11. Cai GX, Yuan FG (1999) Electric current-induced stresses at the crack tip in conductors. Int J Fract 96:279–301
12. Liu TJC (2008) Thermo-electro-structural coupled analyses of crack arrest by Joule heating. Theor Appl Fract Mech 49:171–184
13. Liu TJC (2011) Finite element modeling of melting crack tip under thermo-electric Joule heating. Eng Fract Mech 78:666–684
14. Cheng DK (1983) Field and wave electromagnetics. Addison-Wesley, Reading
15. Incropera FP, DeWitt DP (2002) Fundamentals of heat and mass transfer, 5th edn. Wiley, New York
16. Boresi AP, Chong KP (2000) Elasticity in engineering mechanics, 2nd edn. Wiley, New York
17. ANSYS, Inc. (2005) ANSYS HTML online documentation. SAS IP, Inc., USA
18. Tsai CL, Dai WL, Dickinson DW, Papritan JC (1991) Analysis and development of a real-time control methodology in resistance spot welding. Weld J 70:s339–s351
19. Braunovic M, Konchits VV, Myshkin NK (2007) Electrical contacts: fundamentals, applications and technology. CRC Press, Boca Raton
20. Liu S, Liu J, Ao T, Bai XZ (2004) Electric current density factor and its distribution. J Basic Sci Eng 12:121–126 (in Chinese)
21. Fan HL, Chen P (2005) Crack arrest effect in thin plates. Acta Armamentarii 26:791–794 (in Chinese)
22. Jin XQ, Li H (2006) On the analogy between the current density distribution in a thin conductive plate and the anti-plane shear problem. Mech Eng 28:23–27 (in Chinese)
23. Barsoum RS (1976) On the use of isoparametric finite elements in linear fracture mechanics. Int J Numer Meth Eng 10:25–37
24. Lim IL, Johnston IW, Choi SK (1992) Comparison between various displacement-based stress intensity factor computation techniques. Int J Fract 58:193–210
25. Liu TJC (2012) Local hot region at crack tip or notch tip under electric load and Joule heating effect. In: Proceedings of the 12th international conference on creep and fracture of engineering materials and structures (Creep 2012), Kyoto

Crack Surface Interference

► Crack Closure

Crack Surfaces Contact

► Crack Closure

Cracks in Transversely Isotropic and Inhomogeneous Elastic Solids

Ernian Pan[1,2], Chunying Dong[3], Ali Sangghaleh[2] and Yanfei Zhao[2]
[1]Henan Key Engineering Laboratory for Anti-fatigue Manufacturing Technology and the School of Mechanical Engineering, Zhengzhou University, Zhengzhou, People's Republic of China
[2]Department of Civil Engineering, Computer Modeling and Simulation Group, University of Akron, Akron, OH, USA
[3]Department of Mechanics, School of Aerospace Engineering, Beijing Institute of Technology, Beijing, People's Republic of China

Synonyms

Crack

Overview

Fracture mechanics is essential to the mechanical safety of structures, in which cracks and the corresponding stress intensity factors (SIFs) near their tips (fronts) are important [1]. In 1957, Irwin [2] introduced the SIFs to describe the stress and displacement fields near a crack tip. As it is well known, there are three basic crack modes: opening (mode I), sliding (mode II), and tearing (mode III). Determining the SIFs near the crack tip (or front) in linear elasticity is interesting yet challenging. While most previous studies in SIFs were focused on one or two fracture modes, mixed three-dimensional (3D) modes need to be considered as materials could be mostly failed under combined tensile/compressive, shearing, and tearing loads or the material under consideration is anisotropic (as for most composite materials). For 3D isotropic elastic materials, Singh et al. [3] obtained the SIFs using the concept of a universal crack closure integral. For transversely isotropic (TI), orthotropic, and anisotropic solids, Pan and Yuan [4] presented the general relationship between the SIF and the relative crack opening displacement (COD). Lazarus et al. [5] compared the calculated SIFs with experimental results for brittle solids under mixed mode I-III or I-II-III loadings. The 3D SIFs were also calculated by Zhou et al. [6] using the variable-order singular boundary element. More recently, Yue et al. [7] employed the boundary element method (BEM) [8, 9] in their calculation of the 3D SIFs of an inclined square crack within a finite but bimaterial domain. Other representative works in this direction are those by Liu et al. [10], Blackburn [11], dell'Erba and Aliabadi [12], Partheymüller [13], Hatzigeorgiou and Beskos [14], Popov [15], Ariza and Dominguez [16], Lo et al. [17], and Zhao et al. [18]. The weakly singular and weak-form integral equation method recently proposed by Rungamornrat [19] and Rungamornrat and Mear [20] is also efficient in crack analysis in anisotropic media. Besides the analytical (integral equation) and BEM methods [21], other common methods, such as the finite difference (FD) [22–24] and finite element (FE) [25, 26], were also applied to the 3D SIF analysis. Since both the FD and FE methods require discretization of the whole problem domain, they could be time consuming and more expensive than the BEM in fracture analyses.

While BEM is an excellent choice for fracture mechanics analysis in a linear and homogeneous solid, material heterogeneity or inhomogeneity introduces complexity to this approach. Nevertheless, various progresses have been made in modifying BEM for the inhomogeneity systems, including composites, rock structures, porous and cracked media. Bush [27] investigated the interaction between a crack and a particle cluster in composites using the BEM. Also applying the BEM, Knight et al. [28] analyzed the effect of the constituent material properties, fiber spatial distribution, and microcrack damage on the local behavior of fiber-reinforced composites. Dong et al. [29] presented a general-purpose integral formulation in order to study the interaction between the inhomogeneity and cracks embedded in 3D isotropic matrices. Based on a symmetric Galerkin BEM, Kitey et al. [30] investigated the crack growth behavior in materials embedded with a cluster of inhomogeneities. Lee and Tran [31] applied the Eshelby equivalent inclusion method to carry out the stress analysis when a penny-shaped crack interacts with inhomogeneities and voids. Dong et al. [32] investigated the interaction between cracked TI inhomogeneous solids using a special BEM formulation. Interface cracks in two or more isotropic materials were also studied by Sladek and Sladek [33] and Liu and Xu [34]. Recent representative developments in this direction include the three-step multi-domain BEM solver [35], the subregion-by-subregion approach based on the Krylov solver [36, 37], and the well-known fast multipole BEM [38, 39].

In this entry, we will give a brief account on 3D linear fracture mechanics in TI inhomogeneous materials, based on the BEM approach. The field responses, the relative crack opening displacement (COD), as well as the SIFs will be discussed.

Governing Equations

– Equations of equilibrium

$$\sigma_{ij,j} + b_i = 0, \quad i,j = 1,2,3 \tag{1}$$

where σ_{ij} is the stress tensor; b_i the body force; and the subscript "j" denotes the partial differentiation with respect to the coordinates x, y, and z.

– Strain and displacement relation

$$\varepsilon_{ij} = 0.5\left(u_{i,j} + u_{j,i}\right), \quad i,j = 1,2,3 \tag{2}$$

where u_i is the elastic displacement.

– Constitutive relation

Again, we assume that the material is TI and we let the global z-axis be along the symmetry axis of the material. Then, the constitutive relation for this case can be written as

$$\begin{aligned}
\sigma_{xx} &= c_{11}\varepsilon_{xx} + c_{12}\varepsilon_{yy} + c_{13}\varepsilon_{zz} \\
\sigma_{yy} &= c_{12}\varepsilon_{xx} + c_{11}\varepsilon_{yy} + c_{13}\varepsilon_{zz} \\
\sigma_{zz} &= c_{13}\varepsilon_{xx} + c_{13}\varepsilon_{yy} + c_{33}\varepsilon_{zz} \\
\sigma_{yz} &= 2c_{44}\varepsilon_{yz};\ \sigma_{xz} = 2c_{44}\varepsilon_{xz};\ \sigma_{xy} = 2c_{66}\varepsilon_{xy}
\end{aligned} \tag{3}$$

where c_{ij} are the stiffness coefficients with $c_{66} = (c_{11}\text{-}c_{12})/2$. Thus, in a TI, there are only five independent material coefficients. In terms of the compliance coefficients (the inverse of the stiffness), the physical meanings of the five independent coefficients are: the Young's modulus and Poisson's ratio in the isotropic plane (i.e., the xoy plane), the Young's modulus and Poisson's ratio in the plane normal to the isotropic plane, and the shear modulus in the plane normal to the isotropic plane.

It is noted that since a crack may be oriented in any direction with respect to the TI material system, one usually needs to introduce two orientation angles, for instance, ψ and β [4, 40], to describe the relation between the material system and the crack orientation. Furthermore, one may need extra coordinate transforms between the material systems and the global space-fixed coordinate system if multiple material domains (inhomogeneous materials) are involved and/or the boundary conditions are described in terms of the global system.

The BEM for a Cracked Matrix with a Single Inhomogeneity

We start with a cracked matrix containing only one inhomogeneity. The single crack is located in the matrix. We now present the solution process based on the BEM. First, we discretize the cracked matrix in terms of the single-domain BEM [4]. In other words, we apply the following displacement and traction boundary integral equations [4]

$$\begin{aligned} b_{ij}u_j(y_S) = &\int_S U_{ij}(y_S, x_S)\, t_j(x_S)dS(x_S) \\ &- \int_S T_{ij}(y_S, x_S)\, u_j(x_S)dS(x_S) \\ &- \int_{\Gamma+} T_{ij}(y_S, x_{\Gamma+})\, [u_j(x_{\Gamma+}) \\ &- u_j(x_{\Gamma-})]d\Gamma(x_{\Gamma+}) + u_i^0(y_S) \end{aligned} \tag{4}$$

$$\begin{aligned} &[t_l(y_{\Gamma+}) - t_l(y_{\Gamma-})]/2 + n_m(y_{\Gamma+}) \\ &\int_S c_{lmik}T_{ij,k}(y_{\Gamma+}, x_S)u_j(x_S)dS(x_S) \\ &+ n_m(y_{\Gamma+}) \int_{\Gamma+} c_{lmik}T_{ij,k}(y_{\Gamma+}, x_{\Gamma+}) \\ &[u_j(x_{\Gamma+}) - u_j(x_{\Gamma-})]d\Gamma(x_{\Gamma+}) \\ &= n_m(y_{\Gamma+}) \int_S c_{lmik}U^*_{ij,k}(y_{\Gamma+}, x_S)t_j(x_S)dS(x_S) \\ &+ [t_l^0(y_{\Gamma+}) - t_l^0(y_{\Gamma-})]/2 \end{aligned} \tag{5}$$

to the cracked matrix. In (4) and (5), b_{ij} are the coefficients that depend only on the local geometries of the inhomogeneity–matrix interface S at $\boldsymbol{y}_s$. A point on the positive (negative) side of the cracks is denoted by $\boldsymbol{x}_{\Gamma+}$ ($\boldsymbol{x}_{\Gamma-}$), and on the inhomogeneity–matrix interface S by both $\boldsymbol{x}_s$ and $\boldsymbol{y}_s$; n_m is the unit outward normal of the positive side of the crack surface at $\boldsymbol{y}_{\Gamma+}$; c_{lmik} is the fourth-order stiffness tensor of the TI material; $u_i^0(y_s)$ is the i-th displacement component at point $\boldsymbol{y}_s$ corresponding to the given remote loading, and $t_l^0(y_{\Gamma+})$ and $t_l^0(y_{\Gamma-})$ the corresponding traction components along the l-direction at points $\boldsymbol{y}_{\Gamma+}$ and $\boldsymbol{y}_{\Gamma-}$; u_i and t_i are the displacements and tractions on the inhomogeneity–matrix interface S (or the crack surface Γ); U_{ij} and T_{ij} are the Green's functions of the displacements and tractions; $U_{ij,k}$ and $T_{ij,k}$ are, respectively, the derivatives of the Green's displacements and tractions with respect to the source point. The displacement and traction Green's functions are taken from Pan and Chou [41] while their derivatives are taken from Pan and Yuan [4]. It is noted that the single-domain boundary integral equations similar to (4) and (5) were applied to a cracked homogeneous solid before and it has been demonstrated that this single-domain BEM approach is very efficient. However, if there is also an inhomogeneity in the cracked domain, one needs another BEM equation. In other words, the displacement integral equation needs to be applied to the surface of the inhomogeneity as follows [32]:

$$b_{ij}u_j(y_S) = \int_S U_{ij}(y_S, x_S)\, t_j(x_S) dS(x_S) - \int_S T_{ij}(y_S, x_S)\, u_j(x_S) dS(x_S) \tag{6}$$

Equations (4), (5), and (6) then can be utilized to investigate the effect of the inhomogeneity on the SIFs of the crack in a TI matrix as well as the internal field behaviors both within the inhomogeneity and the matrix. In discretization of these equations, the nine-node quadrilateral curved elements can be applied to the inhomogeneity–matrix interface and the crack surface with the crack front being treated by special elements [4].

Taking each node in turn as the collocation point and performing the involved integrals, we finally obtain the compact forms of the discretized equations from (4), (5), and (6) as

$$\begin{bmatrix} \mathbf{H}_{11} & \mathbf{H}_{12} \\ \mathbf{H}_{21} & \mathbf{H}_{22} \end{bmatrix} \begin{bmatrix} \mathbf{U}_m \\ \Delta\mathbf{U}_c \end{bmatrix} + \begin{bmatrix} \mathbf{B}_1 \\ \mathbf{B}_2 \end{bmatrix} = \begin{bmatrix} \mathbf{G}_{11} & \mathbf{G}_{12} \\ \mathbf{G}_{21} & \mathbf{G}_{22} \end{bmatrix} \begin{bmatrix} \mathbf{T}_m \\ \mathbf{T}_c \end{bmatrix} \tag{7}$$

and

$$\mathbf{H}_i\mathbf{U}_i = \mathbf{G}_i\mathbf{T}_i \tag{8}$$

where the subscripts i and m represent, respectively, the inhomogeneity and matrix; $\mathbf{H}$ and $\mathbf{G}$ are, respectively, the influence coefficient matrices containing integrals of the fundamental Green's function solutions; $\mathbf{B}_1$ and $\mathbf{B}_2$ are, respectively, the displacement and traction vectors induced by the remote loading; $\mathbf{U}_\mathrm{m}$ ($\mathbf{U}_\mathrm{i}$) and $\mathbf{T}_\mathrm{m}$ ($\mathbf{T}_\mathrm{i}$) are, respectively, the nodal displacement and traction vectors on the matrix side (inhomogeneity side) of the inhomogeneity–matrix interface; $\Delta\mathbf{U}_\mathrm{c}$ and $\mathbf{T}_\mathrm{c}$ are, respectively, the discontinuous displacement and traction vectors over the crack surface. In this entry, we assume that the tractions on both sides of the crack are equal and opposite, and thus, $\mathbf{T}_\mathrm{c}$ is equal to zero.

Using the continuity condition of the displacement and traction vectors along the interface, i.e., $\mathbf{U}_\mathrm{m} = \mathbf{U}_\mathrm{i}$ and $\mathbf{T}_\mathrm{m} = -\mathbf{T}_\mathrm{i}$, between the inhomogeneity and matrix, we can combine (7) and (8) into

$$\begin{bmatrix} \mathbf{H}_{11} + \mathbf{G}_{11}\mathbf{G}_i^{-1}\mathbf{H}_i & \mathbf{H}_{12} \\ \mathbf{H}_{21} + \mathbf{G}_{21}\mathbf{G}_i^{-1}\mathbf{H}_i & \mathbf{H}_{22} \end{bmatrix} \begin{Bmatrix} \mathbf{U}_m \\ \Delta\mathbf{U}_c \end{Bmatrix} = -\begin{Bmatrix} \mathbf{B}_1 \\ \mathbf{B}_2 \end{Bmatrix} \tag{9}$$

which can be solved for the unknowns $\mathbf{U}_\mathrm{m}$ and $\Delta\mathbf{U}_\mathrm{c}$. After that, a boundary integral equation similar to (4) or (6) can be applied to find the internal displacements and their gradients (by taking the derivatives) inside the matrix or the inhomogeneity. It is pointed out that in discretizing the boundary and the crack face, besides the regular shape functions, special ones need to be applied. For instance, the discontinuous elements need to be introduced to handle the common edge of the displacement and traction boundary conditions, and the common edge of the displacement/traction boundary and the crack surface. Furthermore, special shape functions have to be utilized to the elements adjacent to the crack front to make sure that the relative COD is proportional to $\sqrt{r}$ where r is the distance behind the crack front. These discontinuous/special elements along with their corresponding shape functions can be found in Pan and Yuan [4].

Once the relative COD $\Delta\mathbf{U}_\mathrm{c}$ is solved in the global coordinates, it can be transformed to the local coordinates (or the crack-tip coordinates) to find the SIFs. Assuming that the crack front is smooth and that the crack tip is away from the possible corner of the problem geometry, then the singular term (in the sense of stresses) in the asymptotic expansion of the displacement field near the crack tip (front) satisfies the generalized plane-strain condition in the local coordinates. Actually, if we let r be the distance behind the crack front, then in terms of the relative CODs in the crack-tip coordinate, the three SIFs can be expressed as follows:

$$\left\{\begin{array}{c} K_{II} \\ K_{I} \\ K_{III} \end{array}\right\} = 2\sqrt{\frac{2r}{\pi}}\mathrm{L}^{-1}\left\{\begin{array}{c} \Delta u_1 \\ \Delta u_2 \\ \Delta u_3 \end{array}\right\} \tag{10}$$

where **L** is the Barnett-Lothe tensors [42] which depends only on the anisotropic properties of the solid in the crack-front coordinates, and Δu_1, Δu_2, and Δu_3 are the relative CODs in the local crack-front coordinates. We also point out that r in (10) was selected to be a very small value as compared to the crack size [4, 32, 40]. For a penny-shaped crack lying in the isotropic plane of the TI material, the SIF can be calculated analytically [43], which can be used as the benchmark for BEM modeling. The result was extended to the bimaterial case where the crack was located on the interface plane [44].

General Inhomogeneity Problems with Multiple Cracks

It is obvious that the approach presented above can be extended to the multi-inhomogeneity case with multiple cracks. However, there are more efficient approaches proposed recently to the problems in inhomogeneous or heterogeneous media, as discussed briefly below.

Three-step Multi-domain BEM

The three-step multi-domain BEM solution technique [35] can be used to effectively solve the problems consisting of any number of arbitrarily distributed sub-domains. In the multi-domain BEM technique, nodes are arranged in the following order: The "self nodes" which are used only by the considered sub-domain itself are collocated first; the "common nodes" which are shared by two adjacent sub-domains are collocated next; and the "internal nodes" which are located inside a sub-domain are arranged in the last step. The three-step multi-domain BEM solution technique will produce condensations by eliminating the internal unknowns (internal nodal displacements) and boundary unknowns (self-nodal quantities) so that the final multi-domain BEM formula only contains the common nodal displacements. Since the number of degrees of freedom in the system is reduced by this technique and the coefficient matrix is blocked sparse, the computational efficiency of large-scale problems can be improved.

Subregion-by-subregion with Krylov Solver

In general, this approach contains two main parts: (1). A robust subregion-by-subregion (SBS) technique, which is necessary for coping with heterogeneous materials. (2). The efficient integration procedures, which are needed for evaluating the singular and nearly singular integrals involved in the BEM. The SBS technique is based on the use of the Krylov solver, which allows the treatment of a large number of inhomogeneities. The diagonal-preconditioned bi-conjugate gradient solver is employed to solve the resulting linear system of equations. A detailed description on this method can be found in the work by Araujo and coworkers [36, 37].

Fast Multipole BEM

With the development of the fast multipole methods (FMMs) [39, 45] for solving boundary integral equations, large models with several million degrees of freedom can be solved readily on a desktop computer. Rokhlin and Greengard [46], who pioneered the FMM, and coworkers [47] have done extensive research on the FMM in the context of potential fields. Fu et al. [48] formulated the boundary integral equations for the 3D elastic inclusion problem using the FMM. Solutions for up to 343 spherical voids in an elastic domain were computed using the parallel FMM BEM code with total degrees of freedom around 400 K. Some other developments of the fast multipole BEM can be found in Pierce and Napier [49] and Popov and Power [50] for general elasticity problems, and in Nishimura et al. [51], Yoshida et al. [52], and Lai and Rodin [53] for crack problems. To develop an FMM for BEM, one needs simple and appropriate expressions of the two-point Green's functions of the associated problem domain, and their suitable expansion, i.e., the multipole expansion.

For most linear systems, the two-point Green's functions can be successfully expanded and therefore, the three key translations in FMM can be achieved (M2M, L2L, and M2L) [38, 39].

Future Directions for Research

Solution to the penny-shaped crack problem in pure elasticity is a benchmark, and it has been extended to the multiphase material couplings [54–57]. For instance, Zhao et al. [54] derived the solution for an ellipsoidal cavity in an infinite TI magneto-electro-elastic medium, and obtained the exact closed-form solution for a penny-shaped crack by letting the minor axis of the ellipsoidal cavity approach zero. Zhao et al. [55] analyzed the planar crack of arbitrary shape in the isotropic plane of a 3D TI magneto-electro-elastic medium by using the hyper-singular integral equation method. Niraula and Wang [56] derived an exact closed-form solution for a penny-shaped crack in an infinite magneto-electro-thermo-elastic medium under a temperature field, where the problem was transformed into the dual integral equations which were solved directly. Wang and Niraula [57] further considered the transient thermal fracture problem of TI magneto-electro-elastic materials, where the problem is reduced to an integral equation which was treated exactly using the Abel's integral equation. The fracture properties of a penny-shaped crack embedded in a magneto-electro-elastic layer of finite height under both thermal flow and radial shear loads were investigated by Feng et al. [58]. Thermally insulated crack surface assumption is adopted. By means of the Hankel transform technique, the problem was reduced to a Fredholm integral equation, which is different to that addressed previously [54–57].

Acknowledgment This work was partially supported by the Dept. of Civil Engineering, University of Akron, Henan Province, National Natural Science Foundation of China (11072034, 11172273), and the Major Program of National Natural Science Foundation of China (11132011).

References

1. Anderson TL (1995) Fracture mechanics – fundamentals and applications. CRC Press, London
2. Irwin GR (1957) Analysis of stresses and strains near the end of a crack traversing a plate. J Appl Mech 24:361–364
3. Singh R, Carter BJ, Wawrzynek PA, Ingraffea AR (1998) Universal crack closure integral for SIF estimation. Eng Fract Mech 60(2):133–146
4. Pan E, Yuan FG (2000) Boundary element analysis of three-dimensional cracks in anisotropic solids. Int J Numer Meth Eng 48:211–237
5. Lazarus V, Leblond JB, Mouchrif SE (2001) Crack front rotation and segmentation in mixed mode I + III or I + II + III. part I: calculation of stress intensity factors. J Mech Phys Solids 49:1399–1420
6. Zhou W, Lim KM, Lee KH, Tay AAO (2005) A new variable-order singular boundary element for calculating stress intensity factors in three-dimensional elasticity problems. Int J Solids Struct 42:159–185
7. Yue ZQ, Xiao HT, Pan E (2007) Stress intensity factors of square crack inclined to interface of transversely isotropic bi-material. Eng Anal Bound Elem 31:50–65
8. Hong HK, Chen JT (1988) Derivations of integral equations of elasticity. J Eng Mech ASCE 114(6):1028–1044
9. Chen JT, Hong HK (1999) Review of dual boundary element methods with emphasis on hypersingular integrals and divergent series. Appl Mech Rev ASME 52(1):17–33
10. Liu Y, Liang LH, Hong QC, Antes H (1999) Nonlinear surface crack analysis by three-dimensional boundary elements with mixed boundary conditions. Eng Fract Mech 63:413–424
11. Blackburn WS (1999) Three dimensional calculation of growth of cracks starting in parallel planes by boundary elements. Int J Fatigue 21:933–939
12. dell'Erba DN, Aliabadi MH (2000) On the solution of three-dimensional thermoelastic mixed-mode edge crack problems by the dual boundary element method. Eng Fract Mech 66:269–285
13. Partheymüller P, Haas M, Kuhn G (2000) Comparison of the basic and the discontinuity formulation of the 3D-dual boundary element method. Eng Anal Bound Elem 24:777–788
14. Hatzigeorgiou GD, Beskos DE (2000) Static analysis of 3D damaged solids and structures by BEM. Eng Anal Bound Elem 26:521–526
15. Popov V, Power H, Walker SP (2003) Numerical comparison between two possible multipole alternatives for the BEM solution of 3D elasticity problems based upon Taylor series expansions. Eng Anal Bound Elem 27:521–531
16. Ariza MP, Dominguez J (2004) Dynamic BE analysis of 3-D cracks in transversely isotropic solids. Comput Meth Appl Mech Eng 193:765–779

17. Lo SH, Dong CY, Cheung YK (2005) Integral equation approach for 3D multiple-crack problems. Eng Fract Mech 72:1830–1840
18. Zhao MH, Fan CY, Yang F, Liu T (2007) Analysis method of planar cracks of arbitrary shape in the isotropic plane of a three-dimensional transversely isotropic magnetoelectroelastic medium. Int J Solids Struct 44:4505–4523
19. Rungamornrat J (2006) Analysis of 3D cracks in anisotropic multi-material domain with weakly singular SGBEM. Eng Anal Bound Elem 30:834–846
20. Rungamornrat J, Mear ME (2008) Weakly-singular, weak-form integral equations for cracks in three-dimensional anisotropic media. Int J Solids Struct 45:1283–1301
21. Noda NA, Xu C (2008) Controlling parameter of the stress intensity factors for a planar interfacial crack in three-dimensional biomaterials. Int J Solids Struct 45(3–4):1017–1031
22. Chen YM (1975) Numerical computation of dynamic stress intensity factors by a Lagrangian finite-difference method (the HEMP code). Eng Fract Mech 7(4):653–660
23. Altus E (1984) The finite difference technique for solving crack problems. Eng Fract Mech 19(5): 947–957
24. Dorogoy A, Banks-Sills L (2005) Effect of crack face contact and friction on Brazilian disk specimens—a finite difference solution. Eng Fract Mech 72(18): 2758–2773
25. Leung AYT, Su RKL (1995) A numerical study of singular stress field of 3D cracks. Finite Elem Anal Des 18:389–401
26. He WJ, Lin Y, Ding HJ (1997) A three-dimensional formula for determining stress intensity factors in finite element analysis of cracked bodies. Eng Fract Mech 57(4):409–415
27. Bush MB (1997) The interaction between a crack and a particle cluster. Int J Fract 88:215–232
28. Knight MG, Wrobel LC, Henshall JL (2003) Fracture response of fibre-reinforced materials with macro/microcrack damage using the boundary element technique. Int J Fract 121:163–182
29. Dong CY, Lo SH, Cheung YK (2003) Numerical analysis of the inclusion-crack interactions using an integral equation. Comput Mech 30:119–130
30. Kitey R, Phan AV, Tippur HV, Kaplan T (2006) Modeling of crack growth through particulate clusters in brittle matrix by symmetric-Galerkin boundary element method. Int J Fract 141:11–25
31. Lee HK, Tran XH (2010) On stress analysis for a penny-shaped crack interacting with inclusions and voids. Int J Solids Struct 47:549–558
32. Dong CY, Yang X, Pan E (2011) Analysis of cracked transversely isotropic and inhomogeneous solids by a special BIE formulation. Eng Anal Bound Elem 35:200–206
33. Sladek J, Sladek V (1995) Boundary element analysis for an interface crack between dissimilar elastoplastic materials. Comput Mech 16:396–405
34. Liu YJ, Xu N (2000) Modeling of interface cracks in fiber-reinforced composites with the presence of interphases using the boundary element method. Mech Mater 32(12):769–783
35. Gao XW, Guo L, Zhang CH (2007) Three-step multi-domain BEM solver for nonhomogeneous material problems. Eng Anal Bound Elem 31:965–973
36. Araujo FC, Gray LJ (2008) Analysis of thin-walled structural elements via 3D standard BEM with generic substructuring. Comp Mech 41:633–645
37. Araujo FC, d'Azevedo EF, Gray LJ (2010) Boundary-element parallel-computing algorithm for the micro-structural analysis of general composites. Comp Struct 88:773–784
38. Nishimura N (2002) Fast multipole accelerated boundary integral equation methods. Appl Mech Rev 55:299–324
39. Liu YJ (2009) Fast multipole boundary element method, Theory and applications in engineering. Cambridge University Press, Cambridge
40. Chen CS, Chen CH, Pan E (2009) Three-dimensional stress intensity factors of a central square crack in a transversely isotropic cuboid with arbitrary material orientations. Eng Anal Bound Elem 33:128–136
41. Pan YC, Chou TW (1976) Point force solution for an infinite transversely isotropic solid. J Appl Mech 43:608–612
42. Ting TCT (1996) Anisotropic elasticity: theory and applications. Oxford University Press, Oxford
43. Hoenig A (1982) Near-tip behavior of a crack in a plane anisotropic elastic body. Eng Fract Mech 16:393–403
44. Qu J, Xue Y (1999) Three-dimensional interface cracks in anisotropic bimaterials – The non-oscillatory case. J Appl Mech 65:1048–1055
45. Liu YJ, Nishimura N, Otani Y, Takahashi T, Chen XL, Munakata H (2005) A fast boundary element method for the analysis of fiber-reinforced composites based on a rigid-inclusion model. J Appl Mech 72:115–128
46. Rokhlin V, Greengard LF (1987) A fast algorithm for particle simulations. J Comput Phys 73:325–348
47. Cheng H, Rokhlin V, Greengard LF (1999) A fast adaptive multipole algorithm in three dimensions. J Comput Phys 155:468–498
48. Fu Y, Klimkowski KJ, Rodin GJ, Berger E, Browne JC, Singer JK, Geijn RAVD, Vemaganti KS (1998) A fast solution method for three-dimensional many-particle problems of linear elasticity. Int J Numer Meth Eng 42:1215–1229
49. Peirce AP, Napier JAL (1995) A spectral multipole method for efficient solution of large-scale boundary element models in elastostatics. Int J Numer Meth Eng 38:4009–4034

50. Popov V, Power H (2001) An O(N) Taylor series multipole boundary element method for three-dimensional elasticity problems. Eng Anal Bound Elem 25:7–18
51. Nishimura N, Yoshida K, Kobayashi S (1999) A fast multipole boundary integral equation method for crack problems in 3D. Eng Anal Bound Elem 23:97–105
52. Yoshida K, Nishimura N, Kobayashi S (2001) Application of fast multipole Galerkin boundary integral equation method to crack problems in 3D. Int J Numer Meth Eng 50:525–547
53. Lai YS, Rodin GJ (2003) Fast boundary element method for three-dimensional solids containing many cracks. Eng Anal Bound Elem 27:845–852
54. Zhao MH, Yang F, Liu T (2006) Analysis of a penny-shaped crack in a magneto-electro-elastic medium. Philos Mag 86:4397–4416
55. Zhao MH, Fan CY, Liu T, Yang F (2007) Extended displacement discontinuity Green's functions for three-dimensional transversely isotropic magneto-electro-elastic media and applications. Eng Anal Bound Elem 31:547–558
56. Niraula OP, Wang BL (2006) Thermal stress analysis in magneto- electro-thermo-elasticity with a penny-shaped crack under uniform heat flow. J Therm Stresses 29:423–437
57. Wang BL, Niraula OP (2007) Transient thermal fracture analysis of transversely isotropic magneto-electro-elastic materials. J Therm Stresses 30: 297–317
58. Feng WJ, Pan E, Wang X (2008) Stress analysis of a penny-shaped crack in magneto-electro-thermo-elastic layer under uniform heat flow and shear loads. J Therm Stresses 31:497–514

Crack-Tip Singular Fields in Functionally Graded Materials

Zhihe Jin
Department of Mechanical Engineering,
University of Maine, Orono, ME, USA

Overview

This entry introduces the asymptotic temperature, thermal flux, stress, and displacement fields near the tip of a crack in a functionally graded material (FGM) with continuous and piecewise differentiable material properties. This entry begins with the introduction of basic equations of heat conduction, thermoelasticity, and thermoplasticity for FGMs. The eigenfunction expansion method is then employed to prove that the governing equations of the crack-tip dominant solutions of temperature and stress functions remain the same as the corresponding equations for homogeneous materials in every differentiable piece near the crack tip. Hence, the inverse square-root singular thermal flux and stress fields still prevail at the crack tip in a thermoelastic FGM, and the near-tip HRR field exists for a power-law hardening FGM. The effects of material property gradients on the dominance of the crack-tip singular fields are also discussed.

Introduction

Functionally graded materials (FGMs) represent a new concept of tailoring materials with microstructural and property gradients to achieve optimized performance. FGMs were originally conceived as high-temperature-resistant materials for aircraft and aerospace applications. The FGM concept has since spread to other areas, for example, tribological coatings, diesel engines, energy conversion systems, biomedical engineering, and so on. An FGM is a multiphase material with volume fractions of the constituents varying gradually in a predetermined (designed) profile, thus yielding a nonuniform microstructure in the material with continuously graded properties. In applications involving severe thermal gradients, FGMs exploit the heat, oxidation, and corrosion resistance typical of ceramics and the strength, ductility, and toughness typical of metals. Damage tolerance and defect assessments for structural integrity of FGM components require knowledge of the fracture behavior of FGMs.

From the fracture mechanics point of view, materials fail by the initiation and unstable growth of macroscopic cracks. Fracture parameters often arise from analyses of the asymptotic stress and deformation fields near the crack tip. The validity of continuum fracture mechanics to predict material failure lies in the fact that the fracture process zone around the crack tip is contained in a singular field of continuum