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Thermal Green's functions in plane anisotropic multiferroic bimaterials with viscous interface

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Abstract Derived in this work are the time-dependent thermal Green's function solutions of a steady line heat source and a steady thermal dislocation near a linear viscous interface between two different anisotropic magneto-electro-thermo-elastic multiferroic half-planes. Our analysis demonstrates that once the thermal Green's functions at the initial time for a perfect interface are known, the corresponding time-dependent thermal Green's functions at any time can be written down immediately. These solutions are obtained in exact closed form.

1 Introduction

At high homologous temperatures, the thermally activated rate-dependent interfacial sliding becomes important and contributes to the plastic deformation in polycrystalline metals and ceramics [1]. Here the thermally activated interfacial sliding can be described by the Newtonian viscous interface: $\dot{\delta} = \tau/\eta$, where $\dot{\delta}$ is the sliding velocity (i.e., the differentiation of the relative sliding with respect to time t), τ is the interfacial shear stress and η is the interfacial viscosity [1–4]. Furthermore the Newtonian viscous interface can also be adopted to describe the microscopically mass diffusion-controlled mechanism along the interface [5–7]. Recently, the isothermal Green's functions in anisotropic magneto-electro-elastic multiferroic bimaterials with a linear viscous interface subjected to an extended line force and an extended line dislocation located in the upper half-plane were obtained [4]. The purpose of this research is to further develop the corresponding thermal Green's functions of a line heat source and a temperature dislocation located in anisotropic multiferroic bimaterials with a planar viscous interface. The thermal Green's functions to be developed in this research together with the previously obtained isothermal Green's functions [4] can be further utilized to solve the magneto-electro-thermo-elastic problems involving cracks or anti-cracks (rigid line inclusions) in anisotropic multiferroic bimaterials with a viscous interface [8–10].

This paper is structured as follows. In Sect. 2, the generalized version of the Stroh formalism suitable for two-dimensional (2D) thermal problems in generally anisotropic multiferroic materials in the presence of viscous interface is presented. In Sect. 3, the full-field solutions for a thermal dislocation and a line heat source located in anisotropic multiferroic bimaterials with a planar viscous interface are derived. Conclusions are drawn in Sect. 4.

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2 Basic formulations

The basic equations for an anisotropic and linearly multiferroic material taking into consideration the thermal effect are [11]

$$\begin{aligned}
&\sigma_{ij} = C_{ijkl}u_{k,l} + e_{kij}\phi_{,k} + q_{kij}\varphi_{,k} - \beta_{ij}T,\\
&D_k = e_{kij}u_{i,j} - \varepsilon_{kl}\phi_{,l} - \alpha_{lk}\varphi_{,l} + \rho_k T,\\
&B_k = q_{kij}u_{i,j} - \alpha_{kl}\phi_{,l} - \mu_{kl}\varphi_{,l} + m_k T,\\
&\sigma_{ij,j} = 0, \quad D_{i,i} = 0, \quad B_{i,i} = 0,
\end{aligned} \tag{1}$$

where repeated indices mean summation, a comma followed by i ($i = 1, 2, 3$) stands for the derivative with respect to the ith spatial coordinate, u_i, ϕ and φ are the elastic displacement, electric potential and magnetic potential, T is the temperature change, σ_{ij}, D_i and B_i are the stress, electric displacement and magnetic induction, C_{ijkl}, ε_{ij} and μ_{ij} are the elastic, dielectric and magnetic permeability coefficients, respectively; e_{ijk}, q_{ijk} and α_{ij} are the piezoelectric, piezomagnetic and magneto-electric coefficients, respectively; β_{ij}, ρ_k and m_k are, respectively, the stress-temperature, pyroelectric and pyromagnetic constants. In writing Eq. (1) we have ignored the inertia effect. In addition the temperature change should satisfy the following differential equation:

$$k_{ij}T_{,ij} = 0, \tag{2}$$

where k_{ij} is the heat conduction coefficient. In this research it is assumed that $k_{ij} = k_{ji}$ is satisfied [12].

For 2D problems in which all quantities depend only on x_1 and x_2, the general solutions can be expressed as [4,10,12]

$$\begin{aligned}
&T = g'(z_0) + \overline{g'(z_0)},\\
&\Theta = \mathrm{i}\gamma\left[g'(z_0) - \overline{g'(z_0)}\right],\\
&\mathbf{u} = \left[u_1\ u_2\ u_3\ \phi\ \varphi\right]^T = \mathbf{A}\mathbf{f}(z,t) + \bar{\mathbf{A}}\overline{\mathbf{f}(z,t)} + \mathbf{c}g(z_0) + \bar{\mathbf{c}}\overline{g(z_0)},\\
&\mathbf{\Phi} = \left[\Phi_1\ \Phi_2\ \Phi_3\ \Phi_4\ \Phi_5\right]^T = \mathbf{B}\mathbf{f}(z,t) + \bar{\mathbf{B}}\overline{\mathbf{f}(z,t)} + \mathbf{d}g(z_0) + \bar{\mathbf{d}}\overline{g(z_0)},
\end{aligned} \tag{3}$$

where the prime denotes differentiation with respect to the complex variable, and

$$\begin{aligned}
&z_0 = x_1 + p_0x_2, \quad p_0 = \frac{-k_{12} + \mathrm{i}\gamma}{k_{22}}, \quad \gamma = \sqrt{k_{11}k_{22} - k_{12}^2} > 0,\\
&\mathbf{A} = \left[\mathbf{a}_1\ \mathbf{a}_2\ \mathbf{a}_3\ \mathbf{a}_4\ \mathbf{a}_5\right], \quad \mathbf{B} = \left[\mathbf{b}_1\ \mathbf{b}_2\ \mathbf{b}_3\ \mathbf{b}_4\ \mathbf{b}_5\right],\\
&\mathbf{f}(z,t) = \left[f_1(z_1,t)\ f_2(z_2,t)\ f_3(z_3,t)\ f_4(z_4,t)\ f_5(z_5,t)\right]^T,\\
&z_i = x_1 + p_ix_2, \quad \mathrm{Im}\{p_i\} > 0, \quad (i = 1-5),
\end{aligned} \tag{4}$$

with

$$\begin{aligned}
&\begin{bmatrix}\mathbf{N}_1 & \mathbf{N}_2\\ \mathbf{N}_3 & \mathbf{N}_1^T\end{bmatrix}\begin{bmatrix}\mathbf{a}_i\\ \mathbf{b}_i\end{bmatrix} = p_i\begin{bmatrix}\mathbf{a}_i\\ \mathbf{b}_i\end{bmatrix}, \quad (i = 1-5),\\
&\begin{bmatrix}\mathbf{N}_1 & \mathbf{N}_2\\ \mathbf{N}_3 & \mathbf{N}_1^T\end{bmatrix}\begin{bmatrix}\mathbf{c}\\ \mathbf{d}\end{bmatrix} = p_0\begin{bmatrix}\mathbf{c}\\ \mathbf{d}\end{bmatrix} - \begin{bmatrix}\mathbf{0} & \mathbf{N}_2\\ \mathbf{I} & \mathbf{N}_1^T\end{bmatrix}\begin{bmatrix}\boldsymbol{\beta}_1\\ \boldsymbol{\beta}_2\end{bmatrix},
\end{aligned} \tag{5}$$

$$\mathbf{N}_1 = -\mathbf{T}^{-1}\mathbf{R}^T, \quad \mathbf{N}_2 = \mathbf{T}^{-1}, \quad \mathbf{N}_3 = \mathbf{R}\mathbf{T}^{-1}\mathbf{R}^T - \mathbf{Q}, \tag{6}$$

$$\begin{aligned}
&\boldsymbol{\beta}_1 = \left[\beta_{11}\ \beta_{21}\ \beta_{31}\ -\rho_1\ -m_1\right]^T,\\
&\boldsymbol{\beta}_2 = \left[\beta_{12}\ \beta_{22}\ \beta_{32}\ -\rho_2\ -m_2\right]^T,
\end{aligned} \tag{7}$$

and

$$\mathbf{Q} = \begin{bmatrix}\mathbf{Q}^E & \mathbf{e}_{11} & \mathbf{q}_{11}\\ \mathbf{e}_{11}^T & -\varepsilon_{11} & -\alpha_{11}\\ \mathbf{q}_{11}^T & -\alpha_{11} & -\mu_{11}\end{bmatrix}, \quad \mathbf{R} = \begin{bmatrix}\mathbf{R}^E & \mathbf{e}_{21} & \mathbf{q}_{21}\\ \mathbf{e}_{12}^T & -\varepsilon_{12} & -\alpha_{21}\\ \mathbf{q}_{12}^T & -\alpha_{12} & -\mu_{12}\end{bmatrix}, \quad \mathbf{T} = \begin{bmatrix}\mathbf{T}^E & \mathbf{e}_{22} & \mathbf{q}_{22}\\ \mathbf{e}_{22}^T & -\varepsilon_{22} & -\alpha_{22}\\ \mathbf{q}_{22}^T & -\alpha_{22} & -\mu_{22}\end{bmatrix}, \tag{8}$$

$$(\mathbf{Q}^E)_{ik} = C_{i1k1}, \quad (\mathbf{R}^E)_{ik} = C_{i1k2}, \quad (\mathbf{T}^E)_{ik} = C_{i2k2}, \quad (\mathbf{e}_{ij})_m = e_{ijm}, \quad (\mathbf{q}_{ij})_m = q_{ijm}. \tag{9}$$

The appearance of the time t in Eq. (3) comes solely from the influence of the viscous interface under quasi-static deformation. p_0 in Eq. (4) can be called the thermal eigenvalue, while p_k $(k = 1 - 5)$ in Eq. (4) are the Stroh eigenvalues [12]. In addition the heat flux function Θ is defined in terms of the heat fluxes h_1 and h_2 $(h_i = -k_{ij}T_{,j})$ as

$$h_1 = \Theta_{,2}, \quad h_2 = -\Theta_{,1}, \tag{10}$$

and the extended stress function vector $\mathbf{\Phi}$ is defined, in terms of the stresses, electric displacements and magnetic inductions, as follows:

$$\begin{aligned} &\sigma_{i1} = -\Phi_{i,2}, \quad \sigma_{i2} = \Phi_{i,1}, \quad (i = 1 - 3), \\ &D_1 = -\Phi_{4,2}, \quad D_2 = \Phi_{4,1}, \\ &B_1 = -\Phi_{5,2}, \quad B_2 = \Phi_{5,1}, \end{aligned} \tag{11}$$

Due to the fact that the two matrices $\mathbf{A}$ and $\mathbf{B}$ satisfy the normalized orthogonal relationship [12]

$$\begin{bmatrix} \mathbf{B}^T & \mathbf{A}^T \\ \bar{\mathbf{B}}^T & \bar{\mathbf{A}}^T \end{bmatrix} \begin{bmatrix} \mathbf{A} & \bar{\mathbf{A}} \\ \mathbf{B} & \bar{\mathbf{B}} \end{bmatrix} = \mathbf{I}, \tag{12}$$

three real Barnett–Lothe tensors $\mathbf{S}$, $\mathbf{H}$ and $\mathbf{L}$ can be introduced [12]:

$$\mathbf{S} = \mathrm{i}(2\mathbf{A}\mathbf{B}^T - \mathbf{I}), \quad \mathbf{H} = 2\mathrm{i}\mathbf{A}\mathbf{A}^T, \quad \mathbf{L} = -2\mathrm{i}\mathbf{B}\mathbf{B}^T. \tag{13}$$

3 Thermal Green's functions

Now let us assume that the anisotropic multiferroic materials 1 and 2 occupy, respectively, the half-planes $x_2 > 0$ and $x_2 < 0$. At the initial moment we introduce at the fixed location $\left[\hat{x}_1 \hat{x}_2\right]$ $(\hat{x}_2 > 0)$ in the upper half-plane a steady heat source of strength Q_0 and a steady temperature dislocation of discontinuity T_0. In the following, the superscripts (1) and (2) (or the subscripts 1 and 2) will be used to identify the quantities in the upper and lower half-planes, respectively. The two anisotropic multiferroic half-planes are bonded together through a linear viscous interface at $x_2 = 0$.

The boundary conditions on the linear viscous interface are given by

$$T_1 = T_2, \ \Theta_1 = \Theta_2, \quad x_2 = 0 \quad \text{and} \quad t \geq 0, \tag{14}$$

$$\mathbf{\Phi}_1 = \mathbf{\Phi}_2, \ \mathbf{u}_1 = \mathbf{u}_2, \quad x_2 = 0 \quad \text{and} \quad t = 0, \tag{15}$$

$$\mathbf{\Phi}_1 = \mathbf{\Phi}_2, \ \dot{\mathbf{u}}_1 - \dot{\mathbf{u}}_2 = \mathbf{\Lambda}\mathbf{\Phi}_{2,1}, \quad x_2 = 0 \text{ and } t > 0, \tag{16}$$

where an overdot denotes the derivative with respect to time t, and

$$\mathbf{\Lambda} = \mathrm{diag}\left[\, \eta_1^{-1} \ 0 \ \eta_3^{-1} \ 0 \ 0 \,\right], \tag{17}$$

with η_1 and η_3 being the viscous coefficients in the x_1 and x_3 directions, respectively. Equation (14) states that the temperature and normal heat flux are always continuous across the interface (i.e., the interface is always thermally perfect); while Eq. (15) implies that at the initial moment the interface is elastically, electrically and magnetically perfect due to the fact that at $t = 0$ the displacements across the interface have no time to experience any jump due to the dashpot [4,13]. Due to the fact that on the interface $x_2 = 0$ we have $z_0 = z_1 = z_2 = z_3 = z_4 = z_5 = z$, $(z = x_1 + \mathrm{i}x_2)$, we can first replace z_k $(k = 0, 1 - 5)$ by the common complex variable z during the following analysis [4,14,15]. After the analysis is finished, we can then change z back to the corresponding complex variables.

Through analytical continuation [15], the two analytic functions in the upper and lower half-planes characterizing the temperature field can easily be obtained as [10]

$$\begin{aligned} &g_1'(z_0) = g_0'(z_0) + \frac{1-\Gamma}{1+\Gamma}\bar{g}_0'(z_0), \quad z_0 = x_1 + p_0 x_2, \\ &g_2'\left(z_0^*\right) = \frac{2}{1+\Gamma} g_0'\left(z_0^*\right), \quad z_0^* = x_1 + p_0^* x_2, \end{aligned} \tag{18}$$

where $\Gamma = \gamma_2/\gamma_1$ is a dimensionless two-phase temperature parameter, and

$$g_0'(z_0) = X_0 \ln(z_0 - \hat{z}_0), \tag{19}$$

with $X_0 = -\frac{Q_0}{4\pi\gamma_1} - \frac{T_0}{4\pi}\mathrm{i}$ and $\hat{z}_0 = \hat{x}_1 + p_0\hat{x}_2$ the complex thermal potential in a homogeneous plane occupied by material 1 [10]. The superscript '*' in Eq. (18) is utilized to distinguish the thermal eigenvalue p_0^* associated with the lower half-plane with p_0 associated with the upper half-plane. The integration of $g_0'(z_0)$ leads to

$$g_0(z_0) = X_0 \left[(z_0 - \hat{z}_0)\ln(z_0 - \hat{z}_0) - (z_0 - \hat{z}_0)\right]. \tag{20}$$

The magneto-electro-elastic boundary conditions on the interface for $t > 0$ in Eq. (16) can be expressed in terms of the two analytic function vectors $\mathbf{f}_1(z,t)$ and $\mathbf{f}_2(z,t)$ as

$$\begin{aligned}&\mathbf{B}_1\mathbf{f}_1^+(x_1,t) + \bar{\mathbf{B}}_1\bar{\mathbf{f}}_1^-(x_1,t) + \mathbf{d}_1 g_1^+(x_1,t) + \bar{\mathbf{d}}_1\bar{g}_1^-(x_1,t)\\ &\quad = \mathbf{B}_2\mathbf{f}_2^-(x_1,t) + \bar{\mathbf{B}}_2\bar{\mathbf{f}}_2^+(x_1,t) + \mathbf{d}_2 g_2^-(x_1,t) + \bar{\mathbf{d}}_2\bar{g}_2^+(x_1,t) \qquad x_2 = 0 \text{ and } t > 0,\end{aligned} \tag{21}$$

$$\begin{aligned}&\mathbf{A}_1\dot{\mathbf{f}}_1^+(x_1,t) + \bar{\mathbf{A}}_1\dot{\bar{\mathbf{f}}}_1^-(x_1,t) - \mathbf{A}_2\dot{\mathbf{f}}_2^-(x_1,t) - \bar{\mathbf{A}}_2\dot{\bar{\mathbf{f}}}_2^+(x_1,t)\\ &\quad = \mathbf{\Lambda}\left[\mathbf{B}_2\mathbf{f}_2'^-(x_1,t) + \bar{\mathbf{B}}_2\bar{\mathbf{f}}_2'^+(x_1,t) + \mathbf{d}_2 g_2'^-(x_1,t) + \bar{\mathbf{d}}_2\bar{g}_2'^+(x_1,t)\right] \qquad x_2 = 0 \text{ and } t > 0.\end{aligned} \tag{22}$$

It follows from Eq. (21) that

$$\begin{aligned}\mathbf{B}_1\mathbf{f}_1(z,t) &= \bar{\mathbf{B}}_2\bar{\mathbf{f}}_2(z,t) + \mathbf{B}_1\mathbf{f}_0(z) - \bar{\mathbf{B}}_1\bar{\mathbf{f}}_0(z) - \mathbf{d}_1 g_1(z) + \bar{\mathbf{d}}_2\bar{g}_2(z) + \mathbf{d}_1 g_0(z) - \bar{\mathbf{d}}_1\bar{g}_0(z),\\ \bar{\mathbf{B}}_1\bar{\mathbf{f}}_1(z,t) &= \mathbf{B}_2\mathbf{f}_2(z,t) - \mathbf{B}_1\mathbf{f}_0(z) + \bar{\mathbf{B}}_1\bar{\mathbf{f}}_0(z) + \mathbf{d}_2 g_2(z) - \bar{\mathbf{d}}_1\bar{g}_1(z) - \mathbf{d}_1 g_0(z) + \bar{\mathbf{d}}_1\bar{g}_0(z),\end{aligned} \tag{23}$$

where $\mathbf{f}_0(z)$ is the analytic function vector in a homogeneous plane occupied by material 1 given by [10]

$$\mathbf{f}_0(z) = \langle (z - \hat{z}_\alpha)\ln(z - \hat{z}_\alpha)\rangle \mathbf{e}, \tag{24}$$

with $\mathbf{e} = -2\mathrm{i}\mathbf{A}_1^T \mathrm{Im}\{\mathbf{d}_1 X_0\} - 2\mathrm{i}\mathbf{B}_1^T \mathrm{Im}\{\mathbf{c}_1 X_0\}$, $\hat{z}_\alpha = \hat{x}_1 + p_\alpha\hat{x}_2$ and $\langle * \rangle$ being a 5×5 diagonal matrix in which each component is varied according to the Greek index α (from 1 to 5).

In view of Eq. (18), Eq. (23) can be further transformed into

$$\begin{aligned}\mathbf{B}_1\mathbf{f}_1(z,t) &= \bar{\mathbf{B}}_2\bar{\mathbf{f}}_2(z,t) + \mathbf{B}_1\mathbf{f}_0(z) - \bar{\mathbf{B}}_1\bar{\mathbf{f}}_0(z) + \left(\frac{2}{1+\Gamma}\bar{\mathbf{d}}_2 + \frac{\Gamma-1}{\Gamma+1}\mathbf{d}_1 - \bar{\mathbf{d}}_1\right)\bar{g}_0(z),\\ \bar{\mathbf{B}}_1\bar{\mathbf{f}}_1(z,t) &= \mathbf{B}_2\mathbf{f}_2(z,t) - \mathbf{B}_1\mathbf{f}_0(z) + \bar{\mathbf{B}}_1\bar{\mathbf{f}}_0(z) + \left(\frac{2}{1+\Gamma}\mathbf{d}_2 + \frac{\Gamma-1}{\Gamma+1}\bar{\mathbf{d}}_1 - \mathbf{d}_1\right) g_0(z).\end{aligned} \tag{25}$$

Inserting the above result into Eq. (22) and applying the Liouville's theorem will finally lead to the following set of partial differential equations:

$$\mathbf{N}\mathbf{B}_2\dot{\mathbf{f}}_2(z,t) + \mathrm{i}\mathbf{\Lambda}\left[\mathbf{B}_2\mathbf{f}_2'(z,t) + \mathbf{d}_2 g_2'(z)\right] = \mathbf{0}, \quad \mathrm{Im}\{z\} < 0, \tag{26}$$

where $\mathbf{N}$ is a 5×5 Hermitian matrix given by

$$\begin{aligned}\mathbf{N} &= \bar{\mathbf{M}}_1^{-1} + \mathbf{M}_2^{-1} = \mathbf{L}_1^{-1} + \mathbf{L}_2^{-1} + \mathrm{i}(\mathbf{S}_1\mathbf{L}_1^{-1} - \mathbf{S}_2\mathbf{L}_2^{-1}),\\ \mathbf{M}_k^{-1} &= \mathrm{i}\mathbf{A}_k\mathbf{B}_k^{-1} = (\mathbf{I} - \mathrm{i}\mathbf{S}_k)\mathbf{L}_k^{-1}, \quad (k = 1, 2).\end{aligned} \tag{27}$$

As in [4], the 5×5 Hermitian matrix $\mathbf{N}$ can be more explicitly written as follows:

$$\mathbf{N} = \bar{\mathbf{N}}^T = \begin{bmatrix} N_{11} & N_{12} & N_{13} & N_{14} & N_{15}\\ \bar{N}_{12} & N_{22} & N_{23} & N_{24} & N_{25}\\ \bar{N}_{13} & \bar{N}_{23} & N_{33} & N_{34} & N_{35}\\ \bar{N}_{14} & \bar{N}_{24} & \bar{N}_{34} & N_{44} & N_{45}\\ \bar{N}_{15} & \bar{N}_{25} & \bar{N}_{35} & \bar{N}_{45} & N_{55}\end{bmatrix}. \tag{28}$$

Even though Eq. (26) is a little bit different from its isothermal counterpart [4], the decoupling methodology adopted in [4] is still valid here. We therefore consider the eigenvalue problem [4]

$$(\mathbf{\Lambda} - \lambda \mathbf{N})\mathbf{v} = \mathbf{0}. \tag{29}$$

It is observed that in total there exist five eigenvalues to the above eigenvalue problem. Furthermore, these five eigenvalues λ_i $(i = 1 - 5)$ can be explicitly determined as

$$\begin{aligned} \lambda_1 &= \frac{a_1 + \sqrt{a_1^2 - 4a_0a_2}}{2a_2} > 0, \\ \lambda_2 &= \frac{a_1 - \sqrt{a_1^2 - 4a_0a_2}}{2a_2} > 0, \\ \lambda_3 &= \lambda_4 = \lambda_5 = 0, \end{aligned} \tag{30}$$

where

$$a_2 = |\mathbf{N}|, \quad a_1 = \frac{\widehat{N}_{11}}{\eta_1} + \frac{\widehat{N}_{33}}{\eta_3}, \quad a_0 = \frac{1}{\eta_1\eta_3}\begin{vmatrix} N_{22} & N_{24} & N_{25} \\ \bar{N}_{24} & N_{44} & N_{45} \\ \bar{N}_{25} & \bar{N}_{45} & N_{55} \end{vmatrix}, \tag{31}$$

with $\widehat{N}_{ij}$ denoting the cofactors of the matrix $\mathbf{N}$.

We specially choose the eigenvectors associated with the three zero eigenvalues $\lambda_3 = \lambda_4 = \lambda_5 = 0$ as

$$\mathbf{v}_3 = \begin{bmatrix} 0 \\ 1 \\ 0 \\ 0 \\ 0 \end{bmatrix}, \quad \mathbf{v}_4 = \begin{bmatrix} 0 \\ -N_{24} \\ 0 \\ N_{22} \\ 0 \end{bmatrix}, \quad \mathbf{v}_5 = \begin{bmatrix} 0 \\ N_{24}N_{45} - N_{25}N_{44} \\ 0 \\ N_{25}\bar{N}_{24} - N_{45}N_{22} \\ N_{22}N_{44} - N_{24}\bar{N}_{24} \end{bmatrix}, \tag{32}$$

so that the following orthogonal relationships with respect to the Hermitian matrix $\mathbf{N}$ and to the real and diagonal matrix $\mathbf{\Lambda}$ hold:

$$\begin{aligned} \bar{\mathbf{\Psi}}^T \mathbf{N} \mathbf{\Psi} &= \mathbf{\Lambda}_0 = \mathrm{diag}\left[\, \delta_1 \ \delta_2 \ \delta_3 \ \delta_4 \ \delta_5 \,\right], \\ \bar{\mathbf{\Psi}}^T \mathbf{\Lambda} \mathbf{\Psi} &= \mathrm{diag}\left[\, \lambda_1\delta_1 \ \lambda_2\delta_2 \ \lambda_3\delta_3 \ \lambda_4\delta_4 \ \lambda_5\delta_5 \,\right], \end{aligned} \tag{33}$$

where $\delta_k = \bar{\mathbf{v}}_k^T \mathbf{N} \mathbf{v}_k (k = 1 - 5)$ are nonzero real values and

$$\mathbf{\Psi} = \left[\, \mathbf{v}_1 \ \mathbf{v}_2 \ \mathbf{v}_3 \ \mathbf{v}_4 \ \mathbf{v}_5 \,\right]. \tag{34}$$

We now introduce the new analytic function vector $\mathbf{\Omega}(z, t) = [\Omega_1(z, t)\ \Omega_2(z, t)\ \Omega_3(z, t)\ \Omega_4(z, t)\ \Omega_5(z, t)]^T$ such that

$$\mathbf{\Psi}\mathbf{\Omega}(z, t) = \mathbf{B}_2\mathbf{f}_2(z, t) + \mathbf{d}_2 g_2(z), \tag{35}$$

where

$$\mathbf{\Psi} = \left[\, \mathbf{v}_1 \ \mathbf{v}_2 \ \mathbf{v}_3 \ \mathbf{v}_4 \ \mathbf{v}_5 \,\right]. \tag{36}$$

Then Eq. (26) can be decoupled into

$$\dot{\Omega}_k(z, t) + \mathrm{i}\lambda_k \Omega_k'(z, t) = 0, \quad k = 1 - 5, \quad \mathrm{Im}\ \{z\} < 0, \tag{37}$$

whose solutions can be conveniently written into

$$\Omega_k(z, t) = \Omega_k(z - \mathrm{i}\lambda_k t, 0), \quad k = 1 - 5, \quad \mathrm{Im}\ \{z\} < 0, \tag{38}$$

which implies that once the initial value $\Omega_k(z,0)$ is known, it is enough to replace z by $z - \mathrm{i}\lambda_k t$ in $\Omega_k(z,0)$ to arrive at the value $\Omega_k(z,t)$ at any time.

Due to the fact that at the initial moment $t = 0$ the interface is perfect (see Eq. (15)), we can obtain the following expression [10]:

$$\mathbf{B}_2\mathbf{f}_2(z,0) + \mathbf{d}_2 g_2(z) = 2\mathbf{N}^{-1}\mathbf{L}_1^{-1}\mathbf{B}_1\mathbf{f}_0(z) + \left(\frac{2}{1+\Gamma}\mathbf{d}_2 - \mathbf{N}^{-1}\mathbf{G}_1\right) g_0(z), \quad \mathrm{Im}\,\{z\} < 0, \tag{39}$$

where

$$\mathbf{G}_1 = \frac{2}{1+\Gamma}\left(\mathrm{i}\mathbf{c}_2 + \bar{\mathbf{M}}_1^{-1}\mathbf{d}_2\right) + \frac{\Gamma-1}{\Gamma+1}\left(\mathrm{i}\bar{\mathbf{c}}_1 + \bar{\mathbf{M}}_1^{-1}\bar{\mathbf{d}}_1\right) - \mathrm{i}\mathbf{c}_1 - \bar{\mathbf{M}}_1^{-1}\mathbf{d}_1. \tag{40}$$

In view of Eqs. (38) and (39), the solution of $\mathbf{\Omega}(z,t)$ can then be easily obtained as

$$\begin{aligned}\mathbf{\Omega}(z,t) = {} & 2\sum_{k=1}^{5}\left\langle (z - \mathrm{i}\lambda_\alpha t - \hat{z}_k)\ln(z - \mathrm{i}\lambda_\alpha t - \hat{z}_k)\right\rangle \mathbf{\Psi}^{-1}\mathbf{N}^{-1}\mathbf{L}_1^{-1}\mathbf{B}_1\mathbf{I}_k\mathbf{e} \\ & + \left\langle (z - \mathrm{i}\lambda_\alpha t - \hat{z}_0)\left[\ln(z - \mathrm{i}\lambda_\alpha t - \hat{z}_0) - 1\right]\right\rangle \mathbf{\Psi}^{-1}\left(\frac{2}{1+\Gamma}\mathbf{d}_2 - \mathbf{N}^{-1}\mathbf{G}_1\right) X_0,\end{aligned} \tag{41}$$

where

$$\begin{aligned}&\mathbf{I}_1 = \mathrm{diag}\left[1\ 0\ 0\ 0\ 0\right], \quad \mathbf{I}_2 = \mathrm{diag}\left[0\ 1\ 0\ 0\ 0\right], \quad \mathbf{I}_3 = \mathrm{diag}\left[0\ 0\ 1\ 0\ 0\right], \\ &\mathbf{I}_4 = \mathrm{diag}\left[0\ 0\ 0\ 1\ 0\right], \quad \mathbf{I}_5 = \mathrm{diag}\left[0\ 0\ 0\ 0\ 1\right].\end{aligned} \tag{42}$$

Consequently we can obtain the expressions of $\mathbf{f}_1(z,t)$ and $\mathbf{f}_2(z,t)$. It is not difficult to write down the full field solutions for $\mathbf{f}_1(z,t)$ and $\mathbf{f}_2(z,t)$ as follows:

$$\begin{aligned}\mathbf{f}_1(z,t) = {} & 2\sum_{m=1}^{5}\sum_{k=1}^{5}\left\langle (z_\alpha + \mathrm{i}\lambda_m t - \overline{\hat{z}_k})\ln(z + \mathrm{i}\lambda_m t - \overline{\hat{z}_k})\right\rangle \mathbf{B}_1^{-1}\bar{\mathbf{\Psi}}\mathbf{I}_m\bar{\mathbf{\Psi}}^{-1}\bar{\mathbf{N}}^{-1}\mathbf{L}_1^{-1}\bar{\mathbf{B}}_1\mathbf{I}_k\bar{\mathbf{e}} \\ & + \sum_{k=1}^{5}\left\langle (z_\alpha + \mathrm{i}\lambda_k t - \overline{\hat{z}_0})\left[\ln(z_\alpha + \mathrm{i}\lambda_k t - \overline{\hat{z}_0}) - 1\right]\right\rangle \mathbf{B}_1^{-1}\bar{\mathbf{\Psi}}\mathbf{I}_k\bar{\mathbf{\Psi}}^{-1}\left(\frac{2}{1+\Gamma}\bar{\mathbf{d}}_2 - \bar{\mathbf{N}}^{-1}\bar{\mathbf{G}}_1\right)\bar{X}_0 \\ & + \left\langle (z_\alpha - \hat{z}_\alpha)\ln(z_\alpha - \hat{z}_\alpha)\right\rangle \mathbf{e} - \sum_{k=1}^{5}\left\langle (z_\alpha - \overline{\hat{z}_k})\ln(z_\alpha - \overline{\hat{z}_k})\right\rangle \mathbf{B}_1^{-1}\bar{\mathbf{B}}_1\mathbf{I}_k\bar{\mathbf{e}} \\ & + \left\langle (z_\alpha - \overline{\hat{z}_0})\left[\ln(z_\alpha - \overline{\hat{z}_0}) - 1\right]\right\rangle \bar{X}_0\mathbf{B}_1^{-1}\left(\frac{\Gamma-1}{\Gamma+1}\mathbf{d}_1 - \bar{\mathbf{d}}_1\right),\end{aligned} \tag{43}$$

$$\begin{aligned}\mathbf{f}_2(z,t) = {} & 2\sum_{m=1}^{5}\sum_{k=1}^{5}\left\langle (z_\alpha^* - \mathrm{i}\lambda_m t - \hat{z}_k)\ln(z_\alpha^* - \mathrm{i}\lambda_m t - \hat{z}_k)\right\rangle \mathbf{B}_2^{-1}\mathbf{\Psi}\mathbf{I}_m\mathbf{\Psi}^{-1}\mathbf{N}^{-1}\mathbf{L}_1^{-1}\mathbf{B}_1\mathbf{I}_k\mathbf{e} \\ & + \sum_{k=1}^{5}\left\langle (z_\alpha^* - \mathrm{i}\lambda_k t - \hat{z}_0)\left[\ln(z_\alpha^* - \mathrm{i}\lambda_k t - \hat{z}_0) - 1\right]\right\rangle \mathbf{B}_2^{-1}\mathbf{\Psi}\mathbf{I}_k\mathbf{\Psi}^{-1}\left(\frac{2}{1+\Gamma}\mathbf{d}_2 - \mathbf{N}^{-1}\mathbf{G}_1\right) X_0 \\ & - \left\langle (z_\alpha^* - \hat{z}_0)\left[\ln(z_\alpha^* - \hat{z}_0) - 1\right]\right\rangle \frac{2X_0}{1+\Gamma}\mathbf{B}_2^{-1}\mathbf{d}_2,\end{aligned} \tag{44}$$

where the superscript '*' is also utilized to distinguish the Stroh eigenvalues associated with the lower half-plane (z_α^*) from those associated with the upper half-plane (z_α). Once $\mathbf{f}_1(z,t)$ and $\mathbf{f}_2(z,t)$ are known, all the

field components can be obtained by using Eq. (3). For example the tractions, normal electric displacement and normal magnetic induction are distributed along the interface $x_2 = 0$ as

$$\begin{aligned}&\left[\sigma_{12}\ \sigma_{22}\ \sigma_{32}\ D_2\ B_2\right]^T\\&= 4\mathrm{Re}\left\{\begin{array}{l}\mathbf{\Psi}\sum\limits_{k=1}^{5} < \ln(x_1 - \mathrm{i}\lambda_\alpha t - \hat{z}_k) > \mathbf{\Psi}^{-1}\mathbf{N}^{-1}\mathbf{L}_1^{-1}\mathbf{B}_1\mathbf{I}_k\mathbf{e}\\+\mathbf{N}^{-1}\mathbf{L}_1^{-1}\mathbf{B}_1\mathbf{e} + \mathbf{\Psi} < \ln(x_1 - \mathrm{i}\lambda_\alpha t - \hat{z}_0) > \mathbf{\Psi}^{-1}\left(\frac{1}{1+\Gamma}\mathbf{d}_2 - \frac{1}{2}\mathbf{N}^{-1}\mathbf{G}_1\right)X_0\end{array}\right\},\\&\qquad -\infty < x_1 < +\infty,\ t \geq 0.\end{aligned} \tag{45}$$

4 Conclusions

By means of the generalized version of the Stroh formalism, we have derived the time-dependent thermal Green's functions for a steady line heat source and a temperature dislocation located in the upper half-plane of an anisotropic multiferroic bimaterial with a planar viscous interface. The original boundary value problem was finally reduced to a set of partial differential equations (26) whose solutions can easily be obtained by using a decoupling methodology [4]. The full-field solutions for $\mathbf{f}_1(z, t)$ and $\mathbf{f}_2(z, t)$ in Eqs. (43) and (45), together with the complex thermal potentials $g_1'(z_0)$ and $g_2'(z_0^*)$ in Eq. (18) are enough to determine the time-dependent magneto-electro-thermo-elastic responses in the two multiferroic half-planes induced by the line heat source and the temperature dislocation.

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