(This is a sample cover image for this issue. The actual cover is not yet available at this time.)

Mechanics Research Communications 38 (2011) 518–523

Contents lists available at ScienceDirect

Mechanics Research Communications

journal homepage: www.elsevier.com/locate/mechrescom

Large deflection of a rectangular magnetoelectroelastic thin plate

C.X. Xue[a,b], E. Pan[b,*], S.Y. Zhang[c], H.J. Chu[b,d]

[a] Department of Mechanics, College of Science, North University of China, Taiyuan 030051, PR China
[b] Department of Civil Engineering, University of Akron, Akron 44325, USA
[c] Institute of Applied Mechanics, Taiyuan University of Technology, Taiyuan 030024, PR China
[d] College of Hydraulic Science and Engineering, Yangzhou University, Yangzhou 225009, PR China

ARTICLE INFO

Article history:
Received 26 January 2011
Received in revised form 21 May 2011

Keywords:
Large deflection
Magnetoelectroelastic
Coupling factor
Von Karman plate theory
Bubnov–Galerkin method

ABSTRACT

Based on the von Karman plate theory of large deflection, we derive the nonlinear partial differential equation for a rectangular magnetoelectroelastic thin plate under the action of a transverse static mechanical load. By employing the Bubnov–Galerkin method, the nonlinear partial differential equation is transformed to a third-order nonlinear algebraic equation for the maximum deflection where a coupling factor is introduced for determining the coupling effect on the deflection. Numerical results are carried out for the thin plate made of piezoelectric $BaTiO_3$ and piezomagnetic $CoFe_2O_4$ materials. Some interesting results are obtained which could be useful to future analysis and design of multiphase composite plates.

1. Introduction

Materials made of piezoelectric and piezomagnetic phases have the ability of converting energy from one form to the other (among magnetic, electric, and mechanical energies). Such materials are currently being used in ultrasound medical devices, automobiles, and aircrafts, among many other areas. Furthermore, composites made of piezoelectric and piezomagnetic materials exhibit a magnetoelectric effect that is not present in the single-phase piezoelectric or piezomagnetic material (Harshe et al., 1993; Nan, 1994; Benveniste, 1995). Avellaneda and Harshe (1994) considered the magnetoelectric effect in multilayer composites. Li and Dunn (1998) investigated the inclusion and inhomogeneity problems in magnetoelectroelastic (MEE) composites. Pan (2001) derived an exact closed-form solution for the simply supported and multilayered plate made of anisotropic piezoelectric and piezomagnetic materials under a static mechanical load. By introducing five potential functions, Wang and Shen (2002) obtained the general solution of the three-dimensional problem in a transversely isotropic MEE media. Chen et al. (2002) established a micromechanical model for the evaluation of the effective properties in layered composites with piezoelectric and piezomagnetic phases. Crack problems were also investigated in order to obtain the crack-tip stress, electric, and magnetic field intensity factors (Feng et al., 2007a,b, 2008; Tupholme, 2008). Recently, Wu et al. (2010) extended the Pagano method for the three-dimensional plate problem to the analysis of a simply-supported, functionally graded rectangular plate under magneto-electro-mechanical loads. Liu and Chang (2010) studied the vibration of a MEE rectangular plate. So far, however, the nonlinear behavior of a MEE plate has not been investigated.

This paper thus proposes a nonlinear (or large-deflection) model for the MEE thin plate deformation under a static mechanical load. Based on the proposed model, the corresponding solutions will be also derived for a simply-supported rectangular MEE plate and numerical results will be further given. This paper is organized as follows. The basic equations are presented in Section 2; The large deflection solution is derived in Section 3; Numerical results are given in Section 4 and conclusions are drawn in Section 5.

2. Basic equations

We consider a rectangular transversely isotropic MEE thin plate in the Cartesian coordinate system (x, y, z), as shown in Fig. 1. The length, width and thickness of the plate are, respectively, L_x, L_y, and h. The coordinate plane Oxy is attached to the middle plane of the plate with a static mechanical load q in z-direction. If z-axis is

* Corresponding author. Tel.: +1 330 972 6739; fax: +1 330 972 6020.
E-mail addresses: xuechunxia@nuc.edu.cn (C.X. Xue), pan2@uakron.edu (E. Pan), syzhang@tyut.edu.cn (S.Y. Zhang), hjchu@yzu.edu.cn (H.J. Chu).

0093-6413/$ – see front matter © 2011 Elsevier Ltd. All rights reserved.
doi:10.1016/j.mechrescom.2011.07.003

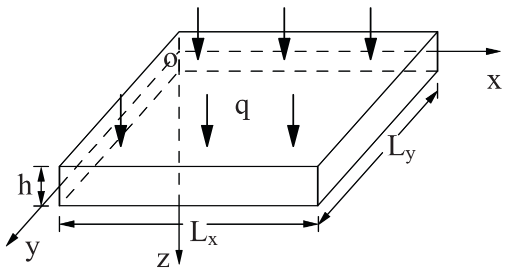

Fig. 1. A rectangular MEE thin plate under the mechanical load q.

normal to the material plane of isotropy, the constitutive relations can be written as (Pan, 2001; Liu and Chang, 2010)

$$\begin{aligned}
\sigma_x &= c_{11}\varepsilon_x + c_{12}\varepsilon_y + c_{13}\varepsilon_z + e_{31}\frac{\partial\phi}{\partial z} + q_{31}\frac{\partial\psi}{\partial z}\\
\sigma_y &= c_{12}\varepsilon_x + c_{11}\varepsilon_y + c_{13}\varepsilon_z + e_{31}\frac{\partial\phi}{\partial z} + q_{31}\frac{\partial\psi}{\partial z}\\
\sigma_z &= c_{13}\varepsilon_x + c_{13}\varepsilon_y + c_{33}\varepsilon_z + e_{33}\frac{\partial\phi}{\partial z} + q_{33}\frac{\partial\psi}{\partial z}\\
\tau_{xz} &= c_{44}\gamma_{xz} + e_{15}\frac{\partial\phi}{\partial x} + q_{15}\frac{\partial\psi}{\partial x}\\
\tau_{yz} &= c_{44}\gamma_{yz} + e_{15}\frac{\partial\phi}{\partial y} + q_{15}\frac{\partial\psi}{\partial y}\\
\tau_{xy} &= c_{66}\gamma_{xy}
\end{aligned}\tag{1}$$

$$\begin{aligned}
D_x &= e_{15}\gamma_{xz} - \kappa_{11}\frac{\partial\phi}{\partial x} - d_{11}\frac{\partial\psi}{\partial x}\\
D_y &= e_{15}\gamma_{yz} - \kappa_{11}\frac{\partial\phi}{\partial y} - d_{11}\frac{\partial\psi}{\partial y}\\
D_z &= e_{31}\varepsilon_x + e_{31}\varepsilon_y + e_{33}\varepsilon_z - \kappa_{33}\frac{\partial\phi}{\partial z} - d_{33}\frac{\partial\psi}{\partial z}
\end{aligned}\tag{2}$$

$$\begin{aligned}
B_x &= q_{15}\gamma_{xz} - d_{11}\frac{\partial\phi}{\partial x} - \mu_{11}\frac{\partial\psi}{\partial x}\\
B_y &= q_{15}\gamma_{yz} - d_{11}\frac{\partial\phi}{\partial y} - \mu_{11}\frac{\partial\psi}{\partial y}\\
B_z &= q_{31}\varepsilon_x + q_{31}\varepsilon_y + q_{33}\varepsilon_z - d_{33}\frac{\partial\phi}{\partial z} - \mu_{33}\frac{\partial\psi}{\partial z}
\end{aligned}\tag{3}$$

where σ_{ij} and τ_{ij} are the normal and shear stresses, and ε_i and γ_{ij} are the normal and shear strains; ϕ, ψ, D_i and B_i are, respectively, the electric potential, magnetic potential, electric displacement components, and magnetic induction components; c_{ij}, κ_{ij}, e_{ij}, q_{ij}, d_{ij} and μ_{ij} are, respectively, the elastic, dielectric, piezoelectric, piezomagnetic, magnetoelectric, and magnetic constants. For a transversely isotropic material, the relation $c_{11} = c_{12} + 2c_{66}$ holds.

The equations of equilibrium (including the balances of the electric and magnetic quantities) are

$$\begin{aligned}
&\frac{\partial\sigma_x}{\partial x} + \frac{\partial\tau_{xy}}{\partial y} + \frac{\partial\tau_{xz}}{\partial z} = 0\\
&\frac{\partial\tau_{xy}}{\partial x} + \frac{\partial\sigma_y}{\partial y} + \frac{\partial\tau_{yz}}{\partial z} = 0\\
&\frac{\partial\tau_{xz}}{\partial x} + \frac{\partial\tau_{yz}}{\partial y} + \frac{\partial\sigma_z}{\partial z} = 0
\end{aligned}\tag{4}$$

$$\frac{\partial D_x}{\partial x} + \frac{\partial D_y}{\partial y} + \frac{\partial D_z}{\partial z} = 0 \tag{5}$$

$$\frac{\partial B_x}{\partial x} + \frac{\partial B_y}{\partial y} + \frac{\partial B_z}{\partial z} = 0 \tag{6}$$

The following basic assumptions are made (see, e.g., Chia, 1980):

1. The plate is thin; i.e., the thickness h is much smaller than the other physical dimensions (thus the normal stress component σ_z is negligible).
2. In order to include in-plane force effects, nonlinear terms in the equations of equilibrium involving products of stresses and plate slopes are retained. All other nonlinear terms are neglected.
3. Points of the plate lying initially on a normal-to-the-middle plane of the plate remain on the normal-to-the-middle plane after deformation. This means that the vertical shear strains γ_{xz} and γ_{yz} are negligible. The deflection of the plate is thus associated principally with the bending strain. It is deduced, therefore, that the normal strain ε_z, resulting from transverse loading, may also be omitted.

Based on these assumptions, the displacement components (u, v, w) should satisfy the following differential relations

$$\frac{\partial w}{\partial z} = 0, \qquad \frac{\partial w}{\partial x} + \frac{\partial u}{\partial z} = 0, \qquad \frac{\partial w}{\partial y} + \frac{\partial v}{\partial z} = 0 \tag{7}$$

In addition, if the MEE plate is thin, the in-plane electric fields and magnetic fields can be ignored, i.e., $E_x = E_y = 0$ and $H_x = H_y = 0$ (Liu and Chang, 2010). This means that only the transverse electric field component E_z and magnetic field component H_z need to be considered in the present study. According to the gradient relations between the electric/magnetic fields and the electric/magnetic potentials, we have:

$$\frac{\partial\phi}{\partial x} = 0, \qquad \frac{\partial\phi}{\partial y} = 0, \qquad E_z = \frac{-\partial\phi}{\partial z}, \tag{8}$$

$$\frac{\partial\psi}{\partial x} = 0, \qquad \frac{\partial\psi}{\partial y} = 0, \qquad H_z = \frac{-\partial\psi}{\partial z}, \tag{9}$$

Therefore, the relations between the stress and strain components for the case of plane stress in the x–y plane can be represented by the following equations:

$$\begin{aligned}
\sigma_x &= c_{11}\varepsilon_x + c_{12}\varepsilon_y + e_{31}\frac{\partial\phi}{\partial z} + q_{31}\frac{\partial\psi}{\partial z}\\
\sigma_y &= c_{12}\varepsilon_x + c_{11}\varepsilon_y + e_{31}\frac{\partial\phi}{\partial z} + q_{31}\frac{\partial\psi}{\partial z}\\
\tau_{xy} &= c_{66}\gamma_{xy}
\end{aligned}\tag{10}$$

$$D_z = e_{31}\varepsilon_x + e_{31}\varepsilon_y - \kappa_{33}\frac{\partial\phi}{\partial z} - d_{33}\frac{\partial\psi}{\partial z} \tag{11}$$

$$B_z = q_{31}\varepsilon_x + q_{31}\varepsilon_y - d_{33}\frac{\partial\phi}{\partial z} - \mu_{33}\frac{\partial\psi}{\partial z} \tag{12}$$

Using the von Karman's theory of large deflection of plates, the Lagrangian strain displacement relations are given by Timoshenko and Woinowsky-Krieger (1959)

$$\begin{aligned}
\varepsilon_x &= \frac{\partial u}{\partial x} + \frac{1}{2}\left(\frac{\partial w}{\partial x}\right)^2\\
\varepsilon_y &= \frac{\partial v}{\partial y} + \frac{1}{2}\left(\frac{\partial w}{\partial y}\right)^2\\
\gamma_{xy} &= \frac{\partial v}{\partial x} + \frac{\partial u}{\partial y} + \frac{\partial w}{\partial x}\cdot\frac{\partial w}{\partial y}
\end{aligned}\tag{13}$$

where again u, v, and w are the elastic displacement components in x-, y-, and z-directions, respectively.

3. Large deflection solutions

Based on the discussion in Section 2, we can assume the unknown elastic displacement vector $\boldsymbol{u}$ as (see, e.g., Chia, 1980):

$$\mathbf{u} = \left(u_0(x,y) - z\frac{\partial w(x,y)}{\partial x}\right)\mathbf{i} + \left(v_0(x,y) - z\frac{\partial w(x,y)}{\partial y}\right)\mathbf{j} + w(x,y)\mathbf{k} \tag{14}$$

where u_0 and v_0 are the tangential displacements in the middle-plane, and again w the displacement in the z-direction of the middle plane of the plate (or the deflection of the plate). Then, substituting Eqs. (11), (12), (13) and (14) into Eqs. (5) and (6) gives

$$-e_{31}\frac{\partial^2 w}{\partial x^2} - e_{31}\frac{\partial^2 w}{\partial y^2} - \kappa_{33}\frac{\partial^2 \phi}{\partial z^2} - d_{33}\frac{\partial^2 \psi}{\partial z^2} = 0 \tag{15}$$

$$-q_{31}\frac{\partial^2 w}{\partial x^2} - q_{31}\frac{\partial^2 w}{\partial y^2} - d_{33}\frac{\partial^2 \phi}{\partial z^2} - \mu_{33}\frac{\partial^2 \psi}{\partial z^2} = 0 \tag{16}$$

These two equations can be further written as

$$\kappa_{33}\frac{\partial^2 \phi}{\partial z^2} + d_{33}\frac{\partial^2 \psi}{\partial z^2} = -e_{31}\left(\frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial y^2}\right) \equiv -e_{31}\nabla^2 w \tag{17}$$

$$d_{33}\frac{\partial^2 \phi}{\partial z^2} + \mu_{33}\frac{\partial^2 \psi}{\partial z^2} = -q_{31}\left(\frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial y^2}\right) \equiv -q_{31}\nabla^2 w \tag{18}$$

Thus, we can solve the electric and magnetic potentials in terms of the deflection of the plate

$$\frac{\partial^2 \phi}{\partial z^2} = -\frac{\Delta_1}{\Delta}\nabla^2 w, \qquad \frac{\partial^2 \psi}{\partial z^2} = -\frac{\Delta_2}{\Delta}\nabla^2 w \tag{19}$$

where

$$\begin{aligned}
\nabla^2 &= \frac{\partial^2}{\partial^2 x} + \frac{\partial^2}{\partial^2 y} \\
\Delta &= \begin{vmatrix} \kappa_{33} & d_{33} \\ d_{33} & \mu_{33} \end{vmatrix} = \kappa_{33}\mu_{33} - d_{33}^2 \\
\Delta_1 &= \begin{vmatrix} e_{31} & d_{33} \\ q_{31} & \mu_{33} \end{vmatrix} = e_{31}\mu_{33} - d_{33}q_{31} \\
\Delta_2 &= \begin{vmatrix} \kappa_{33} & e_{31} \\ d_{33} & q_{31} \end{vmatrix} = q_{31}\kappa_{33} - d_{33}e_{31}
\end{aligned} \tag{20}$$

It can be derived from Eq. (19) that

$$\begin{aligned}
\frac{\partial \phi(x,y,z)}{\partial z} &= -\frac{\Delta_1}{\Delta}z\nabla^2 w(x,y) + \phi_0(x,y) \\
\frac{\partial \psi(x,y,z)}{\partial z} &= -\frac{\Delta_2}{\Delta}z\nabla^2 w(x,y) + \psi_0(x,y)
\end{aligned} \tag{21}$$

where $\phi_0(x,y)$ and $\psi_0(x,y)$ are functions that are independent of variable z.

By taking the second derivatives of Eq. (13) and combing the resulting expressions, it can be shown that

$$\frac{\partial^2 \varepsilon_x}{\partial y^2} + \frac{\partial^2 \varepsilon_y}{\partial x^2} - \frac{\partial^2 \gamma_{xy}}{\partial x \partial y} = \left(\frac{\partial^2 w}{\partial x \partial y}\right)^2 - \frac{\partial^2 w}{\partial x^2}\frac{\partial^2 w}{\partial y^2} \tag{22}$$

where

$$\begin{aligned}
\varepsilon_x &= \frac{c_{12}\sigma_y - c_{11}\sigma_x}{c_{12}^2 - c_{11}^2} - \frac{e_{31}}{c_{12}+c_{11}}\frac{\partial \phi}{\partial z} - \frac{q_{31}}{c_{12}+c_{11}}\frac{\partial \psi}{\partial z} \\
\varepsilon_y &= \frac{c_{12}\sigma_x - c_{11}\sigma_y}{c_{12}^2 - c_{11}^2} - \frac{e_{31}}{c_{12}+c_{11}}\frac{\partial \phi}{\partial z} - \frac{q_{31}}{c_{12}+c_{11}}\frac{\partial \psi}{\partial z}
\end{aligned} \tag{23}$$

The solution of this equation can be greatly simplified by introducing the stress function

$$\sigma_x = \frac{\partial^2 F}{\partial y^2} + e_{31}\frac{\partial \phi}{\partial z} + q_{31}\frac{\partial \psi}{\partial z}, \qquad \sigma_y = \frac{\partial^2 F}{\partial x^2} + e_{31}\frac{\partial \phi}{\partial z} + q_{31}\frac{\partial \psi}{\partial z},$$
$$\tau_{xy} = -\frac{\partial^2 F}{\partial x \partial y} \tag{24}$$

where F is the Airy stress function of x and y. Substituting Eq. (24) into Eq. (23), then into Eq. (22), we obtain

$$\frac{c_{11}}{c_{11}^2 - c_{12}^2}\cdot\left(\frac{\partial^4 F}{\partial x^4} + \frac{\partial^4 F}{\partial y^4}\right) + \left(\frac{1}{c_{66}} - \frac{2c_{12}}{c_{11}^2 - c_{12}^2}\right)\cdot\frac{\partial^4 F}{\partial x^2 \partial y^2} = -\frac{1}{2}L(w,w) \tag{25}$$

where we have introduced the operator $L(\alpha, \beta) = (\partial^2\alpha/\partial x^2)(\partial^2\beta/\partial y^2) + (\partial^2\alpha/\partial y^2)(\partial^2\beta/\partial x^2) - 2(\partial^2\alpha/\partial x\partial y)(\partial^2\beta/\partial x\partial y)$, so that

$$L(w,w) = 2\frac{\partial^2 w}{\partial x^2}\frac{\partial^2 w}{\partial y^2} - 2\left(\frac{\partial^2 w}{\partial x \partial y}\right)^2 \tag{26}$$

We now consider a simply-supported plate. For this case, the mechanical boundary conditions on the sides of the plate are

$$\begin{aligned}
x = 0, x = L_x : w = 0, \ \frac{\partial^2 w}{\partial x^2} = 0 \\
y = 0, y = L_y : w = 0, \ \frac{\partial^2 w}{\partial y^2} = 0
\end{aligned} \tag{27}$$

The elastic displacement w can be assumed as:

$$w(x,y) = hW_m \cdot \sin\left(\frac{\pi x}{L_x}\right)\sin\left(\frac{\pi y}{L_y}\right) \tag{28}$$

in which W_m is the maximum nondimensional deflection at the center of the plate.

Substituting Eq. (28) into Eq. (25), we obtain

$$F = \frac{1}{32}\frac{c_{11}^2 - c_{12}^2}{c_{11}}h^2 W_m^2\left[\left(\frac{L_x}{L_y}\right)^2\cos\frac{2\pi x}{L_x} + \left(\frac{L_y}{L_x}\right)^2\cos\frac{2\pi y}{L_y}\right] \tag{29}$$

Recalling that the resultant force and moment are defined as follows

$$\begin{aligned}
N_x &= \int_{-h/2}^{h/2}\sigma_x\, dz, \quad N_y = \int_{-h/2}^{h/2}\sigma_y\, dz, \quad N_{xy} = \int_{-h/2}^{h/2}\tau_{xy}\, dz \\
M_x &= \int_{-h/2}^{h/2}\sigma_x z dz, \quad M_y = \int_{-h/2}^{h/2}\sigma_y z dz, \quad M_{xy} = \int_{-h/2}^{h/2}\tau_{xy} z dz
\end{aligned} \tag{30}$$

we can now integrate Eq. (4) with respect to z to arrive at the following equation of static equilibrium:

$$\frac{\partial^2 M_x}{\partial x^2} + 2\frac{\partial^2 M_{xy}}{\partial x \partial y} + \frac{\partial^2 M_y}{\partial y^2} + N_x\frac{\partial^2 w}{\partial x^2} + N_y\frac{\partial^2 w}{\partial y^2} + 2N_{xy}\frac{\partial^2 w}{\partial x \partial y} + q = 0 \tag{31}$$

where q is density of the load which is assumed to be uniform over the surface of the plate.

Substituting Eq. (24) into Eq. (30), then into the equation of equilibrium (31) gives us

$$\frac{h^3}{12}\left(c_{11}\left(\frac{\partial^4 w}{\partial x^4} + \frac{\partial^4 w}{\partial y^4}\right) + 2(c_{12} + 2c_{66})\frac{\partial^4 w}{\partial x^2 \partial y^2} + e_{31}\frac{\Delta_1}{\Delta}\nabla^4 w + q_{31}\frac{\Delta_2}{\Delta}\nabla^4 w\right) = hL(w,F) + q \tag{32}$$

where

$$\nabla^4 = \frac{\partial^4 w}{\partial x^4} + \frac{\partial^4 w}{\partial y^4} + 2\frac{\partial^4 w}{\partial x^2 \partial y^2} \tag{33}$$

Substituting Eqs. (28), (29) into Eq. (32), and noticing that $c_{11} = c_{12} + 2c_{66}$, we have

$$\Phi(w) \equiv \frac{h^3}{12}\left[\left(\frac{\pi}{L_x}\right)^2 + \left(\frac{\pi}{L_y}\right)^2\right]^2 \left(c_{11} + e_{31}\frac{\Delta_1}{\Delta} + q_{31}\frac{\Delta_2}{\Delta}\right) \cdot hW_m \cdot \sin\frac{\pi x}{L_x}\sin\frac{\pi y}{L_y}$$
$$-\frac{c_{11}^2 - c_{12}^2}{c_{11}}\left[\frac{h}{8}\left(\frac{\pi}{L_x}\right)^4 \cos\frac{2\pi y}{L_y} + \frac{h}{8}\left(\frac{\pi}{L_y}\right)^4 \cos\frac{2\pi x}{L_x}\right] \cdot h^3 {W_m}^3 \cdot \sin\frac{\pi x}{L_x}\sin\frac{\pi y}{L_y}$$
$$-q = 0 \tag{34}$$

We now apply the Bubnov–Galerkin method to Eq. (34). That is, we integrate both sides of Eq. (34) over the whole plate area by the optimal weighting function *sine* as

$$\int_0^{L_x} dx \int_0^{L_y} \Phi(w)\ \sin\frac{\pi x}{L_x}\ \sin\frac{\pi y}{L_y} dy = 0 \tag{35}$$

Then, the following nonlinear equation for the maximum nondimensional deflection of the plate can be found

$$k_1 \cdot W_m + k_3 \cdot W_m^3 = Q \tag{36}$$

where

$$k_1 = \frac{h^4}{12}\left[\left(\frac{\pi}{L_x}\right)^2 + \left(\frac{\pi}{L_y}\right)^2\right]^2 (1 + k_2),$$
$$k_3 = \frac{(c_{11}^2 - c_{12}^2)h^4}{16c_{11}^2}\left[\left(\frac{\pi}{L_x}\right)^4 + \left(\frac{\pi}{L_y}\right)^4\right], \tag{37}$$
$$Q = \frac{16q_0}{c_{11}\pi^2}$$

The expression for k_1 involves another parameter k_2 defined below, which represents the piezoelectric and piezomagnetic coupling contribution to the purely elastic deflection.

$$k_2 = \frac{e_{31}}{c_{11}}\frac{\Delta_1}{\Delta} + \frac{q_{31}}{c_{11}}\frac{\Delta_2}{\Delta} \tag{38}$$

It can be shown that for a given physical problem, the only real solution of Eq. (36) can be expressed explicitly as

$$W_m = \left[\frac{Q}{2k_3} - \sqrt{\left(\frac{Q}{2k_3}\right)^2 + \left(\frac{k_1}{3k_3}\right)^3}\right]^{1/3}$$
$$+\left[\frac{Q}{2k_3} + \sqrt{\left(\frac{Q}{2k_3}\right)^2 + \left(\frac{k_1}{3k_3}\right)^3}\right]^{1/3} \tag{39}$$

Thus, we have derived an exact close-form expression for the maximum deflection of the nonlinear thin-plate under a static mechanical load. In the following section, we will present some numerical results to show the effect of the coupling factor as well as nonlinearity on the deflection.

4. Numerical results

In the numerical analysis, the coupled MEE plate is assumed to be made of phases $BaTiO_3$ and $CoFe_2O_4$. In the first example, four cases are selected to study the effect of various couplings on the nonlinear deflections: magnetoelectroelastic (MEE, $e_{ij} \neq 0$, $q_{ij} \neq 0$), piezoelectric (PE, derived from MEE by letting $q_{ij} = 0$), piezomagnetic (PM, derived from MEE by letting $e_{ij} = 0$), and purely elastic (Elastic, derived from MEE by letting $e_{ij} = q_{ij} = 0$). The MEE

Table 1
Material coefficients of the MEE material obtained by the simple rule of mixture with 50% $BaTiO_3$ and 50% $CoFe_2O_4$ (elastic constants c_{ij} in 10^9 N/m^2, piezoelectric constants e_{ij} in C/m^2, piezomagnetic constants q_{ij} in N/Am, dielectric constants κ_{ij} in 10^{-9} C^2/Nm^2, magnetic constants μ_{ij} in 10^{-4} Ns^2/C^2).

c_{11}	c_{12}	c_{13}	c_{33}	c_{44}
225	125	124	216	44
e_{31}	e_{33}	e_{15}	κ_{11}	κ_{33}
−2.2	9.3	5.8	5.64	6.35
μ_{11}	μ_{33}	q_{31}	q_{33}	q_{15}
2.97	0.835	290.2	350	275

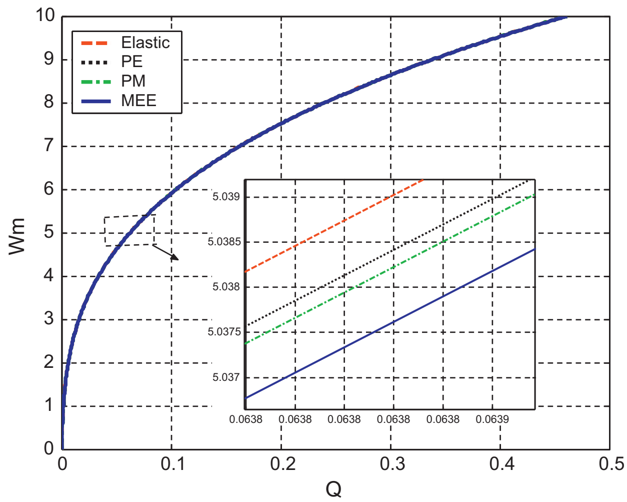

Fig. 2. Nonlinear load–deflection curves for the plate made of elastic, piezoelectric (PE), piezomagnetic (PM) and magnetoelectroelastic (MEE) materials. Other fixed parameters are $L_x/h = 20$ and $L_y/h = 10$.

properties listed in Table 1 are obtained by the simple rule of mixture (Aboudi, 2001; Sih and Chen, 2003; Chen et al., 2010) with 50% $BaTiO_3$ and 50% $CoFe_2O_4$.

Numerical results for the load–deflection curves of the MEE plate are shown in Figs. 2–5. The nonlinear relationship between the central deflection W_m and uniformly distributed (nondimensional) load density Q of the MEE plate is shown in Fig. 2. Other fixed parameters are $L_x/h = 20$ and $L_y/h = 10$. It is observed from Fig. 2, that the load–deflection curves for these four different cases are nearly identical, with the purely elastic result being the same as that in Chia (1980). The zoom-in result in Fig. 2 further indicates that the differences among these different coupling cases are negligible. The reason for this feature is that the coupling factor k_2 for this MEE

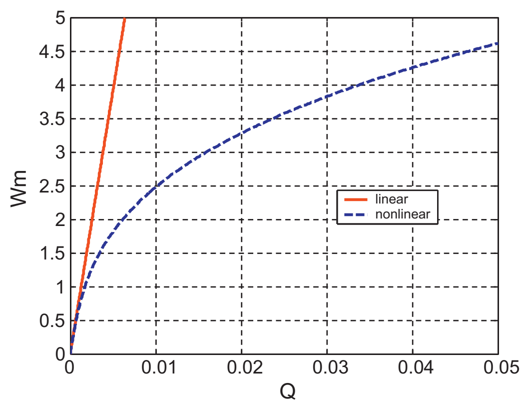

Fig. 3. Linear and nonlinear load–deflection curves for the coupled MEE plate with fixed parameters $L_x/h = 20$ and $L_y/h = 10$.

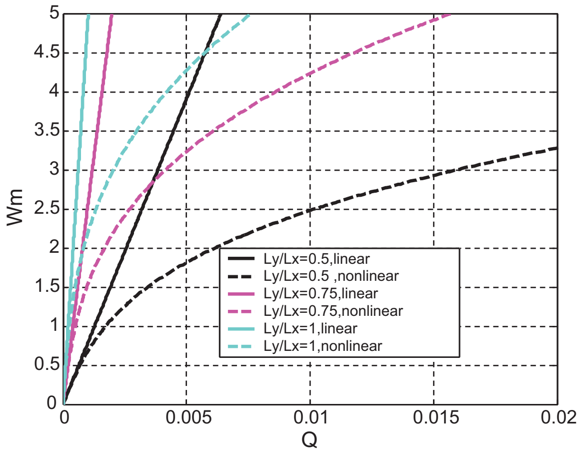

Fig. 4. Linear and nonlinear load–deflection curves for the coupled MEE plate. While $h/L_x = 1/20$ is fixed, the ratio of L_y/L_x varies as 0.5, 0.75, and 1.

composite is very small. In other words, even for this piezoelectric and piezomagnetic coupled plate, the load–deflection curve is still dominated by the elastic part.

Fig. 3 shows the difference of the load–deflection curves between the large (nonlinear) and small (linear) deflection theories. The fixed geometric ratios are $L_x/h = 20$ and $L_y/h = 10$, and the plate is the one made of coupled MEE in Table 1. It is obvious that the small deflection curves, represented by the straight lines, can be very wrong even when the nondimensional load density is very small (say, 10^{-2}).

For the same coupled MEE plate, the lateral geometry influence of the plate on the load–deflection curve is shown in Fig. 4 for both linear and nonlinear cases. While $h/L_x = 1/20$ is fixed, the ratio of L_y/L_x varies as 0.5, 0.75, and 1. It is observed that, for fixed load density Q, the maximum nondimensional deflection increases with increasing L_y/L_x.

Also for the same coupled MEE plate with fixed lateral size, Fig. 5 shows the influence of the plate thickness on the nonlinear load–deflection curve. While the plate thickness ratio h/L_x varies as 0.025, 0.0375, and 0.05, its lateral dimension ratio is fixed at $L_y/L_x = 0.5$. It is very clear that, for a given load density, the deflection decreases with increasing plate thickness.

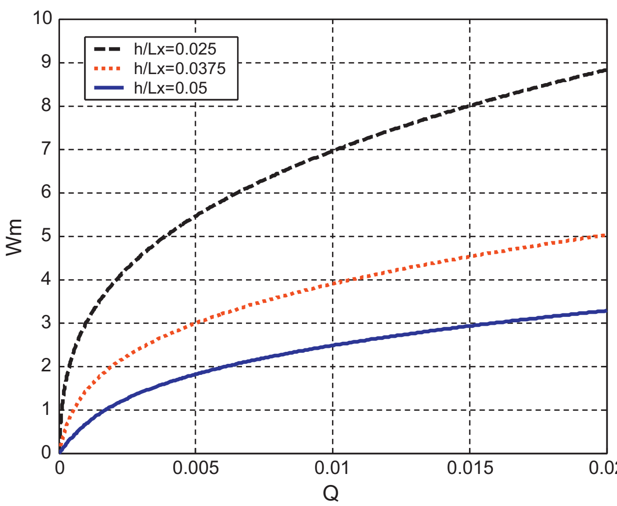

Fig. 5. Nonlinear load–deflection curves for the coupled MEE plate. While the plate thickness ratio h/L_x varies as 0.025, 0.0375, and 0.05, its lateral dimension ratio is fixed at $L_y/L_x = 0.5$.

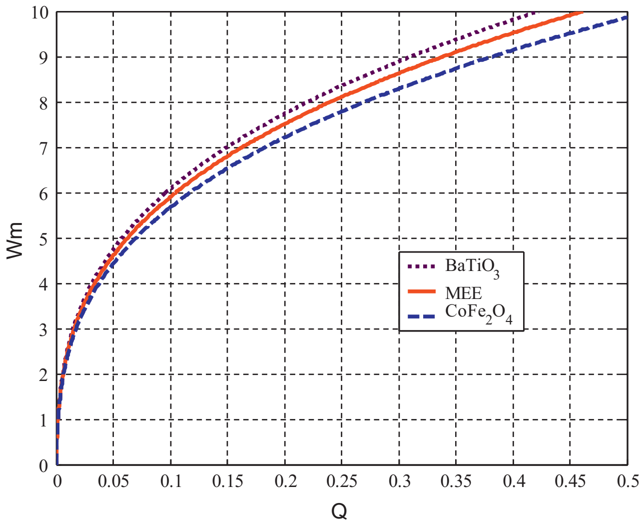

Fig. 6. Nonlinear load–deflection curves for the plate made of MEE, piezoelectric $BaTiO_3$ and piezomagnetic $CoFe_2O_4$ materials. Other fixed parameters are $L_x/h = 20$ and $L_y/h = 10$.

As a second example, the nonlinear load–deflection curves for the plate made of coupled MEE (as listed in Table 1), piezoelectric $BaTiO_3$ and piezomagnetic $CoFe_2O_4$ materials (Pan, 2001) are shown in Fig. 6. Other fixed parameters are $L_x/h = 20$ and $L_y/h = 10$. It is observed that, for fixed load density Q, the deflection corresponding to the piezoelectric $BaTiO_3$ plate is the largest among the three, while that corresponds to the piezomagnetic $CoFe_2O_4$ is the smallest.

5. Conclusions

A nonlinear large-deflection model is proposed for magnetoelectroelastic rectangular thin plates. For a simply-supported plate made of piezoelectric and piezomagnetic materials under a uniform mechanical load, a coupling factor is identified which can be used to characterize the contribution of the multiphase coupling to the plate deflection. Numerical results are carried out for the thin plate made of magnetoelectroelastic (MEE), piezoelectric (PE) and piezomagnetic (PM) materials. It is found that: (1) Even under a relatively small mechanical load, the large-deflection solution should be used for the prediction of the plate deflection since the linear small model could be inaccurate; (2) The nonlinear deflections of the thin plate are different for different coupled materials ($BaTiO_3$, $CoFe_2O_4$ and the composite MEE, as shown in Fig. 6); (3) However, for the MEE made of different volume fractions of piezoelectric and piezomagnetic phases, the coupling effect on the deflection is negligible. This is due to the fact that the coupling factor k_2 is very small for this material system.

Acknowledgements

The work is supported by National Natural Science Foundation of China (10772129) and Science Foundation of North University of China (2009).

References

Aboudi, J., 2001. Micromechanical analysis of fully coupled electro-magneto-thermal-elastic multiphase composite. Smart Mater. Struct. 10, 867–877.
Avellaneda, M., Harshe, G., 1994. Magnetoelectric effect in piezoelectric/magnetostrictive multilayer (2-2) composites. J. Intell. Mater. Syst. Struct. 5, 501–513.
Benveniste, Y., 1995. Magnetoelectric effect in fibrous composites with piezoelectric and piezomagnetic phases. Phys. Rev. B 51, 16424–16427.

Chen, Z.R., Yu, S.W., Meng, L., Lin, Y., 2002. Effective properties of layered magneto-electro-elastic composites. Compos. Struct. 57, 177–182.

Chen, W.Q., Pan, E., Wang, H.M., Zhang, C.Z., 2010. Theory of indentation on multi-ferroic composite materials. J. Mech. Phys. Solids 58, 1524–1551.

Chia, C.Y., 1980. Nonlinear Analysis of Plates. McGraw-hill, New York.

Feng, W.J., Su, R.K.L., Pan, E., 2007a. Fracture analysis of a penny-shaped magnetically dielectric crack in a magnetoelectroelastic material. Int. J. Fracture 146, 125–138.

Feng, W.J., Pan, E., Wang, X., 2007b. Dynamic fracture analysis of a penny-shaped crack in a magnetoelectroelastic layer. Int. J. Solids Struct. 44, 7955–7974.

Feng, W.J., Pan, E., Wang, X., 2008. Stress analysis of a penny-shaped crack in a magneto-electro-thermo-elastic layer under uniform heat flow and shear loads. J. Thermal Stresses 31, 497–514.

Harshe, G., Dougherty, J.P., Newnham, R.E., 1993. Theoretical modeling of multiplayer magnetoelectric composites. Int. J. Appl. Electromagn. Mater. 4, 145–159.

Li, J.Y., Dunn, M.L., 1998. Micromechanics of magnetoelectroelastic composite materials: average fields and effective behavior. J. Intell. Mater. Syst. Struct. 9, 404–416.

Liu, M.F., Chang, T.P., 2010. Closed form expression for the vibration problem of a transversely isotropic magneto-electro-elastic plate. J. Appl. Mech. 77, 24502–24510.

Nan, C.W., 1994. Magnetoelectric effect in composites of piezoelectric and piezomagnetic phases. Phys. Rev. B 50, 6082–6088.

Pan, E., 2001. Exact solution for simply supported and multilayered magneto-electro-elastic plates. J. Appl. Mech. 68, 608–618.

Sih, G.C., Chen, E.P., 2003. Dilatational and distortional behavior of cracks in magnetoelectroelastic materials. Theor. Appl. Fract. Mech. 40, 1–21.

Timoshenko, S., Woinowsky-Krieger, S., 1959. Theory of Plates and Shells. McGraw-Hill, New York.

Tupholme, G.E., 2008. Shear crack in a transversely isotropic magnetoelectroelastic half-space. Mech. Res. Comm. 35, 466–474.

Wang, X., Shen, Y.P., 2002. The general solution of three-dimensional problems in magnetoelectroelastic media. Int. J. Eng. Sci. 40, 1069–1080.

Wu, C.P., Chen, S.J., Chiu, K.H., 2010. Three-dimensional static behavior of functionally graded magneto-electro-elastic plates using the modified Pagano method. Mech. Res. Comm. 37, 54–60.