Conclusion

A brief summary of the explicit versions of the GS4 algorithms and designs has been shown via the implicit counterparts. All explicit time integration schemes are second-order time accurate, and various common schemes are also included within the present frameworks. An *i* Integration Framework (Isochronous Integration Framework) is also described which employs the same framework for integrating both second- and first-order transient systems.

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Extended Displacement Discontinuity Boundary Integral Equation Method for Analysis of Cracks in Smart Materials

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Overview

Fractures greatly affect the integrity and reliability of structures, and crack analysis is one of the main tasks in fracture mechanics. The singularity of stresses near a crack tip and the geometric identity of the two surfaces of a crack have challenged all theoretical and numerical methods.

In 1976, Crouch proposed the displacement discontinuity method (DDM) [1], which is also called the displacement discontinuity boundary integral equation method (DDBIEM). In the DDM, the basic characteristic of a crack, namely, the displacement discontinuity, is automatically contained. In the last 35 years, this method has been studied intensively and extensively in dealing with fracture problems in two- and three-dimensional elastic media.

Along with the increasing usage of smart materials, e.g., the piezoelectric materials and the magnetoelectroelastic (MEE) materials, in various branches of the engineering field, fracture mechanics of these new materials is attracting more and more attention. As one of the key advances, the DDM has been extended to the study of cracks in piezoelectric and MEE media [2–4]. In the extended DDM for the piezoelectric material, the extended displacement discontinuity (DD) includes the elastic displacement discontinuities and the electric potential discontinuity; and for the MEE material, the extended DD includes further the magnetic potential discontinuity.

Extended Displacement Discontinuities

In the absence of body force, electric charge, and electric current, the coupled constitutive equations for a three-dimensional (3D) MEE medium can be expressed in terms of the elastic displacement components u_i ($u_1 = u$, $u_2 = v$ and $u_3 = w$), the electric potential φ and the magnetic potential ψ

$$\sigma_{ij} = c_{ijkl}(u_{k,l} + u_{l,k})/2 + e_{kij}\varphi_{,k} + f_{kij}\psi_{,k} \quad (1a)$$

$$D_i = e_{ikl}(u_{k,l} + u_{l,k})/2 - \varepsilon_{ik}\varphi_{,k} - g_{ik}\psi_{,k} \quad (1b)$$

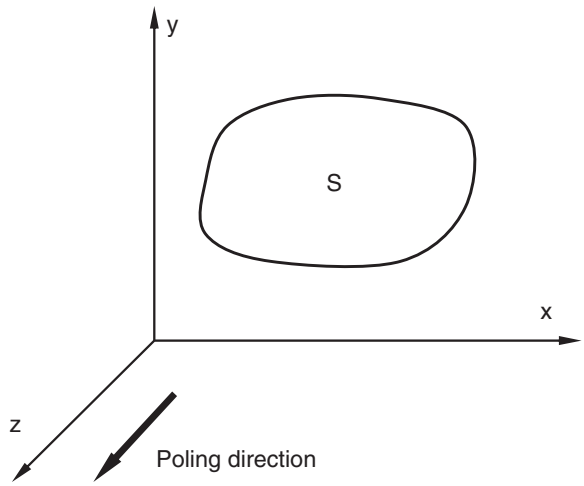
$$B_i = f_{ikl}(u_{k,l} + u_{l,k})/2 - g_{ik}\varphi_{,k} - \mu_{ik}\psi_{,k} \quad (1c)$$

where σ_{ij}, D_i, and B_i denote the stress, electric displacement, and magnetic induction components, respectively, $c_{ijkl}, e_{ijk}, f_{ijk}, \varepsilon_{ij}, g_{ij}$ and μ_{ij} are, respectively, the elastic, piezoelectric, piezomagnetic, dielectric permittivity, electromagnetic, and magnetic permeability coefficients. A subscript comma denotes the partial differentiation with respect to the coordinate. It is noted that by letting $f_{ijk} = 0$, $g_{ij} = 0$ and $\mu_{ij} = 0$, the constitutive equation in (1) is reduced to that for a piezoelectric material, and by further setting $e_{ijk} = 0$ and $\varepsilon_{ij} = 0$, (1) is reduced to the constitutive equation for a purely elastic material.

In the following analyses, we assume that the smart material with cracks is transversely isotropic. As an example, let us consider an arbitrarily shaped planar crack S on the oxy plane, which coincides with the plane of isotropy of an infinite smart medium as shown in Fig. 1. The poling direction is along the z-axis. The front and back faces of the crack S are denoted by S^+ and S^-, respectively. Across the crack faces, the displacement discontinuities $\|u_i\|$ $(i = 1, 2, 3)$, the electric potential discontinuity $\|\varphi\|$, and the magnetic potential discontinuity $\|\psi\|$ are denoted by

$$\begin{aligned} \|u_i\| &= u_i(S^+) - u_i(S^-) \\ \|\varphi\| &= \varphi(S^+) - \varphi(S^-) \\ \|\psi\| &= \psi(S^+) - \psi(S^-) \end{aligned} \quad (2)$$

which are called the extended displacement discontinuities.



Extended Displacement Discontinuity Boundary Integral Equation Method for Analysis of Cracks in Smart Materials, Fig. 1 A crack of arbitrary shape in the isotropic oxy plane of an infinite medium

Boundary Integral Equation Method for Homogeneous Materials

We consider the general case where the applied extended tractions on the crack faces satisfy

$$p_i|_{S^+} = -p_i|_{S^-}, \quad \omega|_{S^+} = -\omega|_{S^-}, \quad \gamma|_{S^+} = -\gamma|_{S^-}, \quad i = 1, 2, 3 \quad \text{or} \quad x, y, z \quad (3)$$

Based on the Green's functions of the unit extended point force [5] and the Somigliana identity, the extended displacement discontinuity boundary integral equations are derived

$$\int_{S^+} \{[L_{11}(1 - 3\cos^2\theta) + L_{12}(1 - 3\sin^2\theta)]\|u\| + L_{13}\cos\theta\sin\theta\|v\|\}\frac{1}{r^3}dS(\xi,\eta) = -p_x(x,y) \quad (4)$$

$$\int_{S^+} \{L_{13}\cos\theta\sin\theta\|u\| + [L_{12}(1 - 3\cos^2\theta) + L_{11}(1 - 3\sin^2\theta)]\|v\|\}\frac{1}{r^3}dS(\xi,\eta) = -p_y(x,y) \quad (5)$$

$$\int_{S^+} [L_{31}\|w\| + L_{32}\|\varphi\| + L_{33}\|\psi\|]\frac{1}{r^3}dS(\xi,\eta) = -p_z(x,y) \quad (6)$$

$$\int_{S^+} [L_{41}||w|| + L_{42}||\varphi|| + L_{43}||\psi||] \frac{1}{r^3} dS(\xi, \eta) = -\omega(x, y) \tag{7}$$

$$\int_{S^+} [L_{51}||w|| + L_{52}||\varphi|| + L_{53}||\psi||] \frac{1}{r^3} dS(\xi, \eta) = -\gamma(x, y) \tag{8}$$

where

$$r^2 = (\xi - x)^2 + (\eta - y)^2, \quad \cos\theta = (\xi - x)/r, \quad \sin\theta = (\eta - y)/r \tag{9}$$

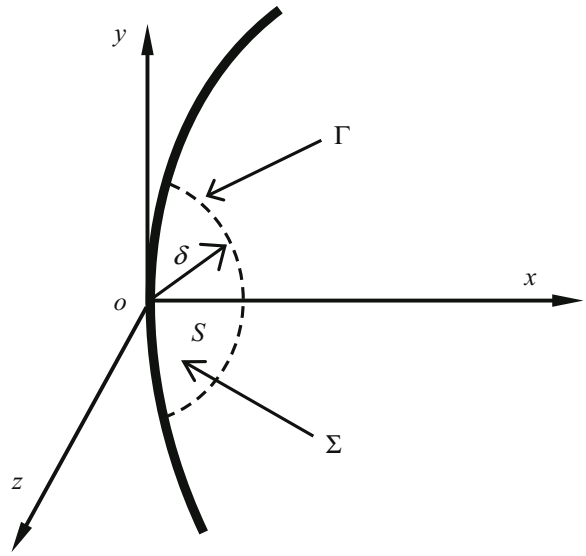
and L_{ij} are the material related constants given in Zhao et al. [4] for MEE media, in Zhao et al. [2] for piezoelectric media, and in Zhao et al. [6] for elastic media.

In (4)–(8), the kernel functions have the singularity of $O(1/r^3)$, and hence the integral equations are hyper-singular. The displacement discontinuities $\|u\|$ and $\|v\|$ on the crack faces are coupled through (4) and (5), while the displacement discontinuity $\|w\|$, the electric potential discontinuity $\|\varphi\|$, and the magnetic potential discontinuity $\|\psi\|$ are coupled through (6)–(8).

Singular Index

The singular behavior of the fields near the crack tip and the corresponding field intensity factors are the keys in fracture mechanics. Based on the extended displacement discontinuity boundary integral equations (4)–(8), the field singularity index and intensity factor in terms of the extended displacement discontinuity can be derived.

We choose an arbitrary but smooth point o on the crack front Γ to analyze the singular behavior (Fig. 2). We assume that the Cartesian coordinate system $oxyz$ is located such that the y -axis and x -axis are tangential and normal to Γ, respectively, while the z -axis is normal to the crack plane S. The infinitesimal δ denotes the radius of a circle Σ centered at point o contained in S.



Extended Displacement Discontinuity Boundary Integral Equation Method for Analysis of Cracks in Smart Materials, Fig. 2 The local coordinate system

Now, we assume that the extended displacement discontinuities at the neighborhood of point o are given by

$$||u|| = A_x(o)x^{\alpha_x}, \quad ||v|| = A_y(o)x^{\alpha_y}, \quad ||w|| = A_z(o)x^{\alpha_z}, \quad ||\varphi|| = A_\varphi(o)x^{\alpha_\varphi}, \quad ||\psi|| = A_\psi(o)x^{\alpha_\psi} \tag{10}$$

where the coefficients A_x, A_y, A_z, A_φ,and A_ψ depend on the location of the point o, and $\alpha_x, \alpha_y, \alpha_z, \alpha_\varphi$, and α_ψ are the singular indices of the extended displacements with their values between (0,1).

Substituting (10) into (4)–(8), letting ε be sufficiently small and taking the limit x → 0, and further making use of the finite-part integral theory, we obtain the conditions for the existence of a nontrivial solution

$$\cot \pi\alpha_x = \cot \pi\alpha_y = \cot \pi\alpha_z = \cot \pi\alpha_\varphi = \cot \pi\alpha_\psi = 0 \tag{11}$$

Finally, one obtains the singular indexes

$$\alpha_x = \alpha_y = \alpha_z = \alpha_\varphi = \alpha_\psi = \frac{1}{2} \tag{12}$$

This result reveals that the extended displacements near the crack tip have the classical singularity $r^{\frac{1}{2}}$ as in the fracture mechanics of conventional elastic materials.

Intensity Factor

Substituting (12) into (10), and using (4)–(8) and the constitutive equation (1), the extended stresses at points $(-\rho, y, 0)$ $(\rho > 0)$ near point o are expressed as

$$\begin{aligned}
\sigma_{zx} &= -L_{11}A_x(o)\pi/\sqrt{\rho} \\
\sigma_{zy} &= -L_{12}A_y(o)\pi/\sqrt{\rho} \\
\sigma_{zz} &= [L_{31}A_z(o) + L_{32}A_\varphi(o) + L_{33}A_\psi(o)]\pi/\sqrt{\rho} \\
D_z &= [L_{41}A_z(o) + L_{42}A_\varphi(o) + L_{43}A_\psi(o)]\pi/\sqrt{\rho} \\
B_z &= [L_{51}A_z(o) + L_{52}A_\varphi(o) + L_{53}A_\psi(o)]\pi/\sqrt{\rho}
\end{aligned} \tag{13}$$

Defining the intensity factors

$$\begin{aligned}
K_{\mathrm{I}}^F &= \lim_{\rho\to 0}\sqrt{2\pi\rho}\,\sigma_{zz}(-\rho, y, 0) \\
K_{\mathrm{I}}^D &= \lim_{\rho\to 0}\sqrt{2\pi\rho}\,D_z(-\rho, y, 0) \\
K_{\mathrm{I}}^B &= \lim_{\rho\to 0}\sqrt{2\pi\rho}\,B_z(-\rho, y, 0) \\
K_{\mathrm{II}}^F &= \lim_{\rho\to 0}\sqrt{2\pi\rho}\,\sigma_{zx}(-\rho, y, 0) \\
K_{\mathrm{III}}^F &= \lim_{\rho\to 0}\sqrt{2\pi\rho}\,\sigma_{zy}(-\rho, y, 0)
\end{aligned} \tag{14}$$

and inserting (13) into (14), and considering (10), the intensity factors can be expressed in terms of the extended displacement discontinuities

$$\begin{aligned}
K_{\mathrm{I}}^{\mathrm{F}} &= \sqrt{2\pi}\pi\lim_{x\to 0}[L_{31}\|w\| + L_{32}\|\varphi\| + L_{33}\|\psi\|]/\sqrt{x} \\
K_{\mathrm{I}}^{\mathrm{D}} &= \sqrt{2\pi}\pi\lim_{x\to 0}[L_{41}\|w\| + L_{42}\|\varphi\| + L_{43}\|\psi\|]/\sqrt{x} \\
K_{\mathrm{I}}^{\mathrm{B}} &= \sqrt{2\pi}\pi\lim_{x\to 0}[L_{51}\|w\| + L_{52}\|\varphi\| + L_{53}\|\psi\|]/\sqrt{x} \\
K_{\mathrm{II}}^{\mathrm{F}} &= -\sqrt{2\pi}\pi\lim_{x\to 0}L_{11}\|u\|/\sqrt{x} \\
K_{\mathrm{III}}^{\mathrm{F}} &= -\sqrt{2\pi}\pi\lim_{x\to 0}L_{12}\|v\|/\sqrt{x}
\end{aligned} \tag{15}$$

Equation (15) demonstrates that once the extended displacement discontinuities are calculated, the intensity factor can be obtained by (15). This conclusion is held for any shape and dimension of the planar crack and for general distribution of the mechanical-electric-magnetic loading. The cracks may be multiple coplanar cracks, and the loading may be point loading.

The singularity of stresses near a crack tip and the intensity factor of a crack in piezoelectric media were given in Zhao et al. [2].

Boundary Integral Equation Method for Two-Phase Materials

Using the same method as given in the previous section, the boundary integral-differential equations of an interface crack in a two-phase MEE material can be further derived [7]

$$\begin{aligned}
&\int_{s^+}\{[K_{11}\cos^2\theta + K_{12}\sin^2\theta]\|u\| \\
&\quad + (K_{11} - K_{12})\sin\theta\cos\theta\|v\|\}\frac{1}{r^3}dS \\
&\quad + 2\pi K_{41}\frac{\partial\|w\|}{\partial x} + 2\pi K_{42}\frac{\partial\|\varphi\|}{\partial x} \\
&\quad + 2\pi K_{43}\frac{\partial\|\psi\|}{\partial x} = -p_x
\end{aligned} \tag{16a}$$

$$\begin{aligned}
&\int_{s^+}(K_{11} - K_{12})\sin\theta\cos\theta\|u\| \\
&\quad + [K_{11}\sin^2\theta + K_{12}\cos^2\theta]\|v\|\}\frac{1}{r^3}dS \\
&\quad + 2\pi K_{41}\frac{\partial\|w\|}{\partial y} + 2\pi K_{42}\frac{\partial\|\varphi\|}{\partial y} \\
&\quad + 2\pi K_{43}\frac{\partial\|\psi\|}{\partial y} = -p_y
\end{aligned} \tag{16b}$$

$$\begin{aligned}
&\int_{s^+}\left[K_{z1}\|w\| + K_{z2}\|\varphi\| + K_{z3}\|\psi\|\right]\frac{1}{r^3}dS \\
&\quad + 2\pi K_1\left(\frac{\partial\|u\|}{\partial x} + \frac{\partial\|v\|}{\partial y}\right) = -p_z
\end{aligned} \tag{16c}$$

$$\int_{s^+} \left[K_{z12}\|w\| + K_{z22}\|\varphi\| + K_{z32}\|\psi\|\right] \frac{1}{r^3} dS + 2\pi K_2 \left(\frac{\partial\|u\|}{\partial x} + \frac{\partial\|v\|}{\partial y}\right) = -\omega \tag{16d}$$

$$\int_{s^+} \left[K_{z13}\|w\| + K_{z23}\|\varphi\| + K_{z33}\|\psi\|\right] \frac{1}{r^3} dS + 2\pi K_3 \left(\frac{\partial\|u\|}{\partial x} + \frac{\partial\|v\|}{\partial y}\right) = -\gamma \tag{16e}$$

where the coefficients K_s with different subscript "s" are material constants given in Zhao et al. [7] for MEE media and in Zhao et al. [8, 9] for piezoelectric media. Note that the kernels in (16) have the singularity order $O(r^{-3})$, and hence the integral-differential equations are hyper-singular. It should be pointed out that the boundary integral-differential equations are applicable to multiple coplanar interface cracks.

When the bimaterial becomes homogeneous, one has

$$K_{41} = K_{42} = K_{43} = K_1 = K_2 = K_3 = 0 \tag{17}$$

and the differential terms in (16) disappear. Therefore, the boundary integral-differential equations are reduced to the hyper-singular boundary integral equations in (4–8).

Solutions of the Boundary Integral Equation for Two-Phase Materials

Making use of (16c–16e), one obtains

$$(K_{z1} - C_3 K_{z12}) \int_{S^+} [\|w\| + C_1\|\varphi\| + C_2\|\psi\|] \frac{1}{r^3} dS = -p_z + C_3\omega \tag{18}$$

where C_i are constants related to the material property given by Zhao et al. [7]. Equation (18) is analogous to the boundary integral equation for the displacement discontinuity in the normal direction of the crack in an elastic medium [10]. Thus, the solution of the combined extended displacement discontinuity $\|w\| + C_1\|\varphi\| + C_2\|\psi\|$ can be directly obtained from the corresponding elastic solution.

The extended stress near the crack tip in the crack plane can be expressed in the following form:

$$\sigma_{zz} - C_3 D_z = (K_{z1} - C_3 K_{z12}) \times \int_{S^+} \left[\|w\| + C_1\|\varphi\| + C_2\|\psi\|\right] \frac{1}{r^3} dS \tag{19}$$

Therefore, the Mode I extended intensity factor in a MEE bimaterial can be defined as

$$\begin{aligned} K_{\mathrm{I}1} &= \lim_{r\to 0} \sqrt{2\pi r}(\sigma_{zz} - C_3 D_z) \\ &= \pi\sqrt{2\pi}(K_{z1} - C_3 K_{z12}) \lim_{\rho\to 0} \frac{\|w\| + C_1\|\varphi\| + C_2\|\psi\|}{\sqrt{\rho}} \end{aligned} \tag{20}$$

Similarly, the other extended stress and the corresponding intensity factor near the crack tip in the crack plane can be written as

$$\sigma_{zz} - C_6 B_z = (K_{z1} - C_6 K_{z13}) \int_{S^+} \left[\|w\| + C_4\|\varphi\| + C_5\|\psi\|\right] \frac{1}{r^3} dS \tag{21}$$

$$\begin{aligned} K_{\mathrm{I}2} &= \lim_{r\to 0} \sqrt{2\pi r}(\sigma_{zz} - C_6 B_z) \\ &= \pi\sqrt{2\pi}(K_{z1} - C_6 K_{z13}) \lim_{\rho\to 0} \frac{\|w\| + C_4\|\varphi\| + C_5\|\psi\|}{\sqrt{\rho}} \end{aligned} \tag{22}$$

Solutions of the Boundary Integral-Differential Equations for Two-Phase Materials

Combining (16c)–(16e) with (16a) and (16b) gives

$$\int_{s^+} \left\{ \left[\frac{LK_{12}}{K_{41}} \frac{1}{r^3} + \frac{L(K_{11} - K_{12})}{K_{41}} \frac{(x-\xi)^2}{r^5}\right] \|u\| + \frac{L(K_{11} - K_{12})}{K_{41}} \frac{(x-\xi)(y-\eta)}{r^5} \|v\| \right\} dS + 2\pi \frac{\partial\|w^*\|}{\partial x} = -\frac{Lp_x}{K_{41}} \tag{23a}$$

$$\int_{s^+}\left\{\frac{L(K_{11}-K_{12})}{K_{41}}\frac{(x-\xi)(y-\eta)}{r^5}\|u\| + \left[\frac{LK_{12}}{K_{41}}+\frac{L(K_{11}-K_{12})}{K_{41}}\frac{(y-\eta)^2}{r^5}\right]\|v\|\right\}dS + 2\pi\frac{\partial\|w^*\|}{\partial y} = -\frac{Lp_y}{K_{41}} \tag{23b}$$

$$\frac{(K_{z1}+C_7K_{z12}+C_8K_{z13})}{L(K_1+C_7K_2+C_8K_3)}\int_{s^+}\|w^*\|\frac{1}{r^3}dS + 2\pi\left(\frac{\partial\|u\|}{\partial x}+\frac{\partial\|v\|}{\partial y}\right) = -\frac{p_z+C_7\omega+C_8\gamma}{K_1+C_7K_2+C_8K_3} \tag{23c}$$

where L and $\|w^*\|$ are given by

$$L=\sqrt{-\frac{3K_{41}(K_{z1}+C_7K_{z12}+C_8K_{z13})}{(2K_{11}+K_{12})(K_1+C_7K_2+C_8K_3)}},\quad \|w^*\| = L(\|w\|+C_9\|\varphi\|+C_{10}\|\psi\|) \tag{23d}$$

Equation (23) is similar to the boundary integral-differential equations for interface crack problem in a 3D isotropic elastic bimaterial system given by Tang et al. [11].

We assume that the displacement discontinuities near the crack tip are

$$\begin{aligned}\|u\| &= A_x(o)x^{\alpha_x}\\ \|v\| &= A_y(o)x^{\alpha_y}\\ \|w^*\| &= A_z(o)x^{\alpha_z}\end{aligned} \tag{24}$$

where $A_y(o)$ is an arbitrary real constant, $A_x(o)$ and $A_z(o)$ are complex constants, and α_i are the singularity indexes [11]. Inserting (24) into (23), and using the integrals given in Zhao et al. [7], we can obtain

$$\alpha_y=\frac{1}{2},\qquad \alpha_x=\alpha_z=\frac{1}{2}\pm \mathrm{i}\varepsilon \tag{25}$$

where ε is a constant related to the bimaterial property. Equation (25) shows that the displacement discontinuity $\|v\|$ has the classical singularity index $1/2$, while $\|u\|$ or $\|w^*(\xi,\eta)\|$ has the oscillating singularity index $1/2\pm \mathrm{i}\varepsilon$ as in the purely elastic bimaterial system case.

The stresses, electric displacement, and magnetic induction at point $(-r,0,0)$ near the crack tip outside of the crack are obtained based on the fundamental solution of the extended displacement discontinuities

$$\int_{-\infty}^{\infty}\int_0^{\infty}\left[\frac{L(K_{11}-K_{12})}{K_{41}}\times\frac{(x+r)^2}{R^5}+\frac{LK_{12}}{K_{41}}\frac{1}{R^3}\right]\|u\|d\xi d\eta = \frac{L}{K_{41}}\sigma_{zy} \tag{26a}$$

$$\int_{-\infty}^{\infty}\int_0^{\infty}\left[\frac{L(K_{11}-K_{12})}{K_{41}}\frac{y^2}{R^5}+\frac{LK_{12}}{K_{41}}\frac{1}{R^3}\right]\|v\|d\xi d\eta = \frac{L}{K_{41}}\sigma_{zy} \tag{26b}$$

$$\frac{(K_{z1}+C_7K_{z12}+C_8K_{z13})}{L(K_1+C_7K_2+C_8K_3)}\int_{-\infty}^{\infty}\int_0^{\infty}\frac{\|w^*\|}{R^3}d\xi d\eta - \frac{L}{K_{41}}C_{12}(\sigma_{zz}+C_7D_z+C_8B_z) \tag{26c}$$

where $R=\sqrt{(x+r)^2+y^2}$

The new intensity factors for an interface crack in a MEE bimaterial are defined as

$$\begin{aligned}K_{\mathrm{I3}} &= \lim_{r\to 0}\sqrt{2\pi r}r^{-i\varepsilon}C_{12}(\sigma_{zz}(-r,0,0) + C_7D_z(-r,0,0)+C_8B_z(-r,0,0))\\ K_{\mathrm{II}} &= \lim_{r\to 0}\sqrt{2\pi r}r^{-i\varepsilon}\sigma_{zx}(-r,0,0)\\ K_{\mathrm{III}} &= \lim_{r\to 0}\sqrt{2\pi r}\sigma_{zy}(-r,0,0)\end{aligned} \tag{27}$$

Finally, the corresponding intensity factors can be expressed in terms of the extended displacement discontinuities

$$\begin{aligned}K_{\mathrm{I3}}+\mathrm{i}K_{\mathrm{II}} &= \sqrt{2\pi}\frac{2\pi K_{41}(1+2\mathrm{i}\varepsilon)e^{\pi\varepsilon}}{(\eta_2-1)}\times\lim_{r\to 0}\left[\frac{(\|w\|+C_9\|\varphi\|+C_{10}\|\psi\|)-\mathrm{i}\|u\|/L}{r^{1/2+\mathrm{i}\varepsilon}}\right]\\ K_{\mathrm{III}} &= \sqrt{\frac{\pi}{2}}\frac{2\pi\eta_1K_{41}}{(\eta_2-1)L}\lim_{r\to 0}\frac{\|v\|}{\sqrt{r}}\end{aligned} \tag{28}$$

Concluding Remarks

The displacement discontinuity method for fracture mechanics in elastic media has been extended to the piezoelectric and MEE media. By using the analogy between the hyper-singular boundary integral/integral-differential equations of elastic media and those of MEE media, the singularity of stresses near a crack tip is studied, and the extended intensity factors are expressed by the extended displacement discontinuities. This result holds for the arbitrarily shaped planar crack and interface crack of any geometric size under the general distribution of the mechanical-electric-magnetic loading. Furthermore, the multiple coplanar cracks and point loading cases can be analyzed.

Fracture of piezoelectric or MEE materials is complex, and thus, various nonlinear models, such as the strip polarization saturation model [12], the strip dielectric breakdown model [13, 14] for piezoelectric media, strip electric-magnetic breakdown model [15], strip electric-magnetic polarization saturation model [16] for MEE media, were proposed to understand the fracture behaviors. However, the extended intensity factors are still the fundamental parameters in these nonlinear models and are important in the corresponding fracture criteria.

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Extended Surfaces (Fins and Pins)

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Overview

Heat transfer from a system can be increased by extending the surface area through the addition of