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Backcalculation of pavement layer elastic modulus and thickness with measurement errors

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This paper presents a backcalculation method for pavement layer elastic modulus and thickness. The effect of deflection measurement errors on the backcalculated results is also considered. The falling weight deflectometer (FWD) data are generated by applying a load to the pavement while calculating deflection at various fixed distances from the load centre. The measurement errors in FWD data are simulated by perturbing the theoretical deflections. Using these data, a backcalculation technique based on the improved genetic algorithm is proposed. In order to deal with the measurement errors, besides the common root mean square, a new objective function called area value with correction factor is introduced to the backcalculation algorithm. Numerical examples for two- and four-layer pavement structures are presented, which show the capability of the proposed method in backcalculation of pavement layer modulus and thickness.

Keywords: deflection measurement error; area value with correction factor; backcalculation; pavement layer modulus and thickness; improved genetic algorithm

1. Introduction

Solutions to the problem of surface loading over an elastic half-space or layered structures are important to various technological and scientific fields including pavement engineering. Numerous analytical and/or numerical methods were proposed in the past to solve the circular loading problem in inhomogeneous elastic isotropic (Pan 1989; Oner 1990; Yue *et al.* 2005) and elastic non-isotropic (Hooper 1975; Rowe and Booker 1981; Kumar 1988; Doherty and Deeks 2003; Wang *et al.* 2006) structures. More recently, Chu *et al.* (2011) have studied the surface-loading problem corresponding to a layered, transversely isotropic magnetoelectroelastic half-space, while Wang *et al.* (2012) have studied the circular surface loading on an anisotropic magnetoelectroelastic half-space. Experimentally, non-destructive tests (NDTs) are commonly carried out on existing pavements to measure the surface deflections, which in turn are used to backcalculate the elastic moduli of the pavement layers. Elastic modulus is an important property of pavement materials. Different methods have been proposed by researchers to estimate the elastic modulus based on laboratory bending tests and empirical Equations (Bonnaure *et al.* 1977; Vennalaganti *et al.* 1994; Saltan *et al.* 2011), wave propagation methods (Szendrei and Freeme 1970; Briggs *et al.* 1992; Benedetto *et al.* 2009) and the falling weight deflectometer (FWD).

Several methods have been developed to backcalculate the mechanical properties of flexible pavement. These methods vary in analysis type, material model and optimisation algorithm. Dong *et al.* (2001) carried out the time-domain backcalculation of pavement structure material properties using 3D finite element method. In a comparative study, Goktepe *et al.* (2006) explained several methods and compared them in terms of modelling precision, computational expense, etc. Although Goktepe *et al.* (2006) considered only the static case, Seo *et al.* (2009) studied the dynamic effects of the deflection on the backcalculation procedure. They found that the DYN-BAL (Dynamic BALMAT), a pseudo-static backcalculation procedure, gave more reliable results than several other computer codes in use. Gopalakrishnan and Papadopoulos (2011) employed a novel machine learning concept called conformal prediction in pavement backcalculation. Backcalculation of pavement layer moduli and Poisson's ratio using data mining method was proposed by Saltan *et al.* (2011).

Since its introduction in 1970s (Ullidtz 1987), the FWD has been widely used in NDT of pavement throughout the world (FHWA-LTPP Technical Support Services Contractor 2000). The FWD test involves applying impact loads to a loading plate while measuring the vertical surface displacement of the pavement at different locations. The measured deflections from the FWD test along the pavement surface are then utilised to backcalculate the modulus of elasticity in each pavement layer. Although numerous approaches have been proposed for the backcalculation of layer modulus and thickness (Khazanovich *et al.* 2001; Von Quintus and Simpson 2002;

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Alkasawneh 2007; Alkasawneh *et al.* 2007a; Pan *et al.* 2008), there exist still some ambiguous factors that could substantially affect the accuracy of the backcalculation. These factors include the inaccuracies in measurement such as FWD deflection measurements, deflection data calibration (Irwin and Richter 2005; Orr *et al.* 2007) and temperature variation (Xu *et al.* 2002; Alkasawneh *et al.* 2007b), as well as the algorithmic issues in the backcalculation procedure. Studstad *et al.* (2000) reported that in the long-term pavement performance (LTPP) database, some FWD deflection sensors were mislocated and these sensors could yield major inaccuracies in backcalculated moduli. Furthermore, the backcalculation results depend significantly on the accurate thickness input. Briggs *et al.* (1992) investigated some LTPP monitoring sites using ground-penetrating radar in the backcalculation program MODULUS (Uzan *et al.* 1989), and found that the backcalculated material properties were extremely sensitive to pavement layer thickness. Jooste *et al.* (1998) found that even allowable and small variation in layer thickness could significantly influence the backcalculated moduli.

Although errors in FWD measurement data are very common in practical pavement engineering (Irwin and Richter 2005), there exist only a few computational approaches in handling them. Irwin *et al.* (1989) analysed the sources of the deflection error and illustrated, through a series of examples, how random errors in pavement deflection and thickness could affect the backcalculated moduli. Vennalaganti *et al.* (1994) investigated the remaining life of flexible pavements based on the predicted strains at the interfaces of different layers and found that errors in NDT load and deflection measurements could significantly affect the accuracy in the strain calculation and thus the predicted pavement remaining life. Meier (1995) developed a backcalculation method using the artificial neural network (ANN) with large volume of synthetic test data generated by static and dynamic pavement response models and concluded that significant errors in the backcalculated pavement moduli could come from errors in thickness. Siddhartan *et al.* (1996) investigated the errors in pavement FWD measurements and found that the corresponding backcalculated pavement layer moduli would vary 5–65%. Sharma and Das (2008) backcalculated the pavement layer moduli using the synthetically derived FWD normal and noisy deflections. They demonstrated that a trained ANN method in backcalculation would give more reliable and accurate results.

The effect of measurement errors on the backcalculation has not been thoroughly investigated. Acknowledging the inevitable existence of measurement errors, we thus propose a new objective function to weaken and even eliminate the effect of measurement errors in backcalculation.

Systematic and random errors are the two types of measurement errors recognised by pavement engineers. Due to the influence of temperature and/or improper operations (Xu *et al.* 2002; Irwin and Richter 2005; Orr *et al.* 2007; Alkasawneh *et al.* 2007b), systematic errors always exist whereas random errors cannot be eliminated. There are several calibration methods to deal with measurement errors. Strategic highway research program (SHRP) calibration procedure can reduce the systematic error to a large extent by periodic calibration of the FWD. However, the usage of this method is limited because it needs a lot of measurement data at a single test point as well as a skilled operator.

Genetic algorithm (GA) as a robust and randomised search algorithm (Goldberg 1989) can be employed to optimise the search domain for backcalculation in pavement engineering (Fwa *et al.* 1997; Reddy *et al.* 2004; Goktepe *et al.* 2006; Alkasawneh 2007). Fwa *et al.* 1997 developed a GA-based backcalculation program which performs comparably well against four other non-GA backcalculation programs. The merits of this method are the capability to overcome the issue of having many local optima in backcalculation procedure and the elimination of dependency of the solution on input seed values. The importance of GA parameters on the backcalculation procedure is undeniable. Optimal GA parameters for backcalculation of pavement layer moduli were conducted by Reddy *et al.* (2004) based on the level of accuracy desired and the corresponding computational effort. There are numerous backcalculation programs listed in Alkasawneh *et al.* (2007b). Most programs can only perform backcalculation for up to 20 layers of pavement due to the limitation associated with the mathematical formulation of their analytical solutions. This limitation restricts the modelling of pavement structures where the temperature variation exists along the depth direction.

BackGenetic3D is a program just developed by the University of Akron group which uses an improved GA and the efficient and accurate forward program *Multi-Smart3D* to backcalculate the thickness as well as the layer moduli of any pavement structure. There is no restriction on the number of layers, thickness, location of the response points, number of loading circles, the shape of the loading area and the type of applied loading. To the best of the authors' knowledge, this program is the first that can backcalculate the pavement moduli with arbitrary number of layers, loading conditions and loading types. Also in this paper, a new objective function, called area value with correction factor (AVCF), is proposed to deal with the measurement error. This paper is organised as follows. In Section 2, the measurement errors are analysed in terms of systematic and random errors, and are discussed based on the objective functions: root mean square (RMS) and AVCF. In Section 3, backcalculation approach based on the *BackGenetic3D* program is presented. In Section 4, parameters in typical pavement models are given, and in Section 5, the corresponding backcalculation results are discussed. Conclusions are drawn in Section 6.

2. Measurement errors and objective functions

Deflection measurement errors are generated by adding random errors to the theoretical deflections (Table 1) calculated from elastic layer theory. In our studies, different random errors are algebraically added to the theoretical deflection at each sensor, and the result is rounded to the nearest whole micrometer to follow the FWD recording format. For the analyses carried out in this paper, we make the following assumptions, some of which are similar to Studstad *et al.* (2000).

Assumption 1: For convenience, we assume that the measurement error ε_i of sensor i can be divided into two parts: systematic error ε_i^s and random error ε_i^r. The measured deflection at sensor i, d_i^m, can be written as

$$\begin{aligned} d_i^m &= d_i^t + \varepsilon_i \quad = d_i^t + (\varepsilon_i^s + \varepsilon_i^r) \\ &= d_i^t(1 + e_i^s) + \varepsilon_i^r \quad = d_i^t(1 + e_i^s + e_i^r) \\ &= d_i^t(1 + e_i), \end{aligned} \tag{1}$$

where d_i^t denotes the true measured deflection (or the measured deflection without any error) at sensor i, $e_i^s (= \varepsilon_i^s/d_i^t)$ is the relative systematic error, $e_i^r (= \varepsilon_i^r/d_i^t)$ is the relative random error and $e_i (= e_i^s + e_i^r)$ is the combination of the relative systematic and random errors. It should be pointed out that, unlike random error, the systematic error depends on the magnitude of the deflection. Thus, in the analysis below, we use the third relation in Equation (1) to express the measured deflection in terms of the relative systematic error e_i^s and random error ε_i^r as

$$d_i^m = d_i^t(1 + e_i^s) + \varepsilon_i^r. \tag{2}$$

Assumption 2: The random error ε_i^r follows a normal distribution with zero mean and shows very small deviation ($< 2\,\mu$m) as in Studstad *et al.* (2000).

Assumption 3: The relative systematic errors e_i^s at each sensor i are identical. Should the relative systematic error be not the same, we can just move the difference into the random error ε_i^r to satisfy:

$$e^s = e_1^s = \cdots = e_n^s, \tag{3}$$

where n denotes the number of sensors in FWD test.

Assumption 4: The centre deflection of FWD d_1^m is more reliable than those at other locations because of the following reasons:

(1) The deflection at different sensors, d_i^m, meets the following inequality:

$$d_i^m > d_{i+1}^m \quad (i = 1, 2, \ldots, n); \tag{4}$$

(2) All random errors, ε_i^r, are very small according to *Assumption 2*;
(3) All relative systematic errors, e_i^s, are identical according to *Assumption 3*.

Table 1. Theoretical deflections (true deflections without any error) of the two-layer pavement (one layer over a half-space), as shown in Figure 2.

Distance from load centre		Deflections	
mm	in.	μm	mil
0.0	0.0	982.2	38.7
203.2	8.0	791.9	31.2
304.8	12.0	669.9	26.4
457.2	18.0	516.4	20.3
609.6	24.0	400.6	15.8
914.4	36.0	257.3	10.1
1524.0	60.0	143.4	5.6

2.1 *Root mean square (RMS)*

A commonly used goodness-of-fit function in existing backcalculation procedures is the RMS.

$$F_{\mathrm{RMS}} = \left[\frac{1}{n}\sum_{i=1}^{n}\left(\frac{d_i^c - d_i^m}{d_i^m}\right)^2\right]^{1/2}, \tag{5}$$

where d_i^c is the backcalculated deflection at sensor i. When all deflections d_i^m are measured exactly without errors, Equation (5) works perfectly. However, we found through a series of numerical simulation that the backcalculated results based on RMS are very sensitive to the measurement errors. In other words, even a slight change in measured deflections could result in a dramatic variation in backcalculated layer moduli. This can be clearly seen from the following analysis.

To see the influence of the measurement errors in backcalculation, the theoretical modulus and thickness are used in the calculation of d_i^c so that the backcalculated d_i^c equals the true measured deflection d_i^t at given sensor i. Thus, making use of the last expression in Equation (1), we have

$$F_{\mathrm{RMS}} = \left[\frac{1}{n}\sum_{i=1}^{n}\left(\frac{e_i}{1 + e_i}\right)^2\right]^{1/2}. \tag{6}$$

It is clear that the relative error of each sensor operates equally in the backcalculation procedure and neither systematic nor random error is weakened or eliminated. Since RMS is unable to treat the measurement errors, we therefore propose a new objective function, which can reduce the error effect significantly.

2.2 *Area value with correction factor (AVCF)*

According to Pierce (1999), the 'area' value represents the normalised area of a slice which means the area divided by the deflection measured at the centre of the test load d_1. The area algorithm has been used extensively to analyse concrete pavement deflection basins since 1980 (Ioannides *et al.* 1989; Hall 1991; Hall *et al.* 1997). To generalise the area value, we define the area value A_k of the first k sensors as

$$A_k = \frac{\sum_{i=1}^{k-1}(d_i + d_{i+1})(r_{i+1} - r_i)}{2d_1}, \quad (k \le n), \tag{7}$$

where d_i denotes the deflection at sensor i and r_i is the distance between load centre and sensor i. In order to consider the error at each sensor, we define a new objective function called AVCF.

$$F_{\mathrm{AVCF}} = \left\{ \frac{1}{n-1} \sum_{k=1}^{n-1} \left(\frac{A_k^c - A_k^m}{A_k^m} \right)^2 \right\}^{1/2} + \left| \frac{d_1^c - d_1^m}{d_1^m} \right|, \tag{8}$$

where A_k^c and A_k^m are, respectively, the backcalculated and measured areas. The first term in Equation (8) not only eliminates the systematic errors and weakens the random errors, but also gives full consideration to the deviation at each sensor. The second term works like a correction factor which can adjust the backcalculated deflection close to the measured value. It is noted that if the calculated deflection at the centre equals the measured value, the second term in Equation (8) equals zero. Therefore, Equation (8) is superior in handling measurement errors than Equation (5). This function can also make the backcalculated result close to the measured value, independent of the backcalculation algorithm used.

In order to understand how the errors are weakened or eliminated in AVCF, we replace the calculated deflection with the true measured deflection while expressing the formula in terms of the relative error. The area term in the first part of Equation (8) can be rewritten as

$$\left| \frac{A_k^c - A_k^m}{A_k^m} \right| = \left| \sum_{i=1}^{n} \Psi_i \right|, \tag{9}$$

where

$$\Psi_i = \frac{d_i^t (e_1^r - e_i^r)(r_{i+1} - r_{i-1})}{\Delta}. \tag{10}$$

We denote

$$\begin{aligned} \Delta &= \sum_{j=1}^{n-1} \left[d_j^t (1 + e_j) + d_{j+1}^t (1 + e_{j+1}) \right] (r_{j+1} - r_j) \\ &= \sum_{j=1}^{n-1} \left[d_j^m + d_{j+1}^m \right] (r_{j+1} - r_j), \end{aligned} \tag{11}$$

which states that Δ is a constant depending only on the measurement data. It is shown in Equations (9) and (10) that all relative systematic errors e^s are eliminated and that the relative random error e_i^r at each sensor i is also weakened by subtracting from e_1^r and dividing a constant. We point out that this analysis is based on the assumption that the measurement error can be divided into systematic and random errors, which could be difficult in practice.

3. The proposed analysis approach

The *BackGenetic3D* program is a new GUI-based program that is capable of backcalculating the elastic modulus and thickness simultaneously. From the analytical point of view, the program searches the elastic modulus and/or thickness domains and determines the optimal solution using the GA search technique. The *MultiSmart3D* program developed by the University of Akron group is used for forward calculations. To consider possible errors in measurements, a series of deflections with errors are generated using a uniform distribution generator.

3.1 *Genetic algorithm*

GAs are robust and randomised search algorithms based on the evolution theory and natural genetics (Goldberg 1989). These algorithms are used to generate useful solutions to optimisation. Alkasawneh (2007) introduced different steps in GA originally established by Mitchell (1999). In this study, we use an improved GA to backcalculate the elastic moduli and thickness. Figure 1 shows the main components of the improved GA and their sequence.

3.2 *Generation of perturbed deflections*

We use *MultiSmart3D* program designed by our group to calculate the theoretical surface responses d_i^t (to mimic the measurement without any error) at sensor i for the given layer moduli, Poisson's ratios and thicknesses. In order to simulate the measured deflections with errors d_i^m, we perturb the theoretical deflection d_i^t 40 times by adding an error term (Equation (2)), which include systematic and random errors. Here, the relative systematic errors e^s are given by a uniform distribution generator with the accuracy within $\pm 8\%$, whereas the random errors ε_i^r are provided by a normal distribution generator with zero mean and 2μ deviations.

3.3 *Backcalculation of layer moduli and thickness based on the perturbed deflections*

With fixed Poisson's ratios, backcalculation of layer moduli and thickness is carried out by using the perturbed

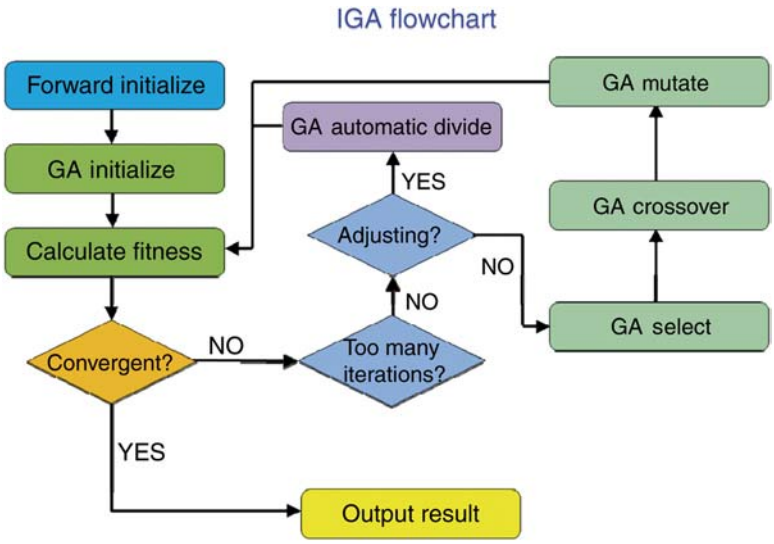

Figure 1. Improved GA flowchart for backcalculation.

deflections as input. Two objective functions, RMS and AVCF, are used. In order to illustrate the performance of the two objective functions, we also present the error and standard derivations of the backcalculated layer moduli and thickness for a two-layer pavement model. We present the numerical examples below.

4. Typical pavement models

Similar to most backcalculation programs, we base our analysis on the layered elastic theory to model real pavement behaviours. A simple two-layer pavement is first analysed. Poisson's ratios and modulus of elasticity of the top layer and subgrade (i.e. the half-space), the thickness of the top layer, the load magnitude and the plate radius are given in Figure 2. Furthermore, we assume that seven sensors ($i = 1–7$, starting from the centre of the loading, Studstad *et al.* 2000) are used in the FWD system with deflections listed in Table 1. To determine the influence of measurement errors on backcalculated layer elastic modulus and thickness, a group of 40 simulated measurement deflection errors are generated. All the simulated measurement deflections d_i^m are given by Equation (2), where the relative systematic errors e_i^s and random errors ε_i^r are provided using the distribution generators. After obtaining the simulated measurement data with errors, the two objective functions in Equations (6) and (8) are used to backcalculate the modulus and thickness of the pavement. The backcalculation error due to the measurement error can be obtained by comparing the backcalculated moduli and thickness to the theoretical values (the ones we used in our forward calculation). A four-layer pavement model is also considered for the backcalculation of elastic moduli and thickness. The geometry and material properties of this four-layer pavement are given in Figure 3.

5. Numerical results and discussions

Table 2 and Figures 4 and 5 show the backcalculated results for elastic moduli and thickness of the two-layer pavement model (Figure 2) using both the RMS and AVCF objective functions. The elastic moduli and thickness are backcalculated using the improved GA developed by the University of Akron group. When there are no systematic and random errors in the measurement

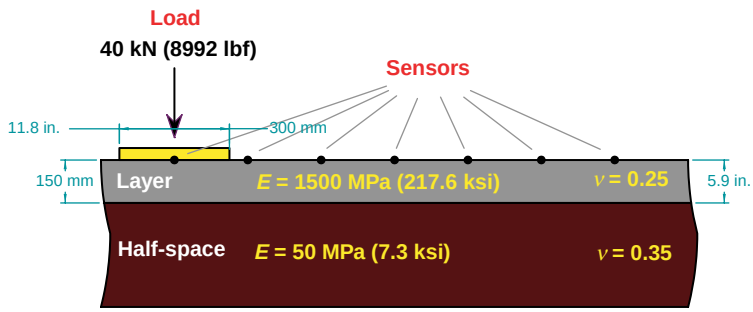

Figure 2. Schematic view of a two-layer pavement model.

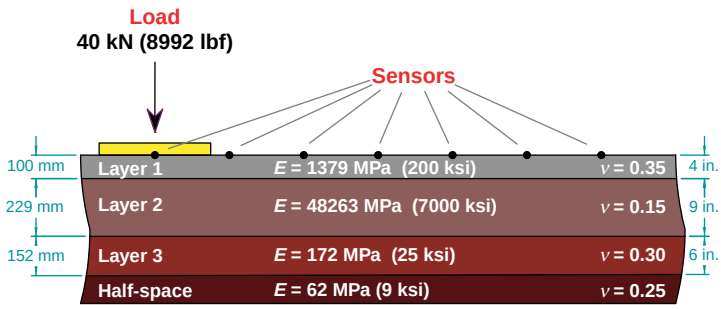

Figure 3. Schematic view of a four-layer pavement model.

data (Figures 4 and 5(a)), the backcalculated values of the thickness of the top layer and the elastic moduli of the top layer and half-space are exactly equal to the measured value with zero error percentage (Table 2). In other words, e^s and ε_i^r are both zero in Equation (2) so that the measurement deflections in FWD are equal to the true values. As can be observed from Table 2 and Figures 4 and 5(a), when there is no error, both objective functions work perfectly in backcalculation of elastic moduli and thickness with only very small standard deviation.

Now, we assume that there are random errors in measurement deflections while no systematic error exists. The random errors are provided by a normal distribution generator with zero mean and 2μ deviations. The backcalculated results for moduli and thickness are presented in Table 2 and Figures 4 and 5(b) for this case. Despite insignificant standard deviation for the half-space, it is obvious that the standard deviation and error percentage for elastic modulus and thickness of the top layer are smaller when we use AVCF as the objective function than RMS. Also at the top layer, the backcalculated modulus by AVCF function is closer to the exact value than that by RMS, which shows that AVCF is more accurate in backcalculation analysis. Although both objective functions can backcalculate accurately the modulus in the subgrade layer, the AVCF function is significantly more accurate than RMS in backcalculating the thickness of the pavement layer.

Table 2. Backcalculation results for the two-layer pavement model as shown in Figure 2 using two objective functions (the exact values for E_1, E_2 and h_1 are, respectively, 1500 MPa, 50 MPa and 0.15 m).

	RMS			AVCF		
	E_1 (MPa)	E_2 (MPa)	h_1 (m)	E_1 (MPa)	E_2 (MPa)	h_1 (m)
Without error						
Min.	1498.40	50.00	0.150	1497.30	50.00	0.150
Max.	1503.22	50.00	0.150	1502.95	50.01	0.150
Mean	1500.04	50.00	0.150	1500.01	50.00	0.150
Standard deviation	0.72	0.00	0.000	1.44	0.00	0.000
% Error	0.00	0.00	0.000	0.00	0.00	0.000
Only random error						
Min.	1237.30	49.35	0.134	1256.88	49.49	0.141
Max.	1985.59	50.51	0.164	1743.23	50.38	0.162
Mean	1538.14	50.00	0.149	1511.90	49.99	0.150
Standard deviation	212.67	0.28	0.008	119.31	0.23	0.005
% Error	2.54	0.00	0.667	0.79	0.02	0.000
Only systematic error						
Min.	1390.01	46.34	0.150	1495.96	50.00	0.150
Max.	1624.12	54.10	0.150	1502.98	50.00	0.150
Mean	1502.99	50.10	0.150	1499.94	49.99	0.150
Standard deviation	68.71	2.29	0.000	1.32	0.00	0.000
% Error	0.20	0.20	0.000	0.00	0.02	0.000
Random and systematic error						
Min.	1168.30	46.17	0.134	1274.35	49.49	0.141
Max.	1986.70	54.32	0.163	1733.25	50.39	0.161
Mean	1542.08	50.11	0.149	1510.83	49.99	0.150
Standard deviation	223.05	2.37	0.008	118.84	0.23	0.005
% Error	2.81	0.22	0.667	0.72	0.02	0.000

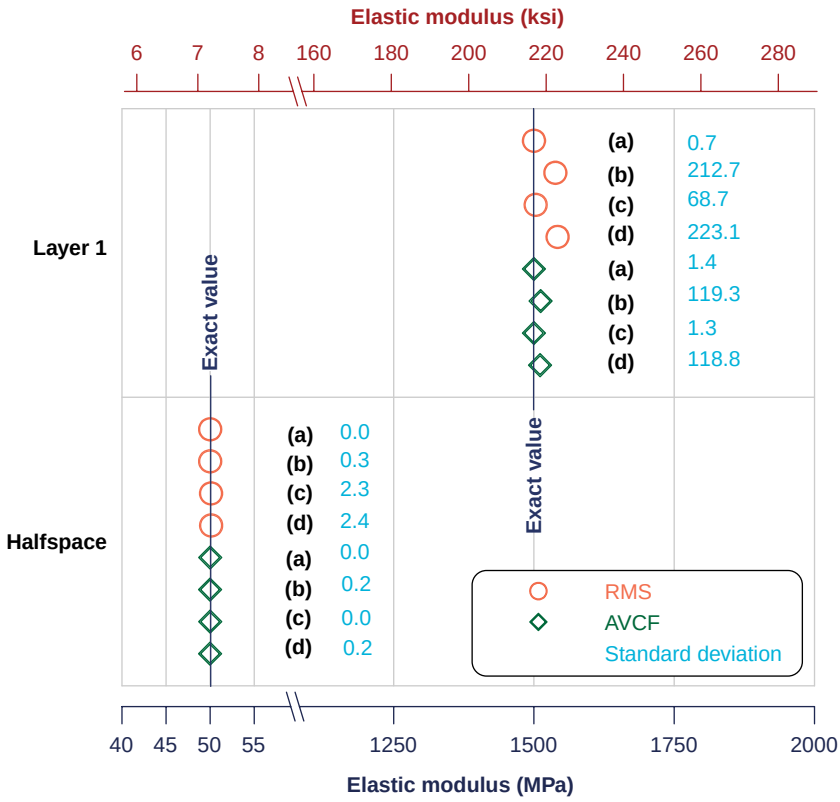

Figure 4. Backcalculated Young's moduli E_1 and E_2 of the two-layer pavement model by two objective functions where (a) no error exists, (b) only random error exists, (c) only systematic error exists and (d) combination of random and systematic errors exists.

Table 2 and Figures 4 and 5(c) also show the backcalculated results for the elastic moduli and thickness of the two-layer pavement shown in Figure 2 using RMS and AVCF when there is a systematic error. The relative systematic errors are given by a uniform distribution generator with the accuracy level of $\pm 8\%$. The backcalculated elastic moduli and thickness are very close to the exact values in this case. Although the standard

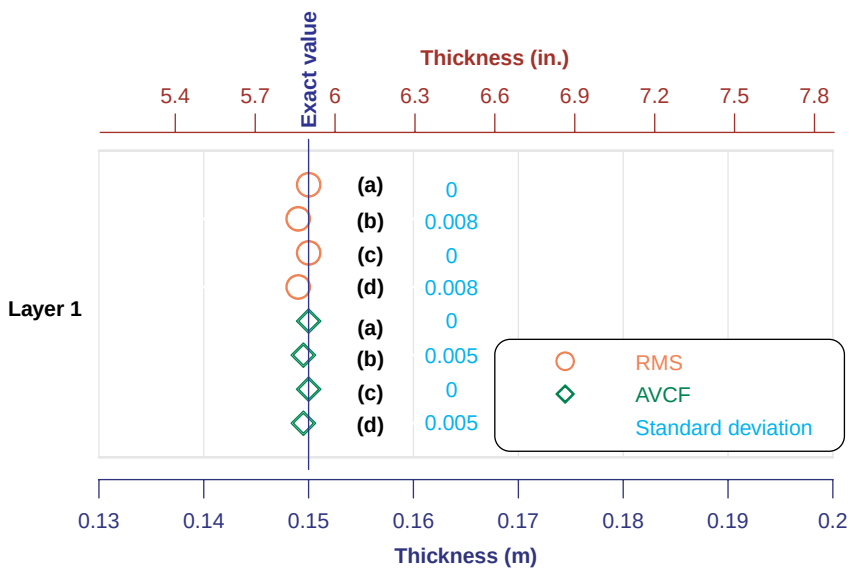

Figure 5. Backcalculated thickness h_1 of the two-layer pavement model by two objective functions where (a) no error exists, (b) only random error exists, (c) only systematic error exists and (d) combination of random and systematic errors exists.

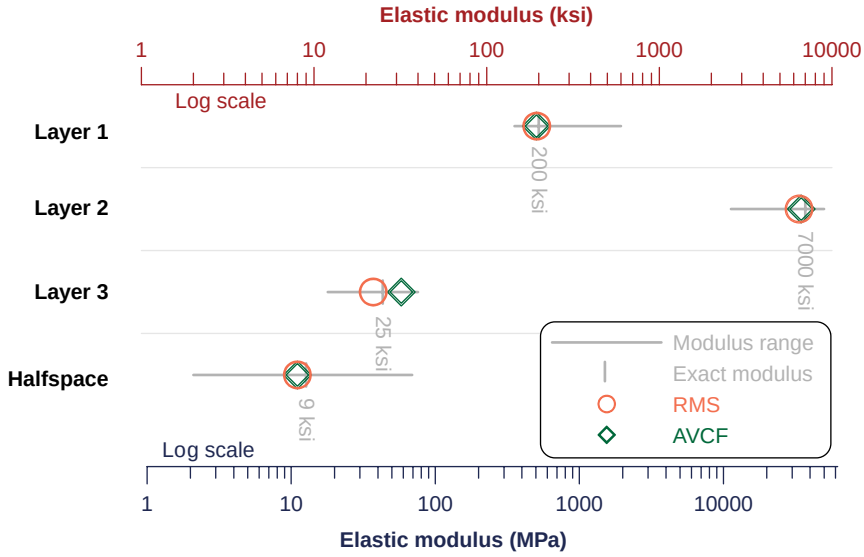

Figure 6. Backcalculated Young's moduli E_i of the four-layer pavement model by two objective functions.

deviation for the backcalculation of the elastic moduli is acceptable using RMS, the standard deviation using AVCF is completely negligible.

We now consider the situation in which not only the systematic errors but also the random errors exist. As shown in Table 2 and Figures 4 and 5(d), under the influence of the combined errors, the backcalculated results using the RMS function are clearly not satisfactory except for the elastic modulus of the second layer of the pavement. However, accurate results can still be obtained using the AVCF objective function. The results for the backcalculated thickness of the top layer in this case also confirm the superiority of the AVCF over RMS.

Figures 6 and 7 show the backcalculated elastic moduli and thicknesses for the four-layer pavement shown in Figure 3. The deflections at the sensor points are exact, obtained by the efficient and accurate forward program *MultiSmart3D*. The range of the seed values for the elastic moduli is based on the recommendation for the composite pavement containing the asphalt concrete, Portland cement concrete, granular base and subgrade layers. When backcalculating the elastic moduli in different layers, the thickness of each layer is fixed at its exact value, whereas in backcalculating the thickness, the elastic moduli are fixed. It is observed from Figures 6 and 7 that the backcalculated results on moduli are acceptable for all

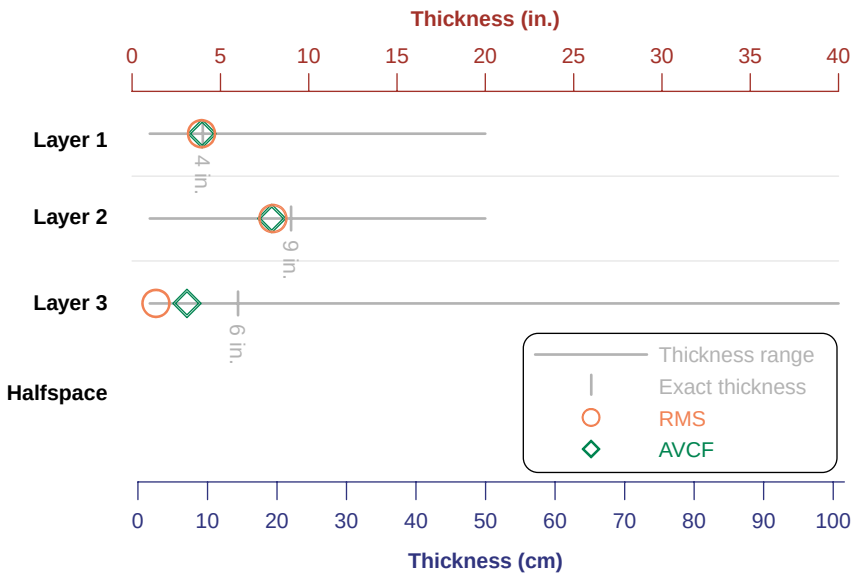

Figure 7. Backcalculated thicknesses h_i of the four-layer pavement model by two objective functions.

Table 3. Comparison of backcalculated elastic moduli of six different pavement structures (including flexible, rigid and composite) using different backcalculation programs.

					Backcalculated moduli (ksi)/relative errors (%)							
Case	Material	Thickness (in.)	Poisson's ratio	Modulus (ksi)	MODCOMP3		WESDEF		MODULUS 4.0		BACK GENETIC3D	
1	AC	8	0.35	1000	1100.4	10.0	960.8	−3.9	969.7	−3.0	1018	1.8
	CTB	6	0.20	2000	1610.1	−19.5	2436.6	21.8	2130.9	6.5	2137	6.9
	SG		0.40	20	19.8	−0.9	18.4	−7.8	20.1	0.5	20	0.0
2	PCC	9	0.15	4000	3572.5	−10.7	3645.3	−8.9	3118.8	−22.0	4051	1.3
	LTB	6	0.20	60	118.7	97.8	177.0	194.9	237.3	295.5	65	8.3
	SG		0.40	20	19.8	−1.0	18.3	−8.7	19.7	−1.5	20	0.0
3	PCC	6	0.15	3000	2622.3	−12.6	3257.2	8.6	2970.8	−1.0	3005	0.2
	SG		0.40	15	15.5	3.3	14.0	−6.8	15.1	0.7	15	0.0
4	PCC	12	0.15	4000	4000.0[a]	–	3757.1	−6.1	4246.4	6.2	3986	−0.4
	CTB	6	0.20	2000	1794.5	−10.3	2406.3	20.3	1829.6	−8.5	2112	5.6
	SG		0.40	10	9.9	−0.9	8.6	−14.1	10.0	0.0	10	0.0
5	AC	3	0.35	300	298.5	−0.5	256.0	−14.7	304.0	1.3	321	7.0
	PCC	9	0.15	4000	3240.6	−19.0	4900.0	22.5	3883.5	−2.9	4102	2.6
	SG		0.40	30	31.4	4.6	26.8	−10.6	30.3	1.0	30	0.0
6	AC	4	0.35	500	500.0[a]	–	493.1	−1.4	447.9	−10.4	526	5.2
	PCC	12	0.15	4000	4000.0[a]	–	3733.9	−6.7	7096.6	77.4	3932	−1.7
	ATB	8	0.20	1000	940.4	−6.0	1300.6	30.1	367.3	−63.3	1317	31.7
	SG		0.40	15	14.8	−1.4	13.0	−13.4	15.1	0.7	14	−6.7

Notes: The exact pavement properties, exact deflections, as well as the backcalculated results by MODCOMP3, WESDEF and MODULUS 4.0 are taken from SHRP (1993). AC, asphalt concrete; CTB, cement-treated base; SG, subgrade; PCC, Portland cement concrete; LTB, lime-treated base; ATB, asphalt-treated base.
[a] Fixed elastic modulus at exact value.

layers using both objective functions. The backcalculated thicknesses are almost acceptable except for the third layer using both objective functions. It is noteworthy that the subgrade material properties in rigid pavements and the base/subbase material properties in flexible and composite pavements could significantly affect the backcalculated results (ASTM 2003).

Table 3 lists the backcalculated elastic moduli for six different and typical pavement structures (including flexible, rigid and composite) using different backcalculation programs. The exact pavement properties, exact deflections, as well as the backcalculated results by MODCOMP3, WESDEF and MODULUS 4.0 are taken from SHRP (1993). It is obvious from Table 3 that the *BackGenetic3D* program is, in general, more reliable than the existing backcalculation software programs.

In general, when there is no error in deflections, RMS is the most commonly used objective function in backcalculation of the elastic modulus and thickness. For deflections with measurement errors, AVCF is more attractive and accurate due to its ability of reducing the systematic error. When the expected value of the relative systematic errors equals to zero, both RMS and AVCF can obtain accurate results. If there are random errors, RMS would perform poorly, but AVCF can still backcalculate accurately. Due to the temperature and/or possible improper operation effect, the systematic errors would be always different from zero. Therefore, only the AVCF function can be used to backcalculate reliable elastic modulus and thickness of the pavement. The new backcalculation program presented here can also be applied to any number of layers to backcalculate the elastic modulus as well as the thickness, whereas the number of pavement layers is limited in other existing programs.

6. Conclusions

Selection of objective functions is very important in backcalculation of pavement layer modulus and thickness. In this paper, we have presented a detailed study on two objective functions applied to a two-layer pavement model containing measurement errors. Besides RMS, the newly introduced objective function AVCF is efficient and accurate in backcalculation of pavement modulus and thickness. It is noted that although RMS is sensitive to measurement errors, AVCF is very accurate even when there are measurement errors. Thus, this new function AVCF could be remarkably helpful in future backcalculation of pavement properties. The proposed backcalculation program *BackGenetic3D* is also applied to a four-layer pavement based on both RMS and AVCF, and the backcalculated results are all acceptable. Our program is further compared to the existing backcalculation approaches for a couple of different and typical pavements and it shows that in general our program is more reliable in backcalculating the layer modulus, not to mention that it can be applied to the pavement structures with any number of layers.

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