2^{-1}(\mathbf{H}^{-1} - \mathbf{M})\hat{\mathbf{b}}$ and an extended force $-2(\mathbf{H}^{-1} - \mathbf{M})\mathbf{C}_2^{-1}\hat{\mathbf{f}}$ located at the moving singular point $\zeta = -\exp(-\lambda_1 t)R\zeta_0^{-1}$. Thus the sum of the extended forces applied at the above seven singularities is $2\hat{\mathbf{f}}$ while the sum of the extended dislocations vanishes.

Once we have obtained the complex potentials, the stress and electric displacement intensity factors at the crack tip can be easily found as

$$\mathbf{K}_{\text{tip}}(t) = \begin{bmatrix} K_{\text{tip}}^{\sigma}(t) \\ K_{\text{tip}}^{\text{D}}(t) \end{bmatrix} = -\sqrt{\frac{2}{\pi r_0}} [\mathbf{H}^{-1} \exp(-\lambda_1 t) + \mathbf{M}[1 - \exp(-\lambda_1 t)]] \left(\hat{\mathbf{b}} \cos\frac{\theta_0}{2} - \mathbf{C}_2^{-1} \hat{\mathbf{f}} \sin\frac{\theta_0}{2} \right), \tag{43}$$

In addition, the image force acting on the screw dislocation can be obtained by employing the Peach–Koehler formula (Ohr et al., 1985; Lee et al., 2000) and the obtained complex potentials above as

$$F_{\mathbf{x}} - \mathbf{i}F_{\mathbf{y}} = (\hat{\mathbf{b}}^{\mathrm{T}}\mathbf{C}_{2} - \mathbf{i}\hat{\mathbf{f}}^{\mathrm{T}})\mathbf{N},\tag{44}$$

where F_x and F_y are, respectively, the x- and y-components of the image force, and N is given by

$$\mathbf{N} = \mathbf{C}_{2}^{-1} \left[\mathbf{H}^{-1} \frac{\exp(-\lambda_{1}t)R}{z_{0}[r_{0} - \exp(-\lambda_{1}t)R]} + \mathbf{M} \frac{Rr_{0}[1 - \exp(-\lambda_{1}t)]}{z_{0}(r_{0} - R)[r_{0} - \exp(-\lambda_{1}t)R]} \right] \frac{\mathbf{b} + i\mathbf{C}_{2}^{-1}\mathbf{f}}{2\pi} + \mathbf{C}_{2}^{-1} \left[\mathbf{H}^{-1} \frac{\exp(-\lambda_{1}t)R}{z_{0}[z_{0} + \exp(-\lambda_{1}t)R]} + \mathbf{M} \frac{R[1 - \exp(-\lambda_{1}t)R]}{(z_{0} + R)[z_{0} + \exp(-\lambda_{1}t)R]} \right] \frac{\mathbf{b} - i\mathbf{C}_{2}^{-1}\mathbf{f}}{2\pi} - \frac{\mathbf{b} - i\mathbf{C}_{2}^{-1}\mathbf{f}}{8\pi} \frac{z_{0} + 3R}{z_{0}(z_{0} + R)} - \frac{\mathbf{b} + i\mathbf{C}_{2}^{-1}\mathbf{f}}{4\pi} \frac{r_{0}(z_{0} + R)}{z_{0}(z_{0} + r_{0})(r_{0} - R)}.$$
(45)

Particularly, when the screw dislocation is located on the positive *x*-axis in the matrix and $\hat{\mathbf{f}} = \mathbf{0}$, the image force on the screw dislocation is reduced to

$$F_{x} = \frac{\mathbf{\hat{b}^{T}H^{-1}\hat{b}}}{\pi} \frac{R\exp(-\lambda_{1}t)}{r_{0}^{2} - \exp(-2\lambda_{1}t)R^{2}} - \frac{\Delta\phi^{2}}{\pi H_{22}} \frac{R[1 - \exp(-\lambda_{1}t)][r_{0}^{2} + \exp(-\lambda_{1}t)R^{2}]}{(r_{0}^{2} - R^{2})[r_{0}^{2} - \exp(-2\lambda_{1}t)R^{2}]} - \frac{\mathbf{\hat{b}^{T}C_{2}\hat{b}}}{4\pi} \frac{r_{0}^{2} + 2Rr_{0} - R^{2}}{r_{0}(r_{0}^{2} - R^{2})},$$

$$F_{y} = 0. \quad (r_{0} \ge R)$$

$$(46)$$

It can be shown that if the inhomogeneity and matrix have the same material property with the same poling direction, at the initial moment *t*=0 the intensity factors at the crack tip and the image force on the dislocation will reduce to those derived by Lee et al. (2000), [Eqs. (34), (35), (39), (40), (41) with $K^{\sigma} = K^{D} = 0$].

5. A piezoelectric screw dislocation in the inhomogeneity

In this section, we consider the loading case of a piezoelectric screw dislocation located in the inhomogeneity. Here, we assume that p = q = 0 (or equivalently $\hat{\mathbf{f}} = \mathbf{0}$). The two analytic function vectors $-\mathbf{f}_1(\zeta, t)$ defined in the inhomogeneity and $\mathbf{f}_2(\zeta, t)$ defined in the matrix - due to a screw dislocation located at $z = z_0$ (or $\zeta = \zeta_0$) in the inhomogeneity can be finally obtained as

$$\mathbf{f}_{1}(\zeta,t) = \frac{\mathbf{C}_{1}^{-1}}{\pi} \left[\mathbf{H}^{-1} \ln \frac{\zeta - \exp(\lambda_{1}t) R \bar{\zeta}_{0}^{-1}}{\zeta + \exp(\lambda_{1}t) R \zeta_{0}^{-1}} + \mathbf{M} \ln \frac{(\zeta - R \bar{\zeta}_{0}^{-1}) [\zeta + \exp(\lambda_{1}t) R \zeta_{0}^{-1}]}{(\zeta + R \zeta_{0}^{-1}) [\zeta - \exp(\lambda_{1}t) R \bar{\zeta}_{0}^{-1}]} \right] \hat{\mathbf{b}}$$

$$+ \frac{\hat{\mathbf{b}}}{2\pi} \ln \frac{\zeta - \zeta_{0}}{\zeta + \bar{\zeta}_{0}} + \frac{\hat{\mathbf{b}}}{2\pi} \ln \frac{\zeta + R \zeta_{0}^{-1}}{\zeta - R \bar{\zeta}_{0}^{-1}},$$
(47)

$$\mathbf{f}_{2}(\zeta,t) = \frac{\mathbf{C}_{2}^{-1}}{\pi} \left[\mathbf{H}^{-1} \ln \frac{\zeta - \exp(-\lambda_{1}t)\zeta_{0}}{\zeta + \exp(-\lambda_{1}t)\overline{\zeta}_{0}} + \mathbf{M} \ln \frac{(\zeta - \zeta_{0})[\zeta + \exp(-\lambda_{1}t)\overline{\zeta}_{0}]}{(\zeta + \overline{\zeta}_{0})[\zeta - \exp(-\lambda_{1}t)\zeta_{0}]} \right] \hat{\mathbf{b}}.$$
(48)

Eq. (47) indicates that the solution in the inhomogeneity in the ζ -plane can be considered as the superposition of the following four static and two moving singularities in a homogeneous infinite piezoelectric plane with material property **C**₁: (i) an extended dislocation $\hat{\mathbf{b}}$ located at the original static singular point $\zeta = \zeta_0$; (ii) an extended dislocation $-\hat{\mathbf{b}}$ located at the static singular point $\zeta = -\bar{\zeta}_0$; (iii) an extended dislocation $(\mathbf{I} - 2\mathbf{C}_1^{-1}\mathbf{M})\hat{\mathbf{b}}$ located at the static singular point $\zeta = -R\zeta_0^{-1}$; (iv) an extended dislocation $(2\mathbf{C}_1^{-1}\mathbf{M} - \mathbf{I})\hat{\mathbf{b}}$ located at the static singular point $\zeta = R\bar{\zeta}_0^{-1}$; (v) an extended dislocation $(2\mathbf{C}_1^{-1}\mathbf{M} - \mathbf{I})\hat{\mathbf{b}}$ located at the static singular point $\zeta = R\bar{\zeta}_0^{-1}$; (v) an extended dislocation $(2\mathbf{C}_1^{-1}\mathbf{M} - \mathbf{I})\hat{\mathbf{b}}$ located at the static singular point $\zeta = R\bar{\zeta}_0^{-1}$; (v) an extended dislocation $(2\mathbf{C}_1^{-1}\mathbf{M} - \mathbf{I})\hat{\mathbf{b}}$ located at the static singular point $\zeta = -R\zeta_0^{-1}$; (iv) an extended dislocation $(2\mathbf{C}_1^{-1}\mathbf{M} - \mathbf{I})\hat{\mathbf{b}}$ located at the static singular point $\zeta = R\bar{\zeta}_0^{-1}$; (v) an extended dislocation $(2\mathbf{C}_1^{-1}\mathbf{M} - \mathbf{I})\hat{\mathbf{b}}$ located at the moving singular point $\zeta = \exp(\lambda_1 t)R\bar{\zeta}_0^{-1}$; (vi) an extended dislocation $-2\mathbf{C}_1^{-1}(\mathbf{H}^{-1} - \mathbf{M})\hat{\mathbf{b}}$ located at the moving singular point $\zeta = -\exp(\lambda_1 t)R\bar{\zeta}_0^{-1}$. Thus the sum of the extended dislocations applied at the above six singularities vanishes.

Similarly, Eq. (48) indicates that the solution in the matrix in the ζ -plane can be considered as the superposition of the following two static and two moving singularities in a homogeneous infinite piezoelectric plane with material property **C**₂: (i) an extended dislocation $2\mathbf{C}_2^{-1}\mathbf{M}\hat{\mathbf{b}}$ located at the static singular point $\zeta = \zeta_0$; (ii) an extended dislocation $-2\mathbf{C}_2^{-1}\mathbf{M}\hat{\mathbf{b}}$ located at the static singular point $\zeta = -\zeta_0$; (iii) an extended dislocation $2\mathbf{C}_2^{-1}(\mathbf{H}^{-1} - \mathbf{M})\hat{\mathbf{b}}$ located at the moving singular point $\zeta = \exp(-\lambda_1 t)\zeta_0$; (iv) an extended dislocation $-2\mathbf{C}_2^{-1}(\mathbf{H}^{-1} - \mathbf{M})\hat{\mathbf{b}}$ located at the moving singular point $\zeta = -\exp(-\lambda_1 t)\zeta_0$. Thus, the sum of the extended dislocation applied at the above four singularities vanishes. It is also observed that the solution structure for a piezoelectric screw dislocation inside the inhomogeneity [see Eqs. (47) and (48)] is different than that for a piezoelectric screw dislocation in the matrix [see Eqs. (41) and (42)]. This is true even when the semi-infinite crack is absent and when the inhomogeneity-matrix interface is perfect (Deng and Meguid, 1999).

The stress and electric displacement intensity factors at the crack tip can be easily obtained as

$$\mathbf{K}_{\text{tip}}(t) = \begin{bmatrix} K_{\text{tip}}^{\sigma}(t) \\ K_{\text{tip}}^{\text{D}}(t) \end{bmatrix} = -\left\{ \sqrt{\frac{2r_0}{\pi R^2}} \Big[\mathbf{H}^{-1} \exp(-\lambda_1 t) + \mathbf{M}[1 - \exp(-\lambda_1 t)] - \frac{\mathbf{C}_1}{2} \Big] + \frac{\mathbf{C}_1}{\sqrt{2\pi r_0}} \right\} \hat{\mathbf{b}} \cos\frac{\theta_0}{2}.$$
(49)

In addition, the image force on the screw dislocation can be obtained by employing the Peach–Koehler formula and the obtained complex potentials above as

$$F_x - \mathbf{i}F_y = \hat{\mathbf{b}}^{\mathrm{T}}\mathbf{C}_1\mathbf{P},\tag{50}$$

where **P** is given by

$$\mathbf{P} = R \frac{z_0 + r_0}{z_0} \frac{\mathbf{C}_1^{-1}}{2\pi} \left[\frac{(\mathbf{H}^{-1} - \mathbf{M}) \exp(\lambda_1 t)}{[r_0 - \exp(\lambda_1 t)R][z_0 + \exp(\lambda_1 t)R]} + \frac{\mathbf{M}}{(r_0 - R)(z_0 + R)} \right] \hat{\mathbf{b}} - \frac{\hat{\mathbf{b}}}{8\pi} \frac{3z_0 + r_0}{z_0(z_0 + r_0)} - \frac{\hat{\mathbf{b}}}{4\pi} \frac{R(z_0 + r_0)}{z_0(r_0 - R)(z_0 + R)}.$$
 (51)

Particularly, when the screw dislocation is located on the positive *x*-axis in the inhomogeneity, the image force on the screw dislocation is given by

$$F_{x} = \frac{\hat{\mathbf{b}}^{\mathrm{T}}\mathbf{H}^{-1}\hat{\mathbf{b}}}{\pi} \frac{R\exp(\lambda_{1}t)}{r_{0}^{2} - \exp(2\lambda_{1}t)R^{2}} - \frac{\Delta\phi^{2}}{\pi H_{22}} \frac{R[1 - \exp(\lambda_{1}t)][r_{0}^{2} + \exp(\lambda_{1}t)R^{2}]}{(r_{0}^{2} - R^{2})[r_{0}^{2} - \exp(2\lambda_{1}t)R^{2}]} - \frac{\hat{\mathbf{b}}^{\mathrm{T}}\mathbf{C}_{1}\hat{\mathbf{b}}}{4\pi} \frac{r_{0}^{2} + 2Rr_{0} - R^{2}}{r_{0}(r_{0}^{2} - R^{2})},$$

$$F_{y} = \mathbf{0}, \quad (\mathbf{0} \le r_{0} \le R)$$
(52)

We observe that it is enough to replace $-\lambda_1$ with λ_1 in Eq. (46) to arrive at Eq. (52).

6. Extension to a wedge crack

In this section, we will look into the more general scenario where a semi-infinite insulating wedge crack of angle α ($0 \le \alpha \le \pi$) is located in the composite system as shown in Fig. 6. When $\alpha = 0$ the wedge crack will reduce to a semi-infinite slit crack studied previously; When $\alpha = \pi$ the wedge crack becomes a straight boundary x = 0. It should be noticed that the wedge crack problem also has some practical implications (Ohr et al., 1985).

The original boundary value problem can be more conveniently discussed by introducing the following conformal mapping function (Ohr et al., 1985)

$$z = m(\zeta) = \zeta^{1/q},\tag{53}$$

where $q = \frac{\pi}{2\pi-\alpha}$. The above mapping function can map the wedge cracked *z*-plane onto the right half-plane in the ζ -plane (Re{ ζ } \geq 0), as shown in Fig. 7. More specifically the wedge cracked circular inhomogeneity is mapped onto the half-circular region $|\zeta| < R^q$ and Re{ ζ } \geq 0 in the ζ -plane; the wedge cracked matrix is mapped onto $|\zeta| > R^q$ and Re{ ζ } \geq 0 in the ζ -plane; the inhomogeneity-matrix interface |z| = R is mapped onto the half- circle $|\zeta| = R^q$ and Re{ ζ } \geq 0 in the ζ -plane.

We first consider the following remote loading

$$\mathbf{f}_2(z,t) \to \frac{z^q}{q(2\pi)^{1-q}} \mathbf{C}_2^{-1} \mathbf{K}, \text{ as } z \to \infty$$
(54)

where $\mathbf{K} = \begin{bmatrix} K^{\sigma} & K^{D} \end{bmatrix}^{T}$. Remember that the remote stress and electric displacement intensity factors K^{σ} and K^{D} are now defined for a wedge crack of angle α (Ohr et al., 1985). At infinity the stresses and electric displacements behave as follows:



Fig. 6. Schematic of a semi-infinite wedge crack of angle α penetrating a piezoelectric circular inhomogeneity bonded to the surrounding piezoelectric matrix through a linear viscous interface.

The boundary conditions on the viscous interface $|\zeta| = R^q$ can be concisely expressed in terms of $\mathbf{f}_1(\zeta, t)$ defined in the inhomogeneity and $\mathbf{f}_2(\zeta, t)$ defined in the matrix as

$$\mathbf{C}_{1}\mathbf{f}_{1}^{+}(\zeta,t) + \mathbf{C}_{1}\bar{\mathbf{f}}_{1}^{-}\left(\frac{R^{2q}}{\zeta},t\right) = \mathbf{C}_{2}\mathbf{f}_{2}^{-}(\zeta,t) + \mathbf{C}_{2}\bar{\mathbf{f}}_{2}^{+}\left(\frac{R^{2q}}{\zeta},t\right),$$

$$\dot{\mathbf{f}}_{2}^{-}(\zeta,t) - \dot{\bar{\mathbf{f}}}_{2}^{+}\left(\frac{R^{2q}}{\zeta},t\right) - \dot{\mathbf{f}}_{1}^{+}(\zeta,t) + \dot{\bar{\mathbf{f}}}_{1}^{-}\left(\frac{R^{2q}}{\zeta},t\right) = \mathbf{\Lambda}\mathbf{C}_{1}\left[\zeta\mathbf{f}_{1}^{\prime+}(\zeta,t) - \frac{R^{2q}}{\zeta}\bar{\mathbf{f}}_{1}^{\prime-}\left(\frac{R^{2q}}{\zeta},t\right)\right], (|\zeta| = R^{q})$$
(56)

where

$$\Lambda = \frac{q}{\eta R} \operatorname{diag}[1 \quad 0].$$
(57)

By ignoring the intermediate steps, the two analytic function vectors – $\mathbf{f}_1(z,t)$ defined in the inhomogeneity and $\mathbf{f}_2(z,t)$ defined in the matrix – can be finally obtained as

$$\mathbf{f}_{1}(z,t) = \frac{2z^{q}}{q(2\pi)^{1-q}} \mathbf{C}_{1}^{-1} [\mathbf{H}^{-1} \exp(-\lambda_{1}t) + \mathbf{M}[1 - \exp(-\lambda_{1}t)]] \mathbf{C}_{2}^{-1} \mathbf{K}, \quad (\mid z \mid < R)$$
(58)

$$\mathbf{f}_{2}(z,t) = \frac{z^{q} \mathbf{C}_{2}^{-1} \mathbf{K}}{q(2\pi)^{1-q}} - \frac{R^{2q} z^{-q}}{q(2\pi)^{1-q}} \mathbf{C}_{2}^{-1} [\mathbf{I} - 2[\mathbf{H}^{-1} \exp(-\lambda_{1} t) + \mathbf{M}[1 - \exp(-\lambda_{1} t)]] \mathbf{C}_{2}^{-1}] \mathbf{K}, \quad (|z| > R)$$
(59)

where **H** and **M** are the same as those defined in previous sections, whereas λ_1 is different and is defined as follows:

$$\lambda_1 = \frac{qH_{22}}{\eta R(H_{11}H_{22} + H_{12}^2)} > 0.$$
(60)



Fig. 7. The mapped ζ -plane by using $z = \zeta^{1/q}$ for the semi-infinite wedge crack problem in *z*-plane.

It is observed that the value of the relaxation time for the wedge cracked composite system is 1/q times than that for the same composite system without wedge crack. The stress and electric displacement intensity factors at the tip of the wedge crack is

$$\mathbf{K}_{\text{tip}}(t) = \begin{bmatrix} K_{\text{tip}}^{o}(t) \\ K_{\text{tip}}^{D}(t) \end{bmatrix} = 2[\mathbf{H}^{-1}\exp(-\lambda_{1}t) + \mathbf{M}[1 - \exp(-\lambda_{1}t)]]\mathbf{C}_{2}^{-1}\mathbf{K}.$$
(61)

It is of interest to observe that the above expression is very similar to Eq. (36) for a semi-infinite crack ($\alpha = 0$) except that now λ_1 is defined by Eq. (60). The intensity factors for a wedge crack decay faster than those for a slit crack due to the fact that the relaxation time for a wedge crack is smaller than that for a slit crack. The displacement jump along the inhomogeneity-matrix interface r = R can be obtained as

$$w^{(2)} - w^{(1)} = \sin(q\theta) [1 - \exp(-\lambda_1 t)] \frac{2R^q}{q(2\pi)^{1-q}} \frac{(H_{22} \in_{11}^{(2)} + H_{12} e_{15}^{(2)})K^{\sigma} + (H_{22} e_{15}^{(2)} - H_{12} C_{44}^{(2)})K^{\mathsf{D}}}{H_{22} \tilde{c}_{44}^{(2)} \in_{11}^{(2)}},$$
(62)

which indicates that at a certain fixed time the magnitude of the displacement jump across the viscous interface attains its maximum when the interface intersects the semi-infinite wedge crack at $\theta = \pm (\pi - \alpha/2)$.

When a screw dislocation is located in the matrix or in the inhomogeneity, it is enough to replace *R* by R^{2q} in Eqs. (41), (42), (47) and (48) to arrive at $\mathbf{f}_1(\zeta, t)$ and $\mathbf{f}_2(\zeta, t)$ for the wedge crack problem while keeping in mind that now $\zeta_0 = z_0^q$ and λ_1 is defined by Eq. (60). It is not difficult to derive the stress and electric displacement intensity factors at the tip of the wedge crack and the image force acting on the screw dislocation.

7. Conclusions

A theoretical analysis was carried out for a semi-infinite crack (or wedge crack of angle α) half-way penetrating a piezoelectric circular inhomogeneity with a viscous interface in the presence of a screw dislocation either in the inhomogeneity or in the surrounding matrix. The characteristic time for the composite system in the presence of a semi-infinite crack is twice as that for the same composite system without crack. In the more general scenario, the characteristic time for the composite system in the presence of a semi-infinite wedge crack of angle α ($0 \le \alpha \le \pi$) is $(2 - \alpha/\pi)$ times than that for the same composite system without wedge crack. The generality of the present model lies in that some previously proposed models (Majumdar and Burns, 1981; Ohr et al., 1985; Steif, 1987; Lee et al., 2000; Chen et al., 2002a) can be considered as special cases of the present model. In this investigation we only addressed the so-called insulating crack or wedge crack. In fact the

problems of a conducting crack (Wang et al., 2003) and a conducting rigid line (anti-crack) (Chen et al., 2002b) can also be discussed similarly. When we discuss crack-tip shielding due to microcracking, the simple inhomogeneity model adopted here is not enough. A more realistic model would have the electromechanical moduli varying smoothly with distance from the crack tip until the uncracked moduli are reached (Steif, 1987). In this case an interesting problem to be solved is how the characteristic time for the composite system is influenced by the radially varying moduli within the circular inhomogeneity.

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