This article was downloaded by: [University of Akron], [Ali Sangghaleh]
On: 08 October 2013, At: 08:36
Publisher: Taylor \& Francis
Informa Ltd Registered in England and Wales Registered Number: 1072954 Registered office: Mortimer House, 37-41 Mortimer Street, London W1T 3J H, UK


## Philosophical Magazine

Publication details, including instructions for authors and subscription information:
http://www.tandfonline.com/loi/tphm20

# Fields induced by three-dimensional dislocation loops in anisotropic magneto-electro-elastic bimaterials 

Xueli Han ${ }^{\text {a }}$, Ernie Pan ${ }^{\text {b }}$ \& Ali Sangghaleh ${ }^{\text {b }}$<br>${ }^{\text {a }}$ Department of Mechanics, School of Aerospace, Beijing Institute of Technology, Beijing, 100081, China<br>${ }^{\mathrm{b}}$ Department of Civil Engineering, University of Akron, Akron, OH , 44325-3905, USA<br>Published online: 14 J un 2013.

To cite this article: Xueli Han , Ernie Pan \& Ali Sangghaleh (2013) Fields induced by threedimensional dislocation loops in anisotropic magneto-electro-elastic bimaterials, Philosophical Magazine, 93:24, 3291-3313, DOI: 10.1080/ 14786435.2013.806830

To link to this article: http:// dx. doi.org/ 10.1080/ 14786435.2013.806830

## PLEASE SCROLL DOWN FOR ARTICLE

Taylor \& Francis makes every effort to ensure the accuracy of all the information (the "Content") contained in the publications on our platform. However, Taylor \& Francis, our agents, and our licensors make no representations or warranties whatsoever as to the accuracy, completeness, or suitability for any purpose of the Content. Any opinions and views expressed in this publication are the opinions and views of the authors, and are not the views of or endorsed by Taylor \& Francis. The accuracy of the Content should not be relied upon and should be independently verified with primary sources of information. Taylor and Francis shall not be liable for any losses, actions, claims, proceedings, demands, costs, expenses, damages, and other liabilities whatsoever or howsoever caused arising directly or indirectly in connection with, in relation to or arising out of the use of the Content.

This article may be used for research, teaching, and private study purposes. Any substantial or systematic reproduction, redistribution, reselling, loan, sub-licensing, systematic supply, or distribution in any form to anyone is expressly forbidden. Terms \& Conditions of access and use can be found at http://www.tandfonline.com/page/terms-and-conditions

# Fields induced by three-dimensional dislocation loops in anisotropic magneto-electro-elastic bimaterials 

Xueli Han ${ }^{\text {a }}$, Ernie Pan ${ }^{\text {b }}$ and Ali Sangghaleh ${ }^{\text {b }}$<br>${ }^{a}$ Department of Mechanics, School of Aerospace, Beijing Institute of Technology, Beijing 100081, China; ${ }^{b}$ Department of Civil Engineering, University of Akron, Akron, OH 44325-3905, USA

(Received 15 January 2013; final version received 1 May 2013)


#### Abstract

The coupled elastic, electric and magnetic fields produced by an arbitrarily shaped three-dimensional dislocation loop in general anisotropic magneto-elec-tro-elastic (MEE) bimaterials are derived. First, we develop line-integral expressions for the fields induced by a general dislocation loop. Then, we obtain analytical solutions for the fields, including the extended Peach-Koehler force, due to some useful dislocation segments such as straight line and elliptic arc. The present solutions contain the piezoelectric, piezomagnetic and purely elastic solutions as special cases. As numerical examples, the fields induced by a square and an elliptic dislocation loop in MEE bimaterials are studied. Our numerical results show the coupling effects among different fields, along with various interesting features associated with the dislocation and interface.


Keywords: three-dimensional dislocation loop; extended Peach-Koehler force; bimaterials; magneto-electro-elastic materials

## 1. Introduction

The magneto-electro-elastic (MEE) materials are a new kind of functional materials. MEE materials are usually composites made of multi-phases or laminae and exhibit magneto-electric coupling effect that is not present in the single-phase piezoelectric or piezomagnetic material [1]. It is particularly important for the energy conversion among the mechanical, electric and magnetic ones, and thus it has potential application as a multifunctional device [2]. In MEE composite materials, the coupling fields are transferred through interfaces; so, the interfaces should have great influence on the properties of MEE materials/devices. At the same time, MEE materials/devices usually contain multi-phase or laminate crystal structures in which dislocations are common defects. Besides mechanical properties, dislocations in them should have effects on their coupled fields and other physical properties.

In the past decade, attention has been paid to predict the effective properties of MEE composites according to the theories of micromechanics [1]. But, for dislocation problems in MEE, relatively little work has been done. Until now, only one-dimensional dislocations in such coupling materials were studied [3-5]. In reality, however, dislocations usually form three-dimensional (3D) loops, and thus it calls for the analysis

[^0]of 3D dislocations in MEE. Recently, the authors developed a method to analyse the fields produced by 3D dislocations in MEE materials [6], in which the MEE materials are taken to be homogeneous in an infinite space and no interface effects were considered.

The fields of dislocations and fracture in bimaterials are fundamental to understand the interaction between a dislocation/crack and the interface in composites or heterostructures $[7,8]$. However, analytical solution is difficult and rare for the fields of 3D dislocations interaction with interfaces. One approach is to use a point-force Green's function and derive the field produced by a dislocation loop in the corresponding materials by integration over the dislocation surface [9]. This method is general, no matter if the dislocation is in a homogeneous or inhomogeneous, elastic or general MEE medium, provided the corresponding point-force Green's function is known [6]. In an infinite homogeneous medium, using the spatial symmetry property of the Green's function, the surface integral can be reduced to a line integral for the field induced by a dislocation loop $[6,10,11]$. In an inhomogeneous medium, however, there is no such spatial symmetry in the Green's function, and thus no line-integral solution is available for a dislocation loop. Since area integration is time-consuming, efforts have been taken for treating some special cases. For examples, Gosling and Willis derived a line-integral expression for the stress field associated with an arbitrary dislocation in an isotropic half-space [12]. Ghoniem and Han proposed an approximate line-integral expression for the elastic field produced by dislocations in multilayered materials of elastic anisotropy [13]. Akarapu and Zbib constructed line-integral expressions for the displacement and stress fields induced by an arbitrarily shaped dislocation in an isotropic bimaterial and derived analytical expressions for the stress field due to a straight dislocation segment in it [14]. Tan and Sun obtained line-integral solutions for the stress fields induced by dislocation loops in an isotropic thin film-substrate system and multilayered heterogeneous thin-film system [15,16]. Recently, the authors derived line-integral expressions for the displacement and stress fields due to a 3D dislocation loop in an anisotropic elastic bimaterial [17] and in a piezoelectric bimaterials [18]. There is, however, no solution available for 3D dislocations in MEE inhomogeneous materials.

In the present paper, we will analyse the field induced by a 3D dislocation loop in an MEE bimaterial system. First, by utilizing the Green's functions and their derivatives in MEE bimaterials, we derive line-integral expressions for the coupled fields induced by an arbitrary 3D dislocation loop in a general anisotropic MEE bimaterial system. Then, we obtain analytical solutions for the dislocation loops made of piecewise straight lines and elliptic arcs. Finally, numerical examples are presented for the fields induced by a square and an elliptic dislocation loop in an MEE composite made of $\mathrm{BaTiO}_{3}-\mathrm{CoFe}_{2} \mathrm{O}_{4} / \mathrm{BaTiO}_{3}$ bimaterial. Our results show clearly the important coupling effects along with various interesting features on the mechanical, electric and magnetic fields and on the dislocation-interface interaction.

## 2. Problem description and line-integral solutions

The problem of interest consists of a dislocation loop in two joined half-spaces with dissimilar MEE (or piezoelectric/piezomagnetic) material properties, see Figure 1. We will derive the fields induced by a dislocation loop in such a bimaterial system. We first write the governing equations for a linear anisotropic MEE solid. Then, based on the Green's function in MEE bimaterials, we derive the dislocation-induced fields.


Figure 1. Schematic for an arbitrary dislocation loop in MEE bimaterials.

The linear constitutive relations for the coupled MEE media can be written as [19]

$$
\left\{\begin{array}{l}
\sigma_{i j}=c_{i j l m} \gamma_{l m}-e_{k j} E_{k}-q_{k j j} H_{k}  \tag{1a}\\
D_{i}=e_{i j k} \gamma_{j k}+\varepsilon_{i j} E_{j}+\alpha_{i j} H_{j} \\
B_{i}=q_{i j k} \gamma_{j k}+\alpha_{i j} E_{j}+\mu_{i j} H_{j}
\end{array}\right.
$$

where $\sigma_{i j}, D_{i}$ and $B_{i}$ are the stress, electric displacement and magnetic induction, respectively; $\gamma_{i j}, E_{i}$ and $H_{i}$ are the strain, electric field and magnetic field, respectively; $c_{i j l m}, e_{i j k}, q_{i j k}$ and $\alpha_{i j}$ are the elastic, piezoelectric, piezomagnetic and magnetoelectric coefficients, respectively; and $\varepsilon_{i j}$ and $\mu_{i j}$ are the dielectric permittivities and magnetic permeabilities, respectively.

Using the extended notation of Barnett and Lothe [20], Equation (1a) can be rewritten in a compact form as [21-23]:

$$
\begin{equation*}
\sigma_{i J}=C_{i J K l} \gamma_{K l} \tag{1b}
\end{equation*}
$$

with a repeated lowercase (uppercase) index taking the summation from 1 to 3 (5) and $\sigma_{i J}, C_{i J K l}, \gamma_{K l}$ being the extended stresses, elastic constants and strains, respectively. Thereafter, for simplicity, the word "extended" will be omitted for all extended quantities, unless otherwise specified.

We now consider the field produced by an extended dislocation loop in an MEE medium. The dislocation loop $L$ is defined as the boundary of a surface $S$ across which the elastic displacement, electric potential and magnetic potential experience discontinuities, which can be described by an extended Burgers vector $\mathbf{b}=\left[b_{1}, b_{2}, b_{3}, \Delta \phi, \Delta \psi\right]^{\mathrm{T}}$. The elastic displacement jump is the traditional dislocation; $\Delta \phi$ corresponds to an electric dipole layer along the surface $S$ [20] and is called the electric potential dislocation [24]; and $\Delta \psi$ is called the magnetic potential dislocation [25]. The extended displacement field produced by a dislocation loop in an MEE medium can be expressed as [6]:

$$
\begin{equation*}
u_{M}(\mathbf{y})=\int_{S} C_{i J K l}(\mathbf{x}) G_{K M, x_{l}}(\mathbf{y} ; \mathbf{x}) b_{J}(\mathbf{x}) n_{i}(\mathbf{x}) \mathrm{d} S(\mathbf{x}) \tag{2a}
\end{equation*}
$$

where $\mathbf{n}$ is the unit normal to the surface $S ; G_{K M}(\mathbf{y} ; \mathbf{x})$ are the Green's functions in the corresponding medium, i.e. the $K$-th displacement component at the field point $\mathbf{x}$ due to the $M$-th unit "point force" component at the source point $\mathbf{y}$; and a subscript comma denotes the partial differentiation with respect to the coordinates, i.e. $G_{K M, x_{l}}=\partial G_{K M} / \partial x_{l}$. The Green's functions and their derivatives in MEE bimaterials are given in Appendix A.

Besides the displacement field, other field, such as the strain and stress, is also important, which requires the derivative of the displacement field. The derivatives of the displacement field can be expressed as:

$$
\begin{equation*}
u_{M, y_{p} \ldots . .}(\mathbf{y})=\int_{S} C_{i J K l}(\mathbf{x}) G_{K M, x y_{j} y_{p} \ldots}(\mathbf{y} ; \mathbf{x}) b_{J}(\mathbf{x}) n_{i}(\mathbf{x}) \mathrm{d} S(\mathbf{x}) \tag{2b}
\end{equation*}
$$

In MEE materials, due to piezoelectric and magnetoelectric effects, the strain field and magnetic field can induce a polarization field $\mathbf{P}$ as:

$$
\begin{equation*}
P_{i}=e_{i j k} \gamma_{j k}+\alpha_{i j} H_{j}=e_{i j k} u_{j, k}+\alpha_{i j} H_{j} \tag{3a}
\end{equation*}
$$

Furthermore, the gradient of $\mathbf{P}$ may induce an electronic polarization charge, with the volume charge density being

$$
\begin{equation*}
\rho=\nabla \cdot \mathbf{P}=e_{i j k} u_{j, k i}+\alpha_{i j} H_{j, i} \tag{3b}
\end{equation*}
$$

In order to obtain the strain/stress field, the polarization field $\mathbf{P}$ and polarization charge field $\rho$, one would need the first and second derivatives of the dislocation-induced displacement field. When a dislocation loop lies on a plane where the material properties are constants or piecewise constants on the dislocation loop surface $S$, the displacement field and its derivatives can be expressed in terms of the surface integrals as:

$$
\begin{equation*}
u_{M, p \ldots}(\mathbf{y})=C_{i J K l} b_{J} n_{i} \int_{S} G_{K M, x_{1} y_{p, . .}}(\mathbf{y} ; \mathbf{x}) \mathrm{d} S(\mathbf{x}) \tag{4}
\end{equation*}
$$

Thus, the key issue is to convert the surface integrals into line integrals. In order to do so, we first analyse the point-force Green's functions involved and their derivatives in Equation (4).

The point-force Green's functions in MEE bimaterials can be separated into two parts:

$$
\begin{equation*}
\mathbf{G}(\mathbf{y} ; \mathbf{x})=\mathbf{G}^{\infty}(\mathbf{y} ; \mathbf{x})+\mathbf{G}^{\text {Image }}(\mathbf{y} ; \mathbf{x}) \tag{5}
\end{equation*}
$$

where $\mathbf{G}^{\infty}(\mathbf{y} ; \mathbf{x})$ corresponds to the full-space part and $\mathbf{G}^{\text {Image }}(\mathbf{y} ; \mathbf{x})$ is called the image or complementary part which is associated with the bimaterial interface. Correspondingly, the derivatives of the Green's function can also be separated into a full-space and an image part. Thus, the integral $\int_{S} G_{K M, x_{1} . . .}(\mathbf{y} ; \mathbf{x}) \mathrm{d} S(\mathbf{x})$ can be also separated into two parts as:

$$
\begin{equation*}
\int_{S} G_{K M, x_{l} \ldots .}(\mathbf{y} ; \mathbf{x}) \mathrm{d} S(\mathbf{x})=\int_{S} G_{K M, x_{l} \ldots}^{\infty}(\mathbf{y} ; \mathbf{x}) \mathrm{d} S(\mathbf{x})+\int_{S} G_{K M, x_{l} \ldots . .}^{\text {Image }}(\mathbf{y} ; \mathbf{x}) \mathrm{d} S(\mathbf{x}) \tag{6}
\end{equation*}
$$

Substituting $G_{K M, x_{1} \ldots}^{\infty}(\mathbf{y} ; \mathbf{x})$ in Appendix A, for the case of a point force at $y_{3}>0$, we have

$$
\int_{L} \mathbf{G}(\mathbf{y} ; \mathbf{x})_{x_{1} \ldots \ldots}^{\infty} \mathrm{d} L(\mathbf{x})= \begin{cases}\frac{-1}{2 \pi^{2}} \int_{0}^{\pi} \overline{\mathbf{A}}^{1}\left[\int_{S}\left(\mathbf{G}_{u p}^{\infty}\right)_{x_{l} \ldots l} \mathrm{~d} L(\mathbf{x})\right]\left(\overline{\mathbf{A}}^{1}\right)^{T} \mathrm{~d} \theta, & x_{3}>y_{3}  \tag{7}\\ \frac{1}{2 \pi^{2}} \int_{0}^{\pi} \mathbf{A}^{1}\left[\int_{S}\left(\mathbf{G}_{u l}^{\infty}\right)_{x_{l} \ldots \ldots} \mathrm{~d} L(\mathbf{x})\right]\left(\mathbf{A}^{1}\right)^{T} \mathrm{~d} \theta, & x_{3}<y_{3}\end{cases}
$$

with

$$
\left\{\begin{array}{l}
\int_{L} \partial^{n}\left(G_{u p}^{\infty}\right)_{I J} / \partial x_{l} \ldots \mathrm{~d} L(\mathbf{x})=H_{1}^{n} \int_{S} \frac{\mathrm{~d} L(\mathbf{x})}{\left[\mathbf{h}\left(\theta, \bar{p}_{I}^{1}\right) \cdot(\mathbf{x}-\mathbf{y})\right]^{n+1}}  \tag{8}\\
\int_{L} \partial^{n}\left(G_{u l}^{\infty}\right)_{I J} / \partial x_{l} \ldots \mathrm{~d} L(\mathbf{x})=H_{2}^{n} \int_{S} \frac{\mathrm{~d} L(\mathbf{x})}{\left[\mathbf{h}\left(\theta, p_{I}^{1}\right) \cdot(\mathbf{x}-\mathbf{y})\right]^{n+1}}
\end{array}\right.
$$

with

$$
\left\{\begin{array}{l}
\int_{L} \partial^{n}\left(G_{u p}\right)_{I J} / \partial x_{l} \ldots \mathrm{~d} L(\mathbf{x})=H_{1}^{n} \int_{S} \frac{\mathrm{~d} L(\mathbf{x})}{\left[-\mathbf{h}\left(\theta, \bar{p}_{I}^{1}\right) \cdot \mathbf{x}+\mathbf{h}\left(\theta, p_{J}^{1}\right) \cdot \mathbf{y}\right]^{n+1}}  \tag{10}\\
\int_{L} \partial^{n}\left(G_{u l}\right)_{I J} / \partial x_{l} \ldots \mathrm{~d} L(\mathbf{x})=H_{2}^{n} \int_{S[-\mathbf{x})}^{\left.S\left(\theta, p_{I}^{2}\right) \cdot x+\mathbf{h}\left(\theta, p_{J}^{1}\right) \cdot \mathbf{y}\right]^{n+1}}
\end{array}\right.
$$

where $H_{1}^{n}$ and $H_{2}^{n}$ are different to those in Equation (8) and their expressions are also given in the Appendix A.

For the case of a point force at $y_{3}<0$, similar results can be obtained.
From Equations (6)-(10), it can be seen that the key problem is to solve the following kind of integration over the dislocation loop surface $S$ :

$$
\begin{equation*}
F_{n}\left(\mathbf{y}, \theta, p_{1}, p_{2}\right)=\int_{S} \frac{\mathrm{~d} S(\mathbf{x})}{\left[-\mathbf{h}\left(\theta, p_{1}\right) \cdot \mathbf{x}+\mathbf{h}\left(\theta, p_{2}\right) \cdot \mathbf{y}\right]^{n}} \quad n=2,3,4 \tag{11}
\end{equation*}
$$

where $p_{1}$ and $p_{2}$ can be assigned to different eigenvalues according to the requirements in the corresponding expressions of the Green's functions.

In order to carry out the surface integration in Equation (11), we first transform the global coordinate system ( $O: x_{1}, x_{2}, x_{3}$ ) to a local one ( $\mathbf{x}_{0}: \xi_{1}, \xi_{2}, \xi_{3}$ ) by $\left[\mathbf{x}-\mathbf{x}_{0}\right]=[\mathbf{D}][\xi]$. Then, the integration in Equation (11) becomes

$$
\begin{equation*}
F_{n}(\mathbf{y}, \theta, p)=\int_{S} \frac{\mathrm{~d} \xi_{1} \mathrm{~d} \xi_{2}}{\left[f_{1}(\mathbf{y}, \theta) \xi_{1}+f_{2}(\mathbf{y}, \theta) \xi_{2}+f_{3}(\mathbf{y}, \theta)\right]^{n}} \quad n=2,3,4 \tag{12}
\end{equation*}
$$

with $f_{\alpha}(\mathbf{y}, \theta)=-D_{k \alpha} h_{k}\left(\theta, p_{1}\right), \alpha=1,2$ and $f_{3}(\mathbf{y}, \theta)=y_{k} h_{k}\left(\theta, p_{2}\right)-x_{0 k} h_{k}\left(\theta, p_{1}\right)$.
By introducing

$$
\begin{align*}
& L_{n}\left(\xi_{1}, \xi_{2}\right)=\int_{-\infty}^{\xi_{2}} \frac{\mathrm{~d} \xi_{2}}{\left(f_{1} \xi_{1}+f_{2} \xi_{2}+f_{3}\right)^{n}}=-\frac{1}{(n-1) f_{2}\left(f_{1} \xi_{1}+f_{2} \xi_{2}+f_{3}\right)^{n-1}}  \tag{13}\\
& \quad n=2,3,4
\end{align*}
$$

we arrive at

$$
\begin{align*}
F_{n} & =\int_{S} \frac{\partial L_{n}\left(\xi_{1}, \xi_{2}\right)}{\partial \xi_{2}} \mathrm{~d} \xi_{1} \mathrm{~d} \xi_{2}=\int_{L} L_{n}\left(\xi_{1}, \xi_{2}\right) \mathrm{d} \xi_{1}=\frac{-1}{(n-1) f_{2}} \int_{L} \frac{\mathrm{~d} \xi_{1}}{\left(f_{1} \xi_{1}+f_{2} \xi_{2}+f_{3}\right)^{n-1}}  \tag{14}\\
& n=2,3,4
\end{align*}
$$

Thus, the surface integrals over the dislocation surface $S$ are reduced to the line integrals along the dislocation loop line $L$.

## 3. Analytical solutions for dislocation segments

In Section 2, the fields produced by an arbitrarily shaped dislocation loop in MEE bimaterials have been expressed by line integrals along the dislocation line. They can be evaluated directly by a numerical integration method. However, for some dislocation lines, the line integral can be carried out analytically, and thus analytical solutions can be obtained for these dislocation segments.

### 3.1. Straight line segment

For the special case of a straight line segment, in the local $\left(\xi_{1}, \xi_{2}\right)$ plane, it can be described by

$$
\begin{equation*}
\boldsymbol{\xi}(t)=(1-t) \mathbf{P}_{1}+t \mathbf{P}_{2}, \quad 0 \leq t \leq 1 \tag{15}
\end{equation*}
$$

with $\mathbf{P}_{1}\left(P_{11}, P_{12}\right), \mathbf{P}_{2}\left(P_{21}, P_{22}\right)$ being the position vectors of the start and end points of the straight line. Substituting it into Equation (14), the integration can be obtained as:

$$
\begin{equation*}
F_{2}=-H_{0} \operatorname{In} \frac{H_{2}}{H_{1}}, \quad F_{3}=\frac{1}{2} H_{0}\left(\frac{1}{H_{2}}-\frac{1}{H_{1}}\right), \quad F_{4}=\frac{1}{6} H_{0}\left[\frac{1}{\left(H_{2}\right)^{2}}-\frac{1}{\left(H_{1}\right)^{2}}\right] \tag{16}
\end{equation*}
$$

with $H_{0}=\frac{1}{f_{2}} \frac{P_{21}-P_{11}}{f_{1}\left(P_{21}-P_{11}\right)+f_{2}\left(P_{22}-P_{12}\right)}, H_{1}=f_{1} P_{11}+f_{2} P_{12}+f_{3}, H_{2}=f_{1} P_{21}+f_{2} P_{22}+f_{3}$.

### 3.2. Elliptic arc segment

For a dislocation curve described by an elliptic arc segment, in the local $\left(\xi_{1}, \xi_{2}\right)$ plane, the corresponding whole ellipse can be expressed as,

$$
\begin{equation*}
\boldsymbol{\xi}(t)=\mathbf{q}_{1} \cos t+\mathbf{q}_{2} \sin t+\mathbf{q}_{0}, \quad-\pi \leq t \leq \pi \tag{17}
\end{equation*}
$$

with $\mathbf{q}_{1}\left(q_{11}, q_{12}\right), \mathbf{q}_{2}\left(q_{21}, q_{22}\right), \mathbf{q}_{0}\left(q_{01}, q_{02}\right)$ being vectors to determine the ellipse, see Figure 2(a).

We denote

$$
\begin{align*}
f_{1} \xi_{1}+f_{2} \xi_{2}+f_{3} & \equiv f_{\alpha} q_{1 \alpha} \cos t+f_{\alpha} q_{2 \alpha} \sin t+f_{\alpha} q_{0 \alpha}+f_{3} \equiv c_{1} \cos t+c_{2} \sin t+c_{3}  \tag{18}\\
\mathrm{~d} \xi_{1} & =\left(-q_{11} \sin t+q_{21} \cos t\right) \mathrm{d} t \equiv\left(c_{4} \cos t+c_{5} \sin t\right) \mathrm{d} t
\end{align*}
$$

Then, substituting Equation (18) into (14), the integration along an elliptic arc $t_{1} \leq t \leq t_{2}$ can be obtained as:

$$
\begin{gather*}
F_{2}=-\left.\frac{1}{f_{2}} \frac{1}{c_{1}^{2}+c_{2}^{2}}\left[d_{1}\left(t-c_{3} I_{1}\right)+\left(c_{2} c_{4}-c_{1} c_{5}\right) \ln \left(c_{1} \cos t+c_{2} \sin t+c_{3}\right)\right]\right|_{t_{1}} ^{t_{2}} \\
F_{3}=-\left.\frac{1}{2 f_{2}} \frac{1}{d_{1}}\left[d_{1} I_{1}+\frac{c_{1} c_{5}-c_{2} c_{4}+c_{3} c_{5} \cos t-c_{3} c_{4} \sin t}{c_{1} \cos t+c_{2} \sin t+c_{3}}\right]\right|_{t_{1}} ^{t_{2}} \\
F_{4}=\left.\frac{1}{6 f_{2}} \frac{1}{d_{2}}\left[\frac{d_{1}}{d_{2}}\left(3 c_{3} I_{1}-\frac{3 c_{2} c_{3}+\left(2 d_{2}+3 c_{3}^{2}\right) \sin t}{c_{1}\left(c_{1} \cos t+c_{2} \sin t+c_{3}\right)}\right)+\frac{c_{2} d_{1}-c_{5} d_{2}+c_{3} d_{1} \sin t}{c_{1}\left(c_{1} \cos t+c_{2} \sin t+c_{3}\right)^{2}}\right]\right|_{t_{1}} ^{t_{2}} \tag{19}
\end{gather*}
$$



Figure 2. (a) Schematic of an elliptic loop. (b) Schematic of representing a smooth dislocation curve segment via a couple of elliptic arcs.
with $d_{1}=c_{1} c_{4}+c_{2} c_{5}, d_{2}=c_{1}^{2}+c_{2}^{2}-c_{3}^{2}$ and $I_{1}=\frac{-2}{\sqrt{c_{3}^{2}-c_{1}^{2}-c_{2}^{2}}} \arctan \frac{\left(c_{1}-c_{3}\right) \tan \frac{t}{2}-c_{2}}{\sqrt{c_{3}^{2}-c_{1}^{2}-c_{2}^{2}}}$. It is noted that when $t_{1}=0$ and $t_{2}=2 \pi$, they are the solutions for an elliptic dislocation loop.

The elliptic arc solution can be a powerful tool in simulating a dislocation loop. Although the elliptic arc range $t$ can be chosen freely, for convenience, we will use a quarter of the elliptic arc with $0 \leq t \leq \pi / 2$ to describe a curved dislocation loop. We assume a smooth curved dislocation segment, with $\mathbf{P}_{1}, \mathbf{P}_{2}$ being the position vectors of the start and end points of the curve and $\mathbf{T}_{1}, \mathbf{T}_{2}$ being the tangent vectors at the two points, see Figure 2(b). In order to describe the curve by a quarter of elliptic arc, first, at point $\mathbf{P}_{1}$, we draw a straight line parallel to $\mathbf{T}_{2}$ and at point $\mathbf{P}_{2}$ we draw another line parallel to $\mathbf{T}_{1}$. Consequently, the intersection point of the two lines is the centre $\mathbf{q}_{0}$ of the ellipse. We then set $\mathbf{q}_{1}=\mathbf{P}_{1}-\mathbf{q}_{0}, \mathbf{q}_{2}=\mathbf{P}_{2}-\mathbf{q}_{0}$ and use the quarter elliptic arc $\boldsymbol{\xi}(t)=\mathbf{q}_{1} \cos t+\mathbf{q}_{2} \sin t+\mathbf{q}_{0}(0 \leq t \leq \pi / 2)$ to model the dislocation curve. In this way, the elliptic arc and the dislocation curve will have the same tangents at the start and end points. When we use this elliptic arc piece by piece, the continuity of the tangent direction of the dislocation curve will be kept, and thus the loop will be smooth.

We point out that a dislocation loop can be described by other kinds of parametric dislocation segments, such as quadratic or cubic spline curves, for which analytical solutions may also be available.

## 4. Dislocation interaction with interfaces in MEE materials

A dislocation loop in an MEE material will induce coupled elastic-electric-magnetic fields and we have the formulae and methods to evaluate them now. On the other hand, when a dislocation is located in an MEE material under an extended stress field, there will also be a force applied on the dislocation loop and this force is known as the Peach-Koehler force for an elastic material. Now, we extend the Peach-Koehler force to dislocations in MEE materials.

We assume that an extended dislocation loop with $\mathbf{b}=\left[b_{1}, b_{2}, b_{3}, \Delta \phi, \Delta \psi\right]^{\mathrm{T}}$ is located in an MEE material. When the dislocation loop is created, work is done by the extended stress field:

$$
\begin{equation*}
W=-\int_{S}\left(b_{j} \sigma_{i j}+D_{i} \Delta \varphi+B_{i} \Delta \psi\right) \mathrm{d} S_{i}=-\int_{S} b_{J} \sigma_{i J} \mathrm{~d} S_{i} \tag{20}
\end{equation*}
$$

As the dislocation expands, assuming every line element $\mathrm{d} \mathbf{l}$ on the loop $L$ has a virtual displacement $\delta \mathbf{r}$, the loop area $S$ will be increased by $\delta \mathbf{r} \times \mathrm{dl}$, and consequently the variation of the work by $\sigma_{i J}$ is

$$
\begin{equation*}
\delta W=\int_{L} \mathrm{~d} \mathbf{F} \cdot \delta \mathbf{r}=-\int_{L} b_{J} \sigma_{i J}(\delta \mathbf{r} \times \mathrm{d} \mathbf{l})_{i}=\int_{L}\left[\left(b_{J} \sigma_{i J}\right) \times \mathrm{d} \mathbf{l}\right] \cdot \delta \mathbf{r} \tag{21}
\end{equation*}
$$

In the last expression in Equation (21), $\left(b_{j} \sigma_{i J}\right)$ is a vector with $i(i=1,2,3)$ being the free index. Thus, the change of the extended Peach-Koehler force dF , i.e. the force acting on a dislocation element $\mathrm{d} \mathbf{l}$ in MEE medium is

$$
\begin{equation*}
\mathrm{d} \mathbf{F}=\left(b_{J} \sigma_{i J}\right) \times \mathrm{d} \mathbf{l} \tag{22}
\end{equation*}
$$

Or writing it in the component form, we have the extended Peach-Koehler force acting on a unit length dislocation element as

$$
\begin{equation*}
F_{l}=\varepsilon_{i k l} b_{J} \sigma_{i J} v_{k} \tag{23}
\end{equation*}
$$

where $\varepsilon_{i k l}$ is the permutation tensor and $\boldsymbol{v}$ is the unit tangent vector along the loop segment.

The stress field $\sigma_{i J}$ in materials may originate from various sources, such as applied force, image stress (due to surfaces/interfaces), other dislocations and even the dislocation itself (self-force). When a dislocation resides in a solid with surfaces/interfaces, the stress field induced by a dislocation is divided into two parts: the full-space stress $\sigma_{i J}^{\infty}$ and the complementary (or called image) stress $\sigma_{i J}^{\text {Image }}$. The stress $\sigma_{i J}^{\infty}$ corresponds to the one induced by the dislocation in a homogeneous and infinite space and this stress will induce a self-force on the dislocation line itself. The Peach-Koehler force induced by the stress $\sigma_{i J}^{\text {Image }}$ reflects the surface/interface effect on modifying the infinite and homogenous medium stress field of the dislocation and will be termed the image force $\mathbf{F}^{\text {Image }}$.

## 5. Numerical examples and results

We have checked the present formulation for a dislocation loop in anisotropic elastic bimaterials. The numerical results are the same as those by the surface integration method [26]. We have also calculated the field produced by a curved dislocation loop in MEE bimaterials using both the straight line solution in Equation (16) and the elliptic arc solution in Equation (19) and obtained the same results. However, we notice that the elliptic arc solution is more powerful for a curved dislocation loop, i.e. one can produce more accurate results using less dislocation segments using Equation (19) than using Equation (16).

As numerical examples, we consider dislocation loops in $\mathrm{BaTiO}_{3}-\mathrm{CoFe}_{2} \mathrm{O}_{4} / \mathrm{BaTiO}_{3}$ bimaterials, with $\mathrm{BaTiO}_{3}-\mathrm{CoFe}_{2} \mathrm{O}_{4}$ having $25 \% \mathrm{BaTiO}_{3}$ and $75 \% \mathrm{CoFe}_{2} \mathrm{O}_{4} . \mathrm{BaTiO}_{3}$ is a piezoelectric material, $\mathrm{CoFe}_{2} \mathrm{O}_{4}$ is a piezomagnetic one and $\mathrm{BaTiO}_{3}-\mathrm{CoFe}_{2} \mathrm{O}_{4}$ is composed as the MEE material as listed in Appendix B. All materials are transversely


Figure 3. Schematic for a square dislocation loop in $\mathrm{BaTiO}_{3}-\mathrm{CoFe}_{2} \mathrm{O}_{4} / \mathrm{BaTiO}_{3}$ bimaterials.

(b)

(c)


Figure 4. (colour online) Contours of extended stress fields (normalized by b/d) on vertical planes $y / d=-0.005$ and $x / d=0$ (a) Stress $\sigma_{12}$ (in Pa ), (b) $|\mathrm{D}|\left(\right.$ in $\mathrm{Cm}^{-2}$ ) and (c) $|\mathrm{B}|$ (in Tm ).
isotropic (or hexagonal) with their poling direction along $z$-axis (i.e. $x_{3}$-axis). The material properties of $\mathrm{BaTiO}_{3}$ are from Han and Pan [6] and those of $\mathrm{BaTiO}_{3}-\mathrm{CoFe}_{2} \mathrm{O}_{4}$ are obtained based on the micromechanics theory [27-28].

In the numerical calculation, the character length $d$ of the loop is used as the unit length of the coordinates. The fields have the following dimension relations:

$$
\left.\begin{array}{c}
\mathbf{U}(\mathbf{u}, \varphi, \psi) \propto b(b, \Delta \varphi, \Delta \psi) \\
\Sigma(\sigma, \mathbf{D}, \mathbf{B}), \boldsymbol{\Gamma}(\gamma, \mathbf{E}, \mathbf{H}), \mathbf{P} \propto b / d(b / d, \Delta \varphi / d, \Delta \psi / d) \\
\rho \propto b / d^{2}\left(b / d^{2}, \Delta \varphi / d^{2}, \Delta \psi / d^{2}\right)  \tag{24}\\
\mathbf{F}
\end{array}\right) b^{2} / R\left(b^{2} / R, \Delta^{2} \varphi / R, \Delta^{2} \psi / R\right)
$$

For instance, $\mathbf{U}(\mathbf{u}, \varphi, \psi)$ in Equation (24) means the extended displacement $\mathbf{U}$ which includes the elastic displacement $\mathbf{u}$, electric potential $\varphi$ and magnetic potential $\psi$. These quantities are proportional to the Burgers value $b$ for a traditional dislocation, or proportional to $\Delta \varphi$ for an electric dislocation, or proportional to $\Delta \psi$ for a magnetic dislocation. Thus, in our numerical results, these field quantities are normalized accordingly. Similar relations can be found in Equation (24) for other important quantities.

In the first numerical example, we assume that the dislocation loop is of a square on the $x-z$ plane (i.e. $x_{1}-x_{3}$ plane), with Burgers vector $\mathbf{b}=b[1,0,0,0,0]^{\mathrm{T}}$. The side length of the square loop is $d$ (the character length of the square) and the distance of the loop centre to the interface is $h=0.7 d$. The upper half-space is MEE $\mathrm{BaTiO}_{3}-\mathrm{CoFe}_{2} \mathrm{O}_{4}$ and the lower half-space is piezoelectric $\mathrm{BaTiO}_{3}$ (Figure 3).

The extended stress fields on the vertical planes $y / d=-0.005$ (on the slip plane) and $x / d=0$ (on the plane normal to the slip plane) are shown in Figure 4(a)-(c). It is observed in Figure 4(a) that there is a large shear stress $\sigma_{12}$ within the dislocation loop


Figure 5. (colour online) Contours of polarization field $|\mathbf{P}|$ (normalized by $b / d$ and in unit $\mathrm{Cm}^{-2}$ ) on horizontal plane $\mathrm{z} / d=0.68$ and vertical plane $x / d=0$.
(a)

(b)

(c)


Figure 6. (colour online) Contours of polarization charge density $\rho$ (normalized by $b / d^{2}$ and in unit $\mathrm{Cm}^{-2}$ ) on different planes (a) on horizontal plane $\mathrm{z} / d=0.68$ and vertical plane $x / d=0$, (b) on horizontal plane $\mathrm{z}=0+$ (above interface) and (c) on horizontal plane $\mathrm{z}=0-$ (below interface).
area and that this shear stress changes its sign outside the dislocation loop on the slip plane. The stress field decreases fast to zero outside the dislocation area. It is noted that an elastic dislocation loop can induce an electric displacement field $\mathbf{D}$ around the edge dislocation segment in the upper MEE half-space and near the interface area of the lower piezoelectric half-space (Figure 4(b)). It also induces a magnetic induction field B around both the edge and screw dislocation loops and near the interface area in the MEE half-space (Figure 4(c)).

The elastic dislocation-induced polarization field on the horizontal plane $z / d=0.68$ and vertical plane $x / d=0$ is shown in Figure 5. It is clear that the edge dislocation will induce a large polarization field along the two sides of the edge dislocation line in the upper MEE half-space and also a large polarization field near the interface area in the lower piezoelectric half-space.

The dislocation-induced polarization charge density $\rho$ is shown in Figure 6. Its contours on the horizontal plane $z / d=0.68$ and vertical plane $x / d=0$ are plotted in Figure 6(a), on horizontal plane $z=0+$ in Figure 6(b) and on horizontal plane $z=0$ in Figure 6(c). It can be clearly seen that the edge dislocation will induce a dipolelike polarization charge along the dislocation line (see $\rho$ on the horizontal plane $\mathrm{z} /$ $d=0.68$ in Figure 6(a)). While there is no evident charge in the lower piezoelectric half-space, a large charge is observed on both sides of the interface directly below the edge dislocation line (Figure 6(b) and (c) on $z= \pm 0$ planes). It is further observed that the charge density contours are different on both sides of the interface due to the material mismatch.

The Peach-Koehler force $\mathbf{F}^{\text {Image }}$ on the dislocation loop induced by the interface image stress field is shown in Figure 7. It can be seen that along the dislocation segment close to the interface, the force is pointing to the interface, attracting the dislocation to the interface (towards the softer material). Along the two vertical segments of the dislocation, the force tends to expand the dislocation loop laterally.


Figure 7. (colour online) The Peach-Koehler force $\mathbf{F}^{\text {Image }}$ (normalized by $b^{2} / d$ and in unit Pa ) on the dislocation loop due to the interface image stress.


Figure 8. (colour online) Contours of extended stress fields (normalized by $b / d$ ) on vertical planes $y / d=0$ and $x / d=0$ induced by the elliptic dislocation loop. (a) $|\mathrm{D}|$ (in $\mathrm{Cm}^{-2}$ ), (b) $|\mathrm{B}|$ (in Tm).

In the second numerical example, we consider that an elliptic dislocation loop lies on the horizontal plane $z / d=0.2$. Its major axis is along $x_{2}$-axis with a length $d$ (the character length of the ellipse) and its minor axis is along $x_{1}$-axis with a length $0.712 d$. The extended Burgers vector is again $\mathbf{b}=b[1,0,0,0,0]^{\mathrm{T}}$.

Shown in Figure 8(a) and (b) are the magnitude contours of the elliptic dislocation loop-induced electric displacement $\mathbf{D}$ and magnetic induction $\mathbf{B}$ on the vertical planes $y / d=0$ and $x / d=0$. It is clear that while this elliptic loop can induce large magnetic induction around its loop, it induces no electric displacement near it. However, it can induce a large electric displacement field in the lower electric half-space immediately below it.


Figure 9. (colour online) Contours of polarization field $|\mathbf{P}|$ (normalized by $b / d$ and in unit $\mathrm{Cm}^{-2}$ ) on vertical planes $y / d=0$ and $x / d=0$ induced by the elliptic dislocation loop.

The contour of polarization field $|\mathbf{P}|$ is shown in Figure 9 on vertical planes $y / d=0$ and $x / d=0$ induced by the elliptic dislocation loop. It is observed that there are four (nearly symmetrically distributed) contour concentrations around the edge part of the dislocation loop and also a large concentration below the loop in the lower piezoelectric half-space. The latter feature is consistent with the large electric displacement field observed in the piezoelectric half-space.

Similar to the square loop case, the elliptical dislocation-induced polarization field will in turn induce a polarization charge density in the bimaterial system. This is presented in Figure 10 where the contours of the charge density on two vertical planes $y / d=0$ and $x / d=0$ are shown in Figure 10(a), those on horizontal plane $z=0+$ in Figure 10(b) and on horizontal plane $z=0-$ in Figure 10(c). Figure 10(a) indicates clearly that on the plane of $y / d=0$, there are two anti-symmetric points which is the edge part of the dislocation loop (on both sides of the plane $x=0$ ) where a density with octal concentration pattern is formed. A large concentration immediately below the loop on both sides of the interface can be also observed, although their corresponding patterns are clearly different (Figure 10(b) vs. Figure 10 (c)).

Finally, the Peach-Koehler force $\mathbf{F}^{\text {Image }}$ on the loop due to the image stress is presented in Figure 11. Similar to the square loop case, this image force is pulling the elliptical dislocation towards the interface (or towards the softer material) with its large magnitude on the minor axis section.

## 6. Conclusions

In this paper, we develop the formulae and methods to analyse the coupled fields induced by an arbitrarily shaped 3D dislocation loop in general anisotropic MEE

(c)


Figure 10. (colour online) Contours of polarization charge density $\rho$ (normalized by $b / d^{2}$ and in unit $\mathrm{Cm}^{-2}$ ) on different planes induced by the elliptic dislocation loop (a) on vertical planes $y / d=0$ and $x / d=0$, (b) on horizontal plane $\mathrm{z}=0+$ (above interface) and (c) on horizontal plane $\mathrm{z}=0$ - (below interface).


Figure 11. (colour online) The Peach-Koehler force $\mathbf{F}^{\text {Image }}$ (normalized by $b^{2} / d$ and in unit Pa ) on the elliptic dislocation loop due to the interface image stress.
bimaterials. The derived solutions are also suitable for dislocations in the corresponding piezoelectric, piezomagnetic and purely elastic solids. As numerical examples, we have analysed the fields induced by dislocations of both straight and curved loops in MEE/ piezoelectric bimaterials and observed the following interesting features:
(1) An elastic dislocation loop can induce electric displacement field $\mathbf{D}$ around the edge dislocation segment in the MEE material; through the interface influence, such a dislocation can also induce $\mathbf{D}$ near the interface of the half-space material with high piezoelectric coefficients.
(2) An elastic dislocation loop can induce magnetic induction field $\mathbf{B}$ around both the edge and screw dislocation loops in the MEE material and can also induce B near the interface area in the MEE half-space or the material with high piezomagnetic coefficients.
(3) An elastic dislocation in MEE material can induce polarization field. Especially, the edge dislocation will induce large polarization field along two sides of the dislocation line in the MEE material and will also induce large polarization field near the interface of the other half-space material with high piezoelectric coefficients.
(4) An edge dislocation in MEE material will induce a dipole-like polarization charge along the dislocation line and also near the interface area of the other half-space material with high piezoelectric coefficients, while a screw dislocation does not induce any polarization charge.
(5) When an edge dislocation is close to the interface of the MEE bimaterial, it will induce a large polarization charge on the interface directly below the edge dislocation line.
(6) The interface image stress field in MEE bimaterials will induce an extended Peach-Koehler force $\mathbf{F}^{\text {Image }}$ acting on the dislocation loop close to the interface.

This force will attract the dislocation towards the softer material, and furthermore the image force tends to expand the dislocation loop laterally.

In the present model, we only considered one dislocation loop and assumed that the dislocation was located entirely within one material. Other kinds of dislocations, such as misfit dislocations, threading dislocations or dislocations piercing through interface, need to be considered in the future.

## Acknowledgements

The work was supported by National Natural Science Foundation of China (No. 10872179, 11172273 and 11272052). The first author is also grateful for the support from the China Scholarship Council.

## References

[1] C.W. Nan, M.I. Bichurin, S.X. Dong, D. Viehland and G. Srinivasan, J. Appl. Phys. 103 (2008) p. 031101.
[2] H.M. Wang, E. Pan, A. Sangghaleh, R. Wang and X. Han, Smart Mater. Struct. 21 (2012) p. 075003.
[3] R.J. Hao and J.X. Liu, Mech. Res. Commun. 33 (2006) p. 415.
[4] C.C. Ma and J.M. Lee, J. Appl. Phys. 101 (2007) p. 123513.
[5] J.M. Lee and C.C. Ma, Eur. J. Mech. A/Solids 29 (2010) p. 420.
[6] X. Han and E. Pan, Mech. Mater. 59 (2013) p. 110.
[7] Y.S. Lia, Z.Y. Caia and W. Wanga, Phil. Mag. 91 (2011) p. 3155.
[8] S. Lartigue-Korineka, O. Hardouin Duparcb, K.P.D. Lagerlöfc, S. Moulahema and A. Halli1a, Phil. Mag. 93 (2013) p. 1182.
[9] T. Mura, The continuum theory of dislocations, in Advances in Materials Research, H. Herman, ed., Interscience, New York, 1968, p. 1.
[10] T. Mura, Micromechanics of Defects in Solids, Martinus Nijhoff, Dordrecht, 1987.
[11] J.P. Hirth and J. Lothe, Theory of Dislocations, John Wiley and Sons, New York, 1982.
[12] T.J. Gosling and J.R. Willis, J. Mech. Phys. Solids 42 (1994) p. 1199.
[13] N.M. Ghoniem and X. Han, Phil. Mag. 85 (2005) p.2809.
[14] S. Akarapu and H.M. Zbib, Phil. Mag. 89 (2009) p. 2149.
[15] E.H. Tan and L.Z. Sun, Model. Simulat. Mater. Sci. Eng. 14 (2006) p. 993.
[16] E.H. Tan and L.Z. Sun, J. Nanomech. Micromech. 1 (2011) p. 91.
[17] H.J. Chu, E. Pan, X. Han, J. Wang and I.J. Beyerlein, J. Mech. Phys. Solids 60 (2012) p. 418.
[18] X. Han and E. Pan, J. Appl. Phys. 112 (2012) p. 103501.
[19] P.C.Y. Lee, J. Appl. Phys. 6 (1991) p. 7470.
[20] D.M. Barnett and J. Lothe, Phys. Status Solidi B 67 (1975) p. 105.
[21] V.I. Alshits, A.N. Darinskii and J. Lothe, Wave Motion 16 (1992) p. 265.
[22] M.Y. Chung and T.C.T. Ting, Phil. Mag. Lett. 72 (1995) p. 405.
[23] E. Pan and Z. Angew, Math. Phys. 53 (2002) p. 815.
[24] Y.E. Pak, J. Appl. Mech. 57 (1990) p. 863.
[25] H.O.K. Kirchner and V.I. Alshits, Phil. Mag. 74 (1996) p. 861.
[26] X. Han and N.M. Ghoniem, Phil. Mag. 85 (2005) p. 1205.
[27] H.Y. Kuo and E. Pan, J. Appl. Phys. 109 (2011) p. 104901.
[28] H.Y. Kuo and Y.L. Wang, Mech. Mater. 50 (2012) p. 88.
[29] E. Pan and F.G. Yuan, Int. J. Solids Struct. 37 (2000) p.5329.
[30] E. Pan and F.G. Yuan, Int. J. Eng. Sci. 38 (2000) p. 1939.

## Appendix A: Extended Green's functions and their derivatives

The 3D Green's functions in anisotropic elastic bimaterials were obtained by Pan and Yuan by virtue of the two-dimensional Fourier transform method [29]. Later on, the extended Green's functions in piezoelectric bimaterials and in MEE bimaterials were also obtained by Pan and Yuan [30] and Pan [23]. We will summarize the Green's functions in general anisotropic bimaterials in a uniform form and will also provide the solutions of their derivatives with respect to the field and/or source coordinates.

Using the 2D Fourier transformation

$$
\begin{equation*}
\tilde{f}\left(\xi_{1}, \xi_{2}, x_{3}\right)=\iint f\left(x_{1}, x_{2}, x_{3}\right) e^{i\left(\xi_{1} x_{1}+\xi_{2} x_{2}\right)} \mathrm{d} x_{1} \mathrm{~d} x_{2} \tag{A1}
\end{equation*}
$$

to the extended equilibrium equation for the extended Green's function problem, we have

$$
\begin{equation*}
-C_{\alpha J K \beta} \xi_{\alpha} \xi_{\beta} \tilde{G}_{K M}-i\left(C_{\alpha J K 3}+C_{3 J K \alpha}\right) \xi_{\alpha} \tilde{G}_{K M, 3}+C_{3 J K 3} \tilde{G}_{K M, 33}+\delta_{J M} e^{i y_{\alpha} \xi_{\alpha}}=0 \tag{A2}
\end{equation*}
$$

where $\alpha, \beta=1,2, i=\sqrt{-1}$. The lowercase index takes values $1 \sim 3$ and uppercase index (such as $J, K$, and $M$ ) takes value $1 \sim N_{\text {index }}$, with $N_{\text {index }}=3$ for elastic materials, $N_{\text {index }}=4$ for piezoelectric materials and $N_{\text {index }}=5$ for MEE materials. Repeated indexes take their corresponding summation.

The general solution of Equation (A2) satisfies the following extended Stroh eigenrelation:

$$
\begin{equation*}
\left[\mathbf{Q}+p\left(\mathbf{R}+\mathbf{R}^{T}\right)+p^{2} \mathbf{T}\right] \mathbf{a}=0 \tag{A3}
\end{equation*}
$$

with

$$
\begin{equation*}
Q_{J K}=C_{i J K s} n_{i} n_{s}, \quad R_{J K}=C_{i J K 3} n_{i}, \quad T_{J K}=C_{3 J K 3}, \quad \mathbf{n}=[\cos \theta, \sin \theta, 0]^{T} \tag{A4}
\end{equation*}
$$

and $p_{I}, \bar{p}_{I}$ are the eigenvalues, $\mathbf{a}_{I}, \overline{\mathbf{a}}_{I}$ are the associated eigenvectors, respectively. We select $\operatorname{Im}\left(p_{I}\right)>0$ and put the associated eigenvectors $\mathbf{a}_{I}$ into a matrix form as $\mathbf{A} \equiv\left[\mathbf{a}_{1}, \mathbf{a}_{2}, \ldots, \mathbf{a}_{N_{\text {index }}}\right]$.

The general solution in each half-space is available. Then, using the interface continuity condition, the Green's function tensor $\mathbf{G}(\mathbf{y} ; \mathbf{x})$ in general bimaterial can be obtained. In the solutions, the eigenvalues $p_{I}$ and associated eigenvectors $\mathbf{a}_{I}$ or matrix $\mathbf{A}$ in corresponding materials will be involved. We summarize them in the following.

When the point force acts at $\mathbf{y}$ (called source point) of the upper half-space of a bimaterial, i.e. $y_{3}>0$, the general Green's function tensor at $\mathbf{x}$ (called field point) is expressed as

$$
\mathbf{G}(\mathbf{y} ; \mathbf{x})=\left\{\begin{array}{rlr}
\mathbf{G}^{\infty}(\mathbf{y} ; \mathbf{x}) & +\mathbf{G}^{\text {Image }}(\mathbf{y} ; \mathbf{x}), & x_{3}>0  \tag{A5}\\
& +\mathbf{G}^{\text {Image }}(\mathbf{y} ; \mathbf{x}), & x_{3}<0
\end{array}\right.
$$

where $\mathbf{G}^{\infty}(\mathbf{y} ; \mathbf{x})$ is the full-space Green's function tensor with material properties in the source point ( $\mathbf{y}$ ) half space. In other words,

$$
\mathbf{G}^{\infty}(\mathbf{y} ; \mathbf{x})= \begin{cases}\frac{-1}{2 \pi^{2}} \int_{0}^{\pi} \overline{\mathbf{A}}^{1} \mathbf{G}_{u p}^{\infty}\left(\overline{\mathbf{A}}^{1}\right)^{T} \mathrm{~d} \theta, & x_{3}>y_{3}  \tag{A6}\\ \frac{1}{2 \pi^{2}} \int_{0}^{\pi} \mathbf{A}^{1} \mathbf{G}_{u l}^{\infty}\left(\mathbf{A}^{1}\right)^{T} \mathrm{~d} \theta, & x_{3}<y_{3}\end{cases}
$$

with superscripts 1 and 2 denoting quantities in materials 1 (upper half space) and 2 (lower half space), respectively, and

$$
\left\{\begin{array}{l}
\left(G_{u p}^{\infty}\right)_{I J}=\frac{\delta_{I J}}{\overline{p_{I}^{1}}\left(x_{3}-y_{3}\right)+\left(x_{1}-y_{1}\right) \cos \theta+\left(x_{2}-y_{2}\right) \sin \theta}=\frac{\delta_{I J}}{\mathbf{h}\left(\theta, \bar{p}_{I}^{1}\right) \cdot(\mathbf{x}-\mathbf{y})},  \tag{A7}\\
\left(G_{u l}^{\infty}\right)_{I J}=\frac{\delta_{I J}}{p_{I}^{1}\left(x_{3}-y_{3}\right)+\left(x_{1}-y_{1}\right) \cos \theta+\left(x_{2}-y_{2}\right) \sin \theta}=\frac{\delta_{1}}{\mathbf{h}\left(\theta, p_{I}^{1}\right) \cdot(\mathbf{x}-\mathbf{y})}
\end{array}\right.
$$

For compaction in writing, the vector $\mathbf{h}(\theta, p)$ is introduced as

$$
\begin{equation*}
\mathbf{h}(\theta, p)=[\cos \theta, \sin \theta, p]^{T} \tag{A8}
\end{equation*}
$$

The complementary part of Green's function added to satisfy the interface continuity condition is

$$
\mathbf{G}^{\text {Image }}(\mathbf{y} ; \mathbf{x})= \begin{cases}\frac{1}{2 \pi^{2}} \int_{0}^{\pi} \overline{\mathbf{A}}^{1} \mathbf{G}_{u p}\left(\mathbf{A}^{1}\right)^{T} \mathrm{~d} \theta, & x_{3}>0  \tag{A9}\\ \frac{1}{2 \pi^{2}} \int_{0}^{\pi} \mathbf{A}^{2} \mathbf{G}_{u l}\left(\mathbf{A}^{1}\right)^{T} \mathrm{~d} \theta, & x_{3}<0\end{cases}
$$

where

$$
\left\{\begin{array}{l}
\left(G_{u p}\right)_{I J}=\frac{\left(G_{1}\right)_{I J}}{-\bar{p}_{I}^{1} x_{3}+p_{J}^{1} y_{3}-\left[\left(x_{1}-y_{1}\right) \cos \theta+\left(x_{2}-y_{2}\right) \sin \theta\right]}=\frac{\left(G_{1}\right)_{I J}}{\left(G_{2}\right)_{I J}}  \tag{A10}\\
\left(G_{u l}\right)_{I J}=\frac{\mathbf{h}\left(\theta, \bar{p}_{I}^{1}\right) \cdot \mathbf{x}+\mathbf{h}\left(\theta, p_{J}^{1}\right) \cdot \mathbf{y}}{\left.-p_{I}^{2} x_{3}+p_{J}^{1} y_{3}-\left[\left(x_{1}-y_{1}\right) \cos \theta+\left(x_{2}-y_{2}\right) \sin \theta\right)\right]}=\frac{\left(G_{2}\right)_{I J}}{-\mathbf{h}\left(\theta, p_{I}^{2}\right) \cdot \mathbf{x}+\mathbf{h}\left(\theta, p_{J}^{1}\right) \cdot \mathbf{y}}
\end{array}\right.
$$

$$
\begin{align*}
& \mathbf{G}_{1}=-\left(\overline{\mathbf{A}}^{1}\right)^{-1}\left(\overline{\mathbf{M}}^{1}+\mathbf{M}^{2}\right)^{-1}\left(\mathbf{M}^{1}-\mathbf{M}^{2}\right) \mathbf{A}^{1} \\
& \mathbf{G}_{2}=-\left(\mathbf{A}^{2}\right)^{-1}\left(\overline{\mathbf{M}}^{1}+\mathbf{M}^{2}\right)^{-1}\left(\mathbf{M}^{1}+\overline{\mathbf{M}}^{1}\right) \mathbf{A}^{1} \quad, \quad \mathbf{M}^{\alpha}=-i \mathbf{B}^{\alpha}\left(\mathbf{A}^{\alpha}\right)^{-1}(\alpha=1,2) \tag{A11}
\end{align*}
$$

Similarly, when the point force $\mathbf{y}$ acts at the lower half-space of a bimaterial, i.e. $y_{3}<0$, the extended Green's function at the field point $\mathbf{x}$ is

$$
\mathbf{G}(\mathbf{y} ; \mathbf{x})=\left\{\begin{align*}
+\mathbf{G}^{\text {Image }}(\mathbf{y} ; \mathbf{x}), & x_{3}>0  \tag{A12}\\
\mathbf{G}^{\infty}(\mathbf{y} ; \mathbf{x})+\mathbf{G}^{\text {Image }}(\mathbf{y} ; \mathbf{x}), & x_{3}<0
\end{align*}\right.
$$

where $\mathbf{G}^{\infty}(\mathbf{y} ; \mathbf{x})$ is similar to that in Equation (A5), but with the correspondent quantities being those in material 2 (lower space). In other words, we have

$$
\mathbf{G}^{\infty}(\mathbf{y} ; \mathbf{x})= \begin{cases}\frac{-1}{2 \pi^{2}} \int_{0}^{\pi} \overline{\mathbf{A}}^{2} \mathbf{G}_{u p}^{\infty}\left(\overline{\mathbf{A}}^{2}\right)^{T} \mathrm{~d} \theta, & x_{3}>y_{3}  \tag{A13}\\ \frac{1}{2 \pi^{2}} \int_{0}^{\pi} \mathbf{A}^{2} \mathbf{G}_{u l}^{\infty}\left(\mathbf{A}^{2}\right)^{T} \mathrm{~d} \theta, & x_{3}<y_{3}\end{cases}
$$

with

$$
\left\{\begin{array}{l}
\left(G_{u p}^{\infty}\right)_{I J}=\frac{\delta_{I J}}{\mathbf{h}\left(\theta, \bar{p}_{I}^{2}\right) \cdot(\mathbf{x}-\mathbf{y})}  \tag{A14}\\
\left(G_{u l}^{\infty}\right)_{I J}=\frac{\delta_{I J}}{\mathbf{h}\left(\theta, p_{I}^{2}\right) \cdot(\mathbf{x}-\mathbf{y})}
\end{array}\right.
$$

The complimentary part is

$$
\mathbf{G}^{\text {Image }}(\mathbf{y} ; \mathbf{x})= \begin{cases}\frac{1}{2 \pi^{2}} \int_{0}^{\pi} \overline{\mathbf{A}}^{1} \mathbf{G}_{u p}\left(\overline{\mathbf{A}}^{2}\right)^{T} \mathrm{~d} \theta, & x_{3}>0  \tag{A15}\\ \frac{1}{2 \pi^{2}} \int_{0}^{\pi} \mathbf{A}^{2} \mathbf{G}_{u l}\left(\overline{\mathbf{A}}^{2}\right)^{T} \mathrm{~d} \theta, & x_{3}<0\end{cases}
$$

where

$$
\left\{\begin{align*}
\left(G_{u p}\right)_{I J}= & \frac{\left(G_{1}\right)_{I J}}{-\mathbf{h}\left(\theta, \bar{p}_{I}^{1}\right) \cdot \mathbf{x}+\mathbf{h}\left(\theta, \bar{p}_{J}^{2}\right) \cdot \mathbf{y}},  \tag{A16}\\
\left(G_{u l}\right)_{I J} & =\frac{\left(G_{2}\right)_{I J}}{-\mathbf{h}\left(\theta, p_{I}^{2}\right) \cdot \mathbf{x}+\mathbf{h}\left(\theta, \bar{p}_{J}^{2}\right) \cdot \mathbf{y}}
\end{align*}\right.
$$

$$
\left.\begin{array}{l}
\mathbf{G}_{1}=\left(\overline{\mathbf{A}}^{1}\right)^{-1}\left(\overline{\mathbf{M}}^{1}+\mathbf{M}^{2}\right)^{-1}\left(\overline{\mathbf{M}}^{2}+\mathbf{M}^{2}\right) \mathbf{A}^{2} \\
\mathbf{G}_{2}=-\left(\mathbf{A}^{2}\right)^{-1}\left(\overline{\mathbf{M}}^{1}+\mathbf{M}^{2}\right)^{-1}\left(\overline{\mathbf{M}}^{1}-\overline{\mathbf{M}}^{2}\right) \mathbf{A}^{2}
\end{array}, \quad \mathbf{M}^{\alpha}=-i \mathbf{B}^{\alpha}\left(\mathbf{A}^{\alpha}\right)^{-1} \quad(\alpha=1,2) \quad \text { A17 }\right)
$$

The derivatives of the extended Green's functions can be also obtained.
For the case of a point force at $y_{3}>0$, we have the infinite part as

$$
\mathbf{G}(\mathbf{y} ; \mathbf{x})_{, x_{l} \ldots y_{p}}^{\infty}= \begin{cases}\frac{-1}{2 \pi^{2}} \int_{0}^{\pi} \overline{\mathbf{A}}^{1}\left(\mathbf{G}_{u p}^{\infty}\right)_{, x_{l} \ldots y_{p}}\left(\overline{\mathbf{A}}^{1}\right)^{T} \mathrm{~d} \theta, & x_{3}>y_{3}  \tag{A18}\\ \frac{1}{2 \pi^{2}} \int_{0}^{\pi} \mathbf{A}^{1}\left(\mathbf{G}_{u l}^{\infty}\right)_{x_{l} \ldots y_{p}}\left(\mathbf{A}^{1}\right)^{T} \mathrm{~d} \theta, & x_{3}<y_{3}\end{cases}
$$

with

$$
\begin{gather*}
\begin{cases}\left(G_{u p}^{\infty}\right)_{I J, x_{l}}=\frac{H_{1}^{1}}{\left[\mathbf{h}\left(\theta, \bar{p}_{I}^{1}\right) \cdot(\mathbf{x}-\mathbf{y})\right]^{2}}, & H_{1}^{1}=-\delta_{I J} h_{l}\left(\theta, \bar{p}_{I}^{1}\right) \\
\left(G_{u l}^{\infty}\right)_{I J, x_{l}}=\frac{H_{2}^{1}}{\left[\mathbf{h}\left(\theta, p_{I}^{1}\right) \cdot(\mathbf{x}-\mathbf{y})\right]^{2}}, & H_{2}^{1}-\delta_{I J} h_{l}\left(\theta, p_{I}^{1}\right)\end{cases}  \tag{A19a}\\
\begin{cases}\left(G_{u p}^{\infty}\right)_{I J, x y_{p}}=\frac{H_{1}^{2}}{\left[\mathbf{h}\left(\theta, \bar{p}_{I}^{1}\right) \cdot(\mathbf{x}-\mathbf{y})\right]^{3}}, & H_{1}^{2}=-2 \delta_{I J} h_{l}\left(\theta, \bar{p}_{I}^{1}\right) h_{p}\left(\theta, \bar{p}_{I}^{1}\right) \\
\left(G_{u l}^{\infty}\right)_{I J, x_{l} y_{p}}=\frac{H_{2}^{2}}{\left[\mathbf{h}\left(\theta, p_{I}^{1}\right) \cdot(\mathbf{x}-\mathbf{y})\right]^{3}}, & H_{2}^{2}=-2 \delta_{I J} h_{l}\left(\theta, p_{I}^{1}\right) h_{p}\left(\theta, p_{I}^{1}\right)\end{cases}  \tag{A19b}\\
\begin{cases}\left(G_{u p}^{\infty}\right)_{I J, x y_{l} y_{p} y_{q}}=\frac{H_{1}^{3}}{\left[\mathbf{h}\left(\theta, \bar{p}_{I}^{1}\right) \cdot(\mathbf{x}-\mathbf{y})\right]^{4}}, & H_{1}^{3}=-6 \delta_{I J} h_{l}\left(\theta, \bar{p}_{I}^{1}\right) h_{p}\left(\theta, \bar{p}_{I}^{1}\right) h_{q}\left(\theta, \bar{p}_{I}^{1}\right) \\
\left(G_{u l}^{\infty}\right)_{I J, x l_{p} y_{p}}=\frac{H_{2}^{3}}{\left[\mathbf{h}\left(\theta, p_{I}^{1}\right) \cdot(\mathbf{x}-\mathbf{y})\right]^{4}}, & H_{2}^{3}=-6 \delta_{I J} h_{l}\left(\theta, p_{I}^{1}\right) h_{p}\left(\theta, p_{I}^{1}\right) h_{q}\left(\theta, p_{I}^{1}\right)\end{cases} \tag{A19c}
\end{gather*}
$$

For the image part, we have

$$
\mathbf{G}(\mathbf{y} ; \mathbf{x})_{x_{1} \ldots y_{p}}^{\text {Image }}= \begin{cases}\frac{1}{2 \pi^{2}} \int_{0}^{\pi} \overline{\mathbf{A}}^{1}\left(\mathbf{G}_{u p}\right)_{x_{l} \ldots y_{p}}\left(\mathbf{A}^{1}\right)^{T} \mathrm{~d} \theta, & x_{3}>0  \tag{A20}\\ \frac{1}{2 \pi^{2}} \int_{0}^{\pi} \mathbf{A}^{2}\left(\mathbf{G}_{u l}\right)_{x_{l} \ldots y_{p}}\left(\mathbf{A}^{1}\right)^{T} \mathrm{~d} \theta, & x_{3}<0\end{cases}
$$

with

$$
\begin{gather*}
\begin{cases}\left(G_{u p}\right)_{I J, x_{l}}=\frac{H_{1}^{1}}{\left[-\mathbf{h}\left(\theta, \bar{p}_{I}^{1}\right) \cdot \mathbf{x}+\mathbf{h}\left(\theta, p_{J}^{1}\right) \cdot \mathbf{y}\right]^{2}}, & H_{1}^{1}=\left(G_{1}\right)_{I J} h_{l}\left(\theta, \bar{p}_{I}^{1}\right) \\
\left(G_{u l}\right)_{I J, x_{l}}=\frac{H_{2}^{1}}{\left[-\mathbf{h}\left(\theta, p_{I}^{2}\right) \cdot \mathbf{x}+\mathbf{h}\left(\theta, p_{J}^{1}\right) \cdot \mathbf{y}\right]^{2}}, & H_{2}^{1}=\left(G_{2}\right)_{I J} h_{l}\left(\theta, p_{I}^{2}\right)\end{cases}  \tag{A21a}\\
\begin{cases}\left(G_{u p}\right)_{I J, x_{l} y_{p}}=\frac{\mathrm{A}_{1}^{2}}{\left[-\mathbf{h}\left(\theta, \bar{p}_{I}^{1}\right) \cdot \mathbf{x}+\mathbf{h}\left(\theta, p_{J}^{1}\right) \cdot \mathbf{y}\right]^{3}}, & H_{1}^{2}=-2\left(G_{1}\right)_{I J} h_{l}\left(\theta, \bar{p}_{I}^{1}\right) h_{p}\left(\theta, p_{J}^{1}\right) \\
\left(G_{u l}\right)_{I J, x y_{l}}=\frac{H_{2}^{2}}{\left[-\mathbf{h}\left(\theta, p_{I}^{2}\right) \cdot \mathbf{x}+\mathbf{h}\left(\theta, p_{J}^{1}\right) \cdot \mathbf{y}\right]^{3}}, & H_{2}^{2}=-2\left(G_{2}\right)_{I J} h_{l}\left(\theta, p_{I}^{2}\right) h_{p}\left(\theta, p_{J}^{1}\right)\end{cases} \tag{A21b}
\end{gather*}
$$

$$
\begin{cases}\left(G_{u p}\right)_{I J, x y_{p} y_{q}}=\frac{H_{1}^{3}}{\left[-\mathbf{h}\left(\theta, \bar{p}_{I}^{1}\right) \cdot \mathbf{x}+\mathbf{h}\left(\theta, p_{J}^{1}\right) \cdot \mathbf{y}\right]^{4}}, & H_{1}^{3}=6\left(G_{1}\right)_{I J} h_{l}\left(\theta, \bar{p}_{I}^{1}\right) h_{p}\left(\theta, p_{J}^{1}\right) h_{q}\left(\theta, p_{J}^{1}\right)  \tag{A21c}\\ \left(G_{u l}\right)_{I J, x y_{p} y_{q}}=\frac{H_{2}^{3}}{\left[-\mathbf{h}\left(\theta, p_{I}^{2}\right) \cdot \mathbf{x}+\mathbf{h}\left(\theta, p_{J}^{1}\right) \cdot \mathbf{y}\right]^{4}}, & H_{2}^{3}=6\left(G_{2}\right)_{I J} h_{l}\left(\theta, p_{I}^{2}\right) h_{p}\left(\theta, p_{J}^{1}\right) h_{q}\left(\theta, p_{J}^{1}\right)\end{cases}
$$

and $\left(G_{1}\right)_{I J}$ and $\left(G_{2}\right)_{I J}$ can be found in Equation (A11).
For the case of a point force at $y_{3}<0$, similar results can be obtained for the derivatives of the Green's functions.

## Appendix B: Material properties

Material properties of $\mathrm{BaTiO}_{3}$ and MEE (with $25 \% \mathrm{BaTiO}_{3}$ and $75 \% \mathrm{CoFe}_{2} \mathrm{O}_{4}$ ), assuming that all materials are transversely isotropic with poling directions along $x_{3}$-axes [27-28].

|  | $C_{11}$ | $C_{12}$ | $C_{13}$ | $C_{33}$ | $C_{44}$ | $e_{31}$ | $e_{33}$ | $e_{15}$ |  |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| $\mathrm{BaTiO}_{3}$ | 166 | 77 | 78 | 162 | 43 | -4.4 | 18.6 | 11.6 |  |
| MEE | 245 | 139 | 138 | 235 | 47.6 | -1.53 | 4.28 | 0.05 |  |
|  | $\varepsilon_{11}$ | $\varepsilon_{33}$ | $q_{31}$ | $q_{33}$ | $q_{15}$ | $\mu_{11}$ | $\mu_{33}$ | $\alpha_{11}$ | $\alpha_{33}$ |
| $\mathrm{BaTiO}_{3}$ | 11.2 | 12.6 | 0 | 0 | 0 | 0.05 | 0.1 | 0 | 0 |
| MEE | 0.13 | 3.24 | 378 | 476 | 331.2 | 3.57 | 1.21 | -3.09 | 2334.15 |

Elastic constants $C_{i j}$ are in GPa, piezoelectric constants $e_{i j}$ in $\mathrm{C} / \mathrm{m}^{2}$, dielectric constants $\varepsilon_{i j}$ in $10^{-9} \mathrm{~F} / \mathrm{m}$ (or $10^{-9} \mathrm{C}^{2} / \mathrm{Nm}^{2}$ ), piezomagnetic coefficients $q_{i j k}$ in $\mathrm{N} / \mathrm{Am}$, magnetic permeabilities $\mu_{i j}$ in $10^{-4} \mathrm{Ns}^{2} / \mathrm{C}^{2}$ and magnetoelectric coefficients $\alpha_{i j}$ in $10^{-12} \mathrm{Ns} / \mathrm{VC}$.


[^0]:    *Corresponding author. Email: pan2@uakron.edu

