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# Rayleigh waves in magneto-electro-elastic half planes 

Received: 23 August 2007 / Revised: 17 February 2008 / Published online: 11 June 2008
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#### Abstract

This paper investigates Rayleigh waves in magneto-electro-elastic half planes. The magneto-electro-elastic materials are assumed to possess hexagonal ( 6 mm ) symmetry. Sixteen sets of boundary conditions are considered and the corresponding frequency equations are derived. It is found that for any of the 16 sets of boundary conditions, the Rayleigh waves, if exist, are always non-dispersive. Numerical results show that both the material coefficients and boundary conditions can significantly influence the Rayleigh wave properties in magneto-electro-elastic half planes.


## 1 Introduction

In recent years, piezoelectric and piezomagnetic materials have been increasingly applied to different engineering structures, especially to smart or intelligent systems as intelligent sensors, damage detectors, etc. Composite materials made of piezoelectric and piezomagnetic phases exhibit a magnetoelectric effect that is absent in either constituent. The magnetoelectric effect of piezoelectric-piezomagnetic composites was first reported in 1972 [1]. Since then, numerous investigations have been devoted to the static deformation, free vibration and fracture behavior of magneto-electro-elastic materials. As for the wave propagation problem, although numerous achievements have been made for piezoelectric ceramics [2-16], research so far on the wave propagation in magneto-electro-elastic materials is still very limited. To the best of the authors' knowledge, Liu et al. [17] studied the electromagnetic wave propagation in piezoelectric-piezomagnetic multilayer with simultaneously negative permeability and permittivity. Feng et al. $[18,19]$ evaluated the scattering properties of arc-shaped interface cracks between piezoelectric-piezomagnetic bimaterials. Soh and Liu [20] analyzed interfacial shear waves in a piezoelectric-piezomagnetic bi-material. Wang et al. [21] investigated surface shear wave in transversely isotropic magneto-electro-elastic materials. Chen et al. [22] presented dispersion curves, natural frequencies, and modal shapes in magneto-electro-elastic multilayered plates by using the propagator matrix and state-vector approaches. Chen and Shen [23] studied the axial shear magneto-electro-elastic waves in piezoelectric-piezomagnetic composites with randomly distributed cylindrical inhomogeneities. Melkumyan [24] discussed 12 surface shear waves guided by clamped/free boundaries in magneto-electro-elastic materials. The propagation properties of Love wave in layered magneto-electro-elastic structures with initial stress were recently discussed by Du et al. [25]. However, the majority of all studies were concentrated on anti-plane problems. Up to now, we have not seen any report on two-dimensional in-plane surface wave study in magneto-electro-elastic materials.

[^0]In this paper, the propagation properties of the Rayleigh wave in transversely isotropic and two-dimensional magneto-electro-elastic half planes are investigated. The boundary conditions at the surface of the magneto-electro-elastic half plane are written by a simple vector equation. By assuming the possible form of the solution, the generalized frequency equations are derived. Numerical examples are also presented to show the influence of the boundary conditions and material properties of the half planes on the Rayleigh wave velocities. It is expected that these results could have potential applications in surface acoustic wave devices made of magneto-electro-elastic materials.

## 2 General equations

Consider a transversely isotropic magneto-electro-elastic half plane with its poling direction along the $y$-axis (Fig. 1). The constitutive equations within the framework of the theory of linear magneto-electro-elastic medium take the form

$$
\begin{align*}
& \left\{\begin{array}{l}
\sigma_{x x} \\
\sigma_{y y} \\
\sigma_{x y}
\end{array}\right\}=\left[\begin{array}{ccc}
c_{11} & c_{13} & 0 \\
c_{13} & c_{33} & 0 \\
0 & 0 & c_{44}
\end{array}\right]\left\{\begin{array}{c}
u_{, x} \\
v_{, y} \\
u_{, y}+v_{, x}
\end{array}\right\}+\left[\begin{array}{cc}
0 & e_{31} \\
0 & e_{33} \\
e_{15} & 0
\end{array}\right]\left\{\begin{array}{l}
\phi_{, x} \\
\phi_{, y}
\end{array}\right\}+\left[\begin{array}{cc}
0 & f_{31} \\
0 & f_{33} \\
f_{15} & 0
\end{array}\right]\left\{\begin{array}{l}
\psi_{, x} \\
\psi_{, y}
\end{array}\right\},  \tag{1a}\\
& \left\{\begin{array}{l}
D_{x} \\
D_{y}
\end{array}\right\}=\left[\begin{array}{ccc}
0 & 0 & e_{15} \\
e_{31} & e_{33} & 0
\end{array}\right]\left\{\begin{array}{c}
u_{, x} \\
v_{, y} \\
u_{, y}+v_{, x}
\end{array}\right\}-\left[\begin{array}{cc}
\varepsilon_{11} & 0 \\
0 & \varepsilon_{33}
\end{array}\right]\left\{\begin{array}{c}
\phi_{, x} \\
\phi_{, y}
\end{array}\right\}-\left[\begin{array}{cc}
g_{11} & 0 \\
0 & g_{33}
\end{array}\right]\left\{\begin{array}{l}
\psi_{, x} \\
\psi, y
\end{array}\right\},  \tag{1b}\\
& \left\{\begin{array}{l}
B_{x} \\
B_{y}
\end{array}\right\}=\left[\begin{array}{ccc}
0 & 0 & f_{15} \\
f_{31} & f_{33} & 0
\end{array}\right]\left\{\begin{array}{c}
u_{, x} \\
v_{, y} \\
u_{, y}+v_{, x}
\end{array}\right\}-\left[\begin{array}{cc}
g_{11} & 0 \\
0 & g_{33}
\end{array}\right]\left\{\begin{array}{l}
\phi_{, x} \\
\phi_{, y}
\end{array}\right\}-\left[\begin{array}{cc}
\mu_{11} & 0 \\
0 & \mu_{33}
\end{array}\right]\left\{\begin{array}{l}
\psi_{, x} \\
\psi_{, y}
\end{array}\right\}, \tag{1c}
\end{align*}
$$

where $u$ and $v$ are the elastic displacement components, respectively; $\phi$ and $\psi$ are the electric and magnetic potentials, respectively; $D_{i}$ and $B_{i}(i=x, y)$ are the electric displacement and magnetic induction, respectively; $c_{i j}, e_{i j}, f_{i j}$ and $g_{i j}(i, j=x, y)$ are elastic, piezoelectric, piezomagnetic and magnetoelectric coefficients, respectively; $\varepsilon_{i j}$ and $\mu_{i j}(i, j=x, y)$ are dielectric permittivities and magnetic permeabilities, respectively.

In the absence of body forces, electric charges and magnetic charges, the governing equations for the elastic displacements $u$ and $v$, electric potential $\phi$, and magnetic potential $\psi$ can be written as follows:

$$
\begin{array}{r}
c_{11} u_{, x x}+c_{44} u_{, y y}+\left(c_{13}+c_{44}\right) v_{, x y}+\left(e_{31}+e_{15}\right) \phi_{, x y}+\left(f_{31}+f_{15}\right) \psi_{, x y}=\rho u_{, t t}, \\
\left(c_{13}+c_{44}\right) u_{, x y}+c_{44} v_{, x x}+c_{33} v_{, y y}+e_{15} \phi_{, x x}+e_{33} \phi_{, y y}+f_{15} \psi_{, x x}+f_{33} \psi_{, y y}=\rho v_{, t t}, \\
\left(e_{31}+e_{15}\right) u_{, x y}+e_{15} v_{, x x}+e_{33} v_{, y y}-\varepsilon_{11} \phi_{, x x}-\varepsilon_{33} \phi_{, y y}-g_{11} \psi_{, x x}-g_{33} \psi_{, y y}=0 \\
\left(f_{31}+f_{15}\right) u_{, x y}+f_{15} v_{, x x}+f_{33} v_{, y y}-g_{11} \phi_{, x x}-g_{33} \phi_{, y y}-\mu_{11} \psi_{, x x}-\mu_{33} \psi_{, y y}=0, \tag{2d}
\end{array}
$$

where $\rho$ is the material density.


Fig. 1 A transversely isotropic magneto-electro-elastic half plane with its poling direction along the $y$-axis and its surface at $y=0$

Following Pan [26], we write the general boundary conditions at the surface of the magneto-electro-elastic half plane by a simple vector equation as follows:

$$
\begin{equation*}
\mathbf{I}_{u} \mathbf{u}+\mathbf{I}_{\tau} \tau=\mathbf{0} \tag{3}
\end{equation*}
$$

where $\mathbf{I}_{u}$ and $\mathbf{I}_{\tau}$ are $4 \times 4$ diagonal matrices whose four diagonal elements are either one or zero, and satisfy the conditions

$$
\begin{equation*}
\mathbf{I}_{u}+\mathbf{I}_{\tau}=\mathbf{I}, \quad \mathbf{I}_{u} \mathbf{I}_{\tau}=\mathbf{0} \tag{4}
\end{equation*}
$$

with I being the identity matrix, and

$$
\begin{equation*}
\mathbf{u}=\{u v \phi \psi\}^{T}, \quad \tau=\left\{\sigma_{x y} \sigma_{y y} D_{y} B_{y}\right\}^{T} \tag{5}
\end{equation*}
$$

are the extended displacement and traction vectors at the plane $y=0$, respectively.
Equation (3) includes a total of 16 different sets of boundary conditions, i.e., Case 1:

$$
\mathbf{I}_{u}=\operatorname{diag}\left[\begin{array}{llll}
0 & 0 & 0 & 0
\end{array}\right], \quad \mathbf{I}_{\tau}=\operatorname{diag}\left[\begin{array}{llll}
1 & 1 & 1 & 1 \tag{6a-1}
\end{array}\right]
$$

Case 2:

$$
\mathbf{I}_{u}=\operatorname{diag}\left[\begin{array}{llll}
0 & 0 & 0 & 1
\end{array}\right], \quad \mathbf{I}_{\tau}=\operatorname{diag}\left[\begin{array}{llll}
1 & 1 & 1 & 0 \tag{6a-2}
\end{array}\right],
$$

Case 3:

$$
\mathbf{I}_{u}=\operatorname{diag}\left[\begin{array}{llll}
0 & 0 & 1 & 0
\end{array}\right], \quad \mathbf{I}_{\tau}=\operatorname{diag}\left[\begin{array}{llll}
1 & 1 & 0 & 1 \tag{6a-3}
\end{array}\right],
$$

Case 4:

$$
\mathbf{I}_{u}=\operatorname{diag}\left[\begin{array}{llll}
0 & 0 & 1 & 1
\end{array}\right], \quad \mathbf{I}_{\tau}=\operatorname{diag}\left[\begin{array}{llll}
1 & 1 & 0 & 0 \tag{6a-4}
\end{array}\right],
$$

Case 5:

$$
\mathbf{I}_{u}=\operatorname{diag}\left[\begin{array}{llll}
1 & 0 & 0 & 0
\end{array}\right], \quad \mathbf{I}_{\tau}=\operatorname{diag}\left[\begin{array}{llll}
0 & 1 & 1 & 1 \tag{6b-1}
\end{array}\right],
$$

Case 6:

$$
\mathbf{I}_{u}=\operatorname{diag}\left[\begin{array}{llll}
1 & 0 & 0 & 1
\end{array}\right], \quad \mathbf{I}_{\tau}=\operatorname{diag}\left[\begin{array}{llll}
0 & 1 & 1 & 0 \tag{6b-2}
\end{array}\right],
$$

Case 7:

$$
\mathbf{I}_{u}=\operatorname{diag}\left[\begin{array}{llll}
1 & 0 & 1 & 0
\end{array}\right], \quad \mathbf{I}_{\tau}=\operatorname{diag}\left[\begin{array}{llll}
0 & 1 & 0 & 1 \tag{6b-3}
\end{array}\right],
$$

Case 8:

$$
\mathbf{I}_{u}=\operatorname{diag}\left[\begin{array}{llll}
1 & 0 & 1 & 1
\end{array}\right], \quad \mathbf{I}_{\tau}=\operatorname{diag}\left[\begin{array}{llll}
0 & 1 & 0 & 0 \tag{6b-4}
\end{array}\right],
$$

Case 9:

$$
\mathbf{I}_{u}=\operatorname{diag}\left[\begin{array}{llll}
1 & 1 & 0 & 0
\end{array}\right], \quad \mathbf{I}_{\tau}=\operatorname{diag}\left[\begin{array}{llll}
0 & 0 & 1 & 1 \tag{6c-1}
\end{array}\right],
$$

Case 10:

$$
\mathbf{I}_{u}=\operatorname{diag}\left[\begin{array}{llll}
1 & 1 & 0 & 1
\end{array}\right], \quad \mathbf{I}_{\tau}=\operatorname{diag}\left[\begin{array}{llll}
0 & 0 & 1 & 0 \tag{6c-2}
\end{array}\right],
$$

Case 11:

$$
\mathbf{I}_{u}=\operatorname{diag}\left[\begin{array}{llll}
1 & 1 & 1 & 0
\end{array}\right], \quad \mathbf{I}_{\tau}=\operatorname{diag}\left[\begin{array}{llll}
0 & 0 & 0 & 1 \tag{6c-3}
\end{array}\right],
$$

Case 12:

$$
\mathbf{I}_{u}=\operatorname{diag}\left[\begin{array}{llll}
1 & 1 & 1 & 1
\end{array}\right], \quad \mathbf{I}_{\tau}=\operatorname{diag}\left[\begin{array}{llll}
0 & 0 & 0 & 0 \tag{6c-4}
\end{array}\right],
$$

## Case 13:

$$
\mathbf{I}_{u}=\operatorname{diag}\left[\begin{array}{llll}
0 & 1 & 0 & 0
\end{array}\right], \quad \mathbf{I}_{\tau}=\operatorname{diag}\left[\begin{array}{llll}
1 & 0 & 1 & 1 \tag{6d-1}
\end{array}\right]
$$

Case 14:

$$
\mathbf{I}_{u}=\operatorname{diag}\left[\begin{array}{llll}
0 & 1 & 0 & 1
\end{array}\right], \quad \mathbf{I}_{\tau}=\operatorname{diag}\left[\begin{array}{llll}
1 & 0 & 1 & 0 \tag{6d-2}
\end{array}\right],
$$

Case 15:

$$
\mathbf{I}_{u}=\operatorname{diag}\left[\begin{array}{llll}
0 & 1 & 1 & 0
\end{array}\right], \quad \mathbf{I}_{\tau}=\operatorname{diag}\left[\begin{array}{llll}
1 & 0 & 0 & 1 \tag{6d-3}
\end{array}\right]
$$

Case 16:

$$
\mathbf{I}_{u}=\operatorname{diag}\left[\begin{array}{llll}
0 & 1 & 1 & 1
\end{array}\right], \quad \mathbf{I}_{\tau}=\operatorname{diag}\left[\begin{array}{llll}
1 & 0 & 0 & 0 \tag{6d-4}
\end{array}\right]
$$

The first to the fourth sets of (6a)-(6d) correspond to the electrically open but magnetically closed, both the electrically and magnetically open, both the electrically and magnetically closed, and the electrically closed but magnetically open conditions, respectively. Mechanically, Eqs. (6a)-(6d) correspond to traction-free, stretchfree, rigid and slippery elastic boundary conditions, respectively.

## 3 Derivation of frequency equation

Consider the possibility of a solution of Eq. (2) in the form

$$
\begin{equation*}
\mathbf{u}=\boldsymbol{\Xi}(y, k, \omega) \exp [i k x-i \omega t] \tag{7}
\end{equation*}
$$

where $k$ and $\omega$ are the wave number and frequency, respectively. For two-dimensional in-plane problems, the extended displacements are generally exponential functions of $y$. Therefore, we can express the solution of Eq. (2), i.e., extended displacements as

$$
\begin{equation*}
\mathbf{u}(x, y, t)=F(k)\left\{A_{1} A_{2} A_{3} A_{4}\right\}^{\mathbf{T}} \exp (\lambda k y) \exp (i k x) \exp (-i k c t) \tag{8}
\end{equation*}
$$

in which, $k c=\omega, c$ is the wave velocity, $F$ an unknown function, $A_{1}, A_{2}, A_{3}, A_{4}$ are four known coefficients (forming an eigenvector), and $\lambda$ satisfies the following characteristic equation:

$$
\begin{equation*}
|\mathbf{M}|=0 \tag{9}
\end{equation*}
$$

with

$$
\mathbf{M}=\left(\begin{array}{cccc}
\rho c^{2}+c_{44} \lambda^{2}-c_{11} & i\left(c_{13}+c_{44}\right) \lambda & i\left(e_{31}+e_{15}\right) \lambda & i\left(f_{31}+f_{15}\right) \lambda  \tag{10}\\
i\left(c_{13}+c_{44}\right) \lambda & \rho c^{2}+c_{33} \lambda^{2}-c_{44} & e_{33} \lambda^{2}-e_{15} & f_{33} \lambda^{2}-f_{15} \\
i\left(e_{31}+e_{15}\right) \lambda & e_{33} \lambda^{2}-e_{15} & \varepsilon_{11}-\varepsilon_{33} \lambda^{2} & g_{11}-g_{33} \lambda^{2} \\
i\left(f_{31}+f_{15}\right) \lambda & f_{33} \lambda^{2}-f_{15} & g_{11}-g_{33} \lambda^{2} & \mu_{11}-\mu_{33} \lambda^{2}
\end{array}\right)
$$

It can be shown from Eq. (10) that there are totally eight roots in Eq. (9), and that if $\lambda_{m}(c)$ is a root of Eq. (9), $-\lambda_{m}(c)$ is also a root. Since all the field quantities must vanish as $y$ approaches infinity, then we can further express the extended displacements as

$$
\begin{equation*}
\mathbf{u}(r, z, t)=\sum_{m=1}^{4} F_{m}(k)\left\{a_{u m} a_{v m} a_{\phi m} a_{\psi m}\right\}^{\mathrm{T}} \exp \left(\lambda_{m} k y\right) \exp (i k x) \exp (-i k c t) \tag{11}
\end{equation*}
$$

where $\lambda_{m}$ is chosen such that $\operatorname{Re}\left(\lambda_{m}\right)$ is less than zero and $a_{u m}\left(\lambda_{m}, c\right), a_{v m}\left(\lambda_{m}, c\right), a_{\phi m}\left(\lambda_{m}, c\right)$ and $a_{\psi m}\left(\lambda_{m}, c\right)$ are known material constants (again, forming an eigenvector) satisfying the following relations:

$$
\begin{equation*}
\mathbf{M}\left\{a_{u m} a_{v m} a_{\phi m} a_{\psi m}\right\}^{\mathrm{T}}=\mathbf{0} \tag{12}
\end{equation*}
$$

From the constitutive equations, the expressions for the stresses, electric displacement and magnetic induction (i.e., extended traction) in terms of $F_{m}$ are also obtained. They are

$$
\begin{equation*}
\tau(x, y, t)=\sum_{m=1}^{4} F_{m}(k)\left\{G_{1 m} G_{2 m} G_{3 m} G_{4 m}\right\}^{\mathrm{T}} \exp \left(\lambda_{m} k y\right) \exp (i k x) \exp (-i k c t) \tag{13}
\end{equation*}
$$

in which

$$
\begin{align*}
G_{1 m}\left(k, c, \lambda_{m}\right) & =c_{44} \lambda_{m} k a_{u m}+i k\left(c_{44} a_{v m}+e_{15} a_{\phi m}+f_{15} a_{\psi m}\right)  \tag{14a}\\
G_{2 m}\left(k, c, \lambda_{m}\right) & =c_{13} i k a_{u m}+\lambda_{m} k\left(c_{33} a_{v m}+e_{33} a_{\phi m}+f_{33} a_{\psi m}\right)  \tag{14b}\\
G_{3 m}\left(k, c, \lambda_{m}\right) & =e_{31} i k a_{u m}+\lambda_{m} k\left(e_{33} a_{v m}-\varepsilon_{33} a_{\phi m}-g_{33} a_{\psi m}\right)  \tag{14c}\\
G_{4 m}\left(k, c, \lambda_{m}\right) & =f_{31} i k a_{u m}+\lambda_{m} k\left(f_{33} a_{v m}-g_{33} a_{\phi m}-\mu_{33} a_{\psi m}\right) . \tag{14d}
\end{align*}
$$

Substituting Eqs. (13) and (11) into Eq. (3) yields

$$
\begin{equation*}
\boldsymbol{\Omega} \mathbf{F}=\mathbf{0} \tag{15}
\end{equation*}
$$

where

$$
\mathbf{F}=\left\{\begin{array}{llll}
F_{1} & F_{2} & F_{3} & F_{4} \tag{16}
\end{array}\right\}
$$

and $\boldsymbol{\Omega}$ is a $4 \times 4$ matrix. For the 16 different sets of boundary conditions, $\Omega_{k m}(k, m=1,2,3,4)$ are, respectively,
Case 1:

$$
\begin{equation*}
\Omega_{1 m}=G_{1 m}, \quad \Omega_{2 m}=G_{2 m}, \quad \Omega_{3 m}=G_{3 m}, \quad \Omega_{4 m}=G_{4 m}, \tag{17a-1}
\end{equation*}
$$

Case 2:

$$
\begin{equation*}
\Omega_{1 m}=G_{1 m}, \quad \Omega_{2 m}=G_{2 m}, \quad \Omega_{3 m}=G_{3 m}, \quad \Omega_{4 m}=a_{\psi m} \tag{17a-2}
\end{equation*}
$$

Case 3:

$$
\begin{equation*}
\Omega_{1 m}=G_{1 m}, \quad \Omega_{2 m}=G_{2 m}, \quad \Omega_{3 m}=a_{\phi m}, \quad \Omega_{4 m}=G_{4 m}, \tag{17a-3}
\end{equation*}
$$

Case 4:

$$
\begin{equation*}
\Omega_{1 m}=G_{1 m}, \quad \Omega_{2 m}=G_{2 m}, \quad \Omega_{3 m}=a_{\phi m}, \quad \Omega_{4 m}=a_{\psi m} \tag{17a-4}
\end{equation*}
$$

Case 5:

$$
\begin{equation*}
\Omega_{1 m}=a_{u m}, \quad \Omega_{2 m}=G_{2 m}, \quad \Omega_{3 m}=G_{3 m}, \quad \Omega_{4 m}=G_{4 m} \tag{17b-1}
\end{equation*}
$$

Case 6:

$$
\begin{equation*}
\Omega_{1 m}=a_{u m}, \quad \Omega_{2 m}=G_{2 m}, \quad \Omega_{3 m}=G_{3 m}, \quad \Omega_{4 m}=a_{\psi m} \tag{17b-2}
\end{equation*}
$$

Case 7:

$$
\begin{equation*}
\Omega_{1 m}=a_{u m}, \quad \Omega_{2 m}=G_{2 m}, \quad \Omega_{3 m}=a_{\phi m}, \quad \Omega_{4 m}=G_{4 m} \tag{17b-3}
\end{equation*}
$$

Case 8:

$$
\begin{equation*}
\Omega_{1 m}=a_{u m}, \quad \Omega_{2 m}=G_{2 m}, \quad \Omega_{3 m}=a_{\phi m}, \quad \Omega_{4 m}=a_{\psi m} \tag{17b-4}
\end{equation*}
$$

Case 9:

$$
\begin{equation*}
\Omega_{1 m}=a_{u m}, \quad \Omega_{2 m}=a_{v m}, \quad \Omega_{3 m}=G_{3 m}, \quad \Omega_{4 m}=G_{4 m} \tag{17c-1}
\end{equation*}
$$

Case 10:

$$
\begin{equation*}
\Omega_{1 m}=a_{u m}, \quad \Omega_{2 m}=a_{v m}, \quad \Omega_{3 m}=G_{3 m}, \quad \Omega_{4 m}=a_{\psi m} \tag{17c-2}
\end{equation*}
$$

Case 11:

$$
\begin{equation*}
\Omega_{1 m}=a_{u m}, \quad \Omega_{2 m}=a_{v m}, \quad \Omega_{3 m}=a_{\phi m}, \quad \Omega_{4 m}=G_{4 m} \tag{17c-3}
\end{equation*}
$$

Case 12:

$$
\begin{equation*}
\Omega_{1 m}=a_{u m}, \quad \Omega_{2 m}=a_{v m}, \quad \Omega_{3 m}=a_{\phi m}, \quad \Omega_{4 m}=a_{\psi m} \tag{17c-4}
\end{equation*}
$$

Case 13:

$$
\begin{equation*}
\Omega_{1 m}=G_{1 m}, \quad \Omega_{2 m}=a_{v m}, \quad \Omega_{3 m}=G_{3 m}, \quad \Omega_{4 m}=G_{4 m}, \tag{17~d-1}
\end{equation*}
$$

Case 14:

$$
\begin{equation*}
\Omega_{1 m}=G_{1 m}, \quad \Omega_{2 m}=a_{v m}, \quad \Omega_{3 m}=G_{3 m}, \quad \Omega_{4 m}=a_{\psi m}, \tag{17d-2}
\end{equation*}
$$

Case 15:

$$
\begin{equation*}
\Omega_{1 m}=G_{1 m}, \quad \Omega_{2 m}=a_{v m}, \quad \Omega_{3 m}=a_{\phi m}, \quad \Omega_{4 m}=G_{4 m}, \tag{17d-3}
\end{equation*}
$$

Case 16:

$$
\begin{equation*}
\Omega_{1 m}=G_{1 m}, \quad \Omega_{2 m}=a_{v m}, \quad \Omega_{3 m}=a_{\phi m}, \quad \Omega_{4 m}=a_{\psi m} \tag{17d-4}
\end{equation*}
$$

For a non-trivial solution of $\mathbf{F}$, the determinant of its coefficient matrix in Eq. (15) must be zero, which leads to

$$
\begin{equation*}
|\boldsymbol{\Omega}|=0 \tag{18}
\end{equation*}
$$

Equation (18) is the frequency equation, which reveals the frequency dispersion characteristics of the wave propagating along the surface of the magneto-electro-elastic half plane.

It is easily observed from Eq. (17), together with Eqs. (14) and (12), that the wave velocity $c$ satisfying Eq. (18) is independent of the wave number $k$, which implies that the Rayleigh wave in a magneto-electro-elastic half plane is non-dispersive.

## 4 Numerical results and discusses

Equation (18) is complicated, and thus, it is impossible for us to derive an analytical solution for the Rayleigh wave velocity. Therefore, in this section, numerical results are presented and discussed for $\mathrm{BaTiO}_{3}-\mathrm{CoFe}_{2} \mathrm{O}_{4}$ composites under different surface conditions. Non-zero material properties of $\mathrm{BaTiO}_{3}-\mathrm{CoFe}_{2} \mathrm{O}_{4}$ are listed in Table 1 [27].

In the calculation, the dimensionless Rayleigh wave velocity is taken as $c_{0}=c / c_{\mathrm{s}}$, where $c_{\mathrm{s}}=\sqrt{\widetilde{c}_{44} / \rho}, \tilde{c}_{44}=c_{44}+\frac{\varepsilon_{11} f_{15}^{2}-2 e_{15} f_{15} g_{11}+\mu_{11} e_{15}^{2}}{\mu_{11} \varepsilon_{11}-g_{11}^{2}}$. Actually, $c_{\mathrm{s}}$ denotes the corresponding shear wave velocity in magneto-electro-elastic materials. Considering that for purely elastic materials, the dimensionless Rayleigh wave velocity is less than one, our numerical examinations are limited to the range of $0 \leq c_{0} \leq 1$. Numerical results are tabulated in Table 2.

Table 1 Material properties ( $c_{i j}$ in $10^{10} \mathrm{~N} / \mathrm{m}^{2}, e_{i j}$ in $\mathrm{C} / \mathrm{m}^{2}, f_{i j}$ in $\mathrm{N} / \mathrm{Am}, \varepsilon_{i j}$ in $10^{-10} C^{2} / \mathrm{Nm}^{2}, \mu_{i j}$ in $\mathrm{Ns}^{2} / \mathrm{C}^{2}, \rho$ in $\mathrm{kg} / \mathrm{m}^{3}$ )

| $c_{11}$ | $c_{12}$ | $c_{13}$ | $c_{33}$ | $c_{44}$ | $e_{31}$ | $e_{33}$ | $e_{15}$ | $f_{31}$ | $f_{33}$ | $f_{15}$ | $\varepsilon_{11}$ | $\varepsilon_{33}$ | $\mu_{11}$ |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| 22.6 | 12.5 | 12.4 | 21.6 | 4.4 | -2.2 | 9.3 | 5.8 | 290.2 | 350 | 275 | 56.4 | 63.5 | 8.1 |

Table 2 Normalized Rayleigh wave velocity $c_{0}$ (material parameters are the same as those given in Table 1)

|  | Case 1 | Case 2 | Case 3 | Case 4 | Cases 5-16 |
| :--- | :--- | :--- | :--- | :--- | :--- |
| $c_{01}$ | 0.93570 | 0.93445 | 0.93067 | 0.92926 | - |
| $c_{02}$ | 0.37694 | 0.37694 | 0.37694 | 0.37694 | 0.37694 |

Table 3 Normalized Rayleigh wave velocity $c_{0}\left(e_{31}=e_{33}=e_{15}=0\right.$, all the other material parameters are the same as those given in Table 1)

|  | Case 1 | Case 2 | Case 3 | Case 4 | Cases 5-16 |
| :--- | :--- | :--- | :--- | :--- | :--- |
| $c_{01}$ | 0.94528 | 0.94391 | 0.94528 | 0.94391 | - |
| $c_{02}$ | 0.08646 | 0.08646 | 0.08646 | 0.08646 | 0.08646 |

Table 4 Normalized Rayleigh wave velocity $c_{0}\left(f_{31}=f_{33}=f_{15}=0\right.$, all the other material parameters are the same as those given in Table 1)

|  | Case 1 | Case 2 | Case 3 | Case 4 | Cases 5 and 6 | Cases 7 and 8 | Cases 9-16 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| $c_{01}$ | 0.93831 | 0.93831 | 0.93313 | 0.93313 | - | - | - |
| $c_{02}$ | - | - | - | - | - | 0.99417 | - |

Table 5 Normalized Rayleigh wave velocity $c_{0}\left(e_{31}=e_{33}=e_{15}=0, f_{31}=f_{33}=f_{15}=0\right.$, all the other material parameters are the same as those given in Table 1)

|  | Cases 1-4 | Cases 5-16 |
| :--- | :--- | :--- |
| $c_{01}$ | 0.94788 | - |
| $c_{02}$ | - | - |

As shown in Table 2, there are totally two Rayleigh wave velocities denoting by $c_{01}$ and $c_{02}$, respectively, for Cases 1-4. However, there is only one Rayleigh wave velocity denoting by $c_{02}$ for the other cases. Moreover, for the traction-free surface conditions (i.e., Eq. (6a)), the first Rayleigh wave velocity $c_{01}$ is the largest for the electrically open but magnetically closed magnetoelectric surface conditions (i.e., Case 1 ), and is the smallest for the electrically closed but magnetically open magnetoelectric surface conditions (i.e., Case 4). In addition, it is interesting to note that the Rayleigh wave velocity $c_{02}$ depends only on the material properties of the magneto-electro-elastic composites. In other words, it is independent of boundary conditions (including both extended traction and extended displacement conditions).

In order to further illustrate the propagation properties of the Rayleigh wave in magneto-electro-elastic half planes, numerical results for some reduced uncoupled materials are given in Tables 3, 4, 5 .

Table 3 shows that if $e_{31}=e_{33}=e_{15}=0$ (piezoelectric uncoupling), there are also two Rayleigh wave velocities for Cases $1-4$, and there is only one Rayleigh wave velocity for Cases $5-16$. It should also be noted that $c_{01}$ is the same for both Cases 1 and 3 , and $c_{01}$ is the same for both Cases 2 and 4 . Furthermore, for the 16 different sets of surface conditions, there is also only one identical Rayleigh wave velocity $c_{02}$, which is much smaller than $c_{01}$.

Table 4 indicates that if $f_{31}=f_{33}=f_{15}=0$ (piezomagnetic uncoupling), there is only one Rayleigh wave velocity for Cases $1-4$, and Cases 7 and 8 . In other words, there are no Rayleigh waves for the other ten sets of surface conditions. Table 4 also indicates that for this piezomagnetic-uncoupled material half plane $\left(f_{31}=f_{33}=f_{15}=0\right), c_{01}$ is the same for Cases 1 and $2, c_{01}$ is the same for Cases 3 and 4 , and that $c_{02}$ is the same for Cases 7 and 8 . It is interesting, however, that $c_{02}>c_{01}$ !

Table 5 shows that if $e_{31}=e_{33}=e_{15}=0$ and $f_{31}=f_{33}=f_{15}=0$ (both piezoelectric and piezomagnetic uncoupled), there is only one identical Rayleigh wave velocity for Cases 1 to 4; no Rayleigh wave exists for the other 12 sets of surface conditions.

Thus, Tables 2, 3, 4, 5 further reveal that for an magneto-electro-elastic half plane, the first Rayleigh wave velocity $c_{01}$ mainly results from the elastic properties of magneto-electro-elastic materials; whilst the second Rayleigh wave $c_{02}$ results from the piezoelectric and piezomagnetic effects of the magneto-electro-elastic body. Moreover, $c_{01}$ exists only in the case of traction-free surface conditions. While the magnitude of $c_{02}$ depends only on material properties, whether it exists or not depends strongly on the boundary conditions.

## 5 Conclusions

For a magneto-electro-elastic half plane, the Rayleigh wave is a non-dispersive surface wave. In general, both material properties and boundary conditions can significantly affect the existence and velocity magnitude of the Rayleigh waves. For the $\mathrm{BaTiO}_{3}-\mathrm{CoFe}_{2} \mathrm{O}_{4}$ composites, there are at most two Rayleigh wave velocities.

One exists only in the case of traction-free boundary conditions, and both the piezoelectric and piezomagnetic coefficients have only slight influence on its magnitude. On the other hand, if the second Rayleigh wave appears, its normalized wave velocity depends on the material properties only.

Acknowledgments The work was supported by Natural Science Fund of China (10772123), and partly by AFOSR/AFRL.

## References

1. Van Suchtelen, J.: Product properties: a new application of composite materials. Philips Res. Rep. 27, 28-37 (1972)
2. Wislon, L.O., Morrison, J.A.: Wave propagation in piezoelectric rods of hexagonal crystal symmetry. Q. J. Mech. Appl. Math. 30, 387-395 (1977)
3. Guzellsu, N., Saha, S.: Electro-mechanical wave propagation in long bones. J. Biomech. 14, 19-33 (1981)
4. Paul, H.S., Venkatesan, M.: Wave propagation in a piezoelectric bone with a cylindrical cavity of arbitrary shape. Int. J. Eng. Sci. 29, 1601-1607 (1991)
5. Honein, B., Herrmann, G.: Wave propagation in nonhomogeneous piezoelectric materials. Dyn. Syst. Cont. Division 38, 105-112 (1992)
6. Chai, J.F.: Propagation of surface waves in a prestressed piezoelectric material. J. Acoust. Soc. Am. 100, 2112-2122 (1996)
7. Jin, F., Wang, Z., Wang, T.: The Bleustein-Gulyaev (B-G) wave in a piezoelectric layered half-space. Int. J. Eng. Sci. 39, 12711285 (2001)
8. Liu, H., Wang, Z.K., Wang, T.J.: Effect of initial stress on the propagation behavior of Love waves in a layered piezoelectric structure. Int. J. Solids Struct. 38, 37-51 (2001)
9. Wang, Q.: Wave propagation in a piezoelectric coupled solid medium. J. Appl. Mech. 69, 819-824 (2002)
10. Wang, Q., Varadan, V.K.: Longitudinal wave propagation in piezoelectric coupled rods. Smart Mater. Struct. 11, 48-54 (2002)
11. Ashida, F., Tauchert, T.R.: Thermally-induced wave propagation in a piezoelectric plate. Acta Mech. 161, 1-16 (2003)
12. Qian, Z., Jin, F., Wang, Z., Kishimoto, K.: Love wave propagation in a piezoelectric layered structure with initial stresses. Acta Mech. 171, 41-57 (2004)
13. Li, X.F., Yang, J.S., Jiang, Q.: Spatial dispersion of short surface acoustic waves in piezoelectric ceramics. Acta Mech. 180, 11-20 (2005)
14. Wang, J., Lin, J.B.: Two-dimensional theory for surface acoustic wave propagation in finite piezoelectric solids. J. Intell. Mater. Syst. Struct. 16, 623-629 (2005)
15. Wei, J.P., Su, X.Y.: Wave propagation in a piezoelectric rod of 6 mm symmetry. Int. J. Solids Struct. 42, 3644-3654 (2005)
16. Darloyan, Z.N., Posian, G.T.: Surface electro-elastic Love waves in a layered structure with a piezoelectric substrate and a dielectric layer. Int. J. Solids Struct. 44, 5829-5847 (2007)
17. Liu, H., Zhu, S.N., Zhu, Y.Y., Chen, Y.F., Ming, N.B., Zhang, X.: Piezoelectric-piezomagnetic multilayer with simultaneously negative permeability and permittivity. Appl. Phys. Lett. 86, 102904 (2005)
18. Feng, W.J., Hao, R.J., Liu, J.X., Duan, S.M.: Scattering of SH waves by arc-shaped interface cracks between a cylindrical magneto-electro-elastic inclusion and matrix: near fields. Arch. Appl. Mech. 74, 649-663 (2005)
19. Feng, W.J., Su, R.K.L., Liu, Y.Q.: Scattering of SH waves by an arc-shaped interface crack between a cylindrical magneto-electro-elastic inclusion and matrix with the symmetry of 6 mm . Acta Mech. 183, 81-102 (2006)
20. Soh, A.K., Liu, J.X.: Interfacial shear horizontal waves in a piezoelectric-piezomagnetic bi-material. Philos. Mag. Lett. 86, 31-35 (2006)
21. Wang, B.L., Mai, Y.W., Niraula, O.P.: A horizontal shear surface wave in magnetoelectroelastic materials. Philos. Mag. Lett. 87, 53-58 (2007)
22. Chen, J.Y., Pan, E., Chen, H.L.: Wave propagation in magneto-electro-elastic multilayered plates. Int. J. Solids Struct. 44, 1073-1085 (2007)
23. Chen, P., Shen, Y.P.: Propagation of axial shear magneto-electro-elastic waves in piezoelectric-piezomagnetic composites with randomly distributed cylindrical inhomogeneities. Int. J. Solids Struct. 44, 1511-1532 (2007)
24. Melkumyan, A.: Twelve shear surface waves guided by clamped/free boundaries in magneto-electro-elastic materials. Int. J. Solids Struct. 44, 3594-3599 (2007)
25. Du, J., Jin, X., Wang, J.: Love wave propagation in layered magneto-electro-elastic structures with initial stress. Acta Mech. 192, 169-189 (2007)
26. Pan, E.: Mindlin's problem for an anisotropic piezoelectric half-space with general boundary conditions. Proc. R. Soc. Lond. A 458, 181-208 (2002)
27. Wang, B.L., Mai, Y.W.: Applicability of the crack-face electromagnetic boundary conditions for fracture of magnetoelectroelastic materials. Int. J. Solids Struct. 44, 387-398 (2007)

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