Modeling and experimental study of coupled porous/channel flow

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Introduction

- The coexistence of free flow and porous flow has been witnessed in a variety of natural phenomena and industrial processing including filtration technology.
- The Brinkman extension to Darcy's law equation includes the effect of wall. The introduction of 2nd order shear stress terms ensures the variables like velocity and pressure to be continuous across the interface between channel/porous regions to achieve numerically solvable field.

Objective

- The aim of this research is to develop a two-dimensional model to simulate the fluid dynamics of a steady state creeping, isothermal flow that is incompressible and viscous passing through a free channel with porous media placed in the middle of whole regime.
- Experiment will be performed to validate the model prediction in terms of pressure drop.

Mathematical model

- **Flow model**

Momentum equations

Start from multiphase momentum balance equations of multiphase continuum theory

$$\frac{\partial}{\partial t}\varepsilon^\alpha\rho^\alpha\underline{\vec{v}}^\alpha+\underline{\nabla}\cdot\varepsilon^\alpha\rho^\alpha\underline{\vec{v}}^\alpha\underline{\vec{v}}^\alpha+\underline{\nabla}\cdot\varepsilon^\alpha\underline{\tau}^\alpha+\varepsilon^\alpha\underline{\nabla}P+\underline{F}^d+E^\alpha+G^\alpha=0 \qquad (1)$$

In the absence of any body forces, neglect heterogeneous reaction or mechanical transport across the interface and take into account the steady state flow with negligible inertial force, the overall balance equation reduces to:

$$-\mu\varepsilon^\alpha\nabla^2\underline{\vec{v}}^\alpha+\varepsilon^\alpha\underline{\nabla}P+\underline{F}^d=0 \qquad (2)$$

In which is ε is the porosity of the homogeneous media, α refers to the fluid phase. $\underline{F}^d$ is the drag force on fluid exerted by porous media. It should be noted that $\underline{\vec{v}}^\alpha$ represents the physical velocity within the porous region. According to Darcy's law,

$$\varepsilon^\alpha\underline{\vec{v}}^\alpha=-k\frac{\underline{\nabla}P}{\mu} \qquad (3) \qquad \underline{\nabla}P=-\frac{\mu\varepsilon^\alpha\underline{\vec{v}}^\alpha}{k} \qquad (4)$$

Consider only the resistance from the porous media

$$\underline{F}^d=\frac{\mu\varepsilon^\alpha\varepsilon^\alpha\underline{\vec{v}}^\alpha}{k} \qquad (5)$$

Substitution of Eq.(3) into Eq.(2) gives:

$$-\mu\varepsilon^\alpha\nabla^2\underline{\vec{v}}^\alpha+\varepsilon^\alpha\underline{\nabla}P+\frac{\mu\varepsilon^\alpha\varepsilon^\alpha\underline{\vec{v}}^\alpha}{k}=0 \qquad (6)$$

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EQUATIONS
vz: fluid_part*(dz(P)-visc*div( grad( vz))) +(1-fluid_part)*(visc/eps*div( grad( vz))-dz(P)-(visc)*vz/kperm)=0
{Coupled flow model of Stokes and Brinkman Equation, vz is the superficial velocity in porous media}
vr:  fluid_part*( dr(P)-visc*div( grad( vr))) +(1-fluid_part)*(visc/eps*div( grad(vr))-dr(P)-(visc)*vr/kperm)=0
!assume fiber size as characteristic length, Re is very small renders the non-linear term neglected
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Fig.1 FlexPDE scripter of momentum equations

Consider the physical velocity $\underline{\vec{v}}^\alpha$ and superficial velocity $\underline{\vec{q}}^\alpha$ that is continuous across the boundary, $\underline{\vec{v}}^\alpha=\underline{\vec{q}}^\alpha/\varepsilon^\alpha$, the Eq.(4) becomes: $-\frac{\mu}{\varepsilon^\alpha}\nabla^2\underline{\vec{q}}^\alpha+\nabla P+\frac{\mu_e\underline{\vec{q}}^\alpha}{k}=0$

Eq(7) becomes Brinkman's equation in which k refers to permeability of porous media. μ_e refers to the effective Brinkman viscosity.

Eq.(7) will be coded in FlexPDE scripter in 2-D Cylindrical coordinate system as:

z-component: *r-component:*

$$\frac{\mu}{\varepsilon^\alpha}\left(\nabla^2\underline{\vec{q}}^\alpha\right)_z-\frac{\partial P}{\partial z}-\frac{\mu_e}{k}\left(\underline{\vec{q}}^\alpha\right)_z=0 \qquad (8) \qquad \frac{\mu}{\varepsilon^\alpha}\left(\nabla^2\underline{\vec{q}}^\alpha\right)_r-\frac{\partial P}{\partial r}-\frac{\mu_e}{k}\left(\underline{\vec{q}}^\alpha\right)_r=0 \qquad (9)$$

In the channel flow region, the behavior of a laminar, incompressible creeping flow can be necessarily modeled by the Stokes equations:

r-component: *z-component:*

$$-\frac{\partial P}{\partial r}+\mu\left(\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial v_r}{\partial r}\right)+\frac{\partial^2 v_r}{\partial z^2}-\frac{v_r}{r^2}\right)=0 \qquad (10) \qquad -\frac{\partial P}{\partial z}+\mu\left(\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial v_z}{\partial r}\right)+\frac{\partial^2 v_z}{\partial z^2}\right)=0 \qquad (11)$$

Where μ refers to the dynamic viscosity of the fluid.

Continuity equation

$$\frac{1}{r}\frac{\partial}{\partial r}(rv_r)+\frac{\partial v_z}{\partial z}=0 \qquad (12)$$

Assuming constant density, conservation of mass can be expressed in Eq.(12)

Model solution domain and assumptions

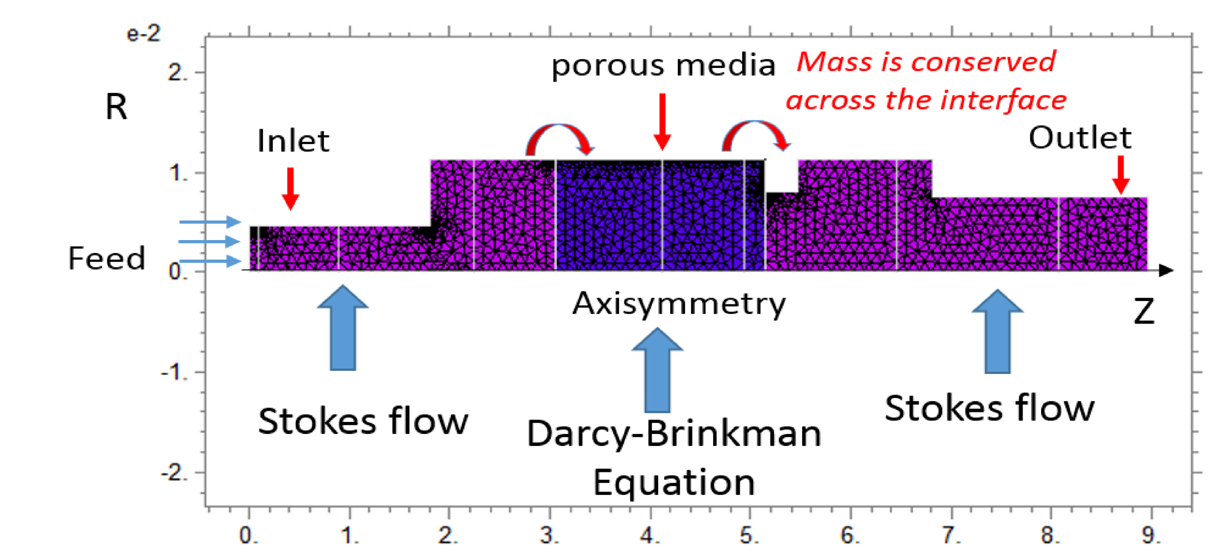

Fig.2 Computational domain

Assumptions
1) The whole process is a steady state isothermal creeping flow and fluid is incompressible and viscous.
2) Filter media is incompressible and the interface between the free/porous flow is rigid and stationary.
3) The porosities and permeabilities of filter media is uniform and constant
4) Oil viscosity and density are constant
5) No chemical reaction or mass transfer occurs during the flow
6) Wall effect is neglected and no property transfer due to slip
7) The property of flow system including boundary conditions is axisymmetric

Fig.3 Model assumption

Experimental

- A unit of three hollow molds made of Plexiglas are linked to create a continuous flow channel in which the middle chamber is filled with porous media as shown in Fig.4.
- The effect of permeabilities on pressure loss was explored , a range of volumetric flow rates are chosen to obtain a plot between feed flow velocity and pressure drop.
- Diesel with measured density 832 kg/m3 and viscosity 2.52×10^{-3} pa · s is used as fluid while the permeability and porosity are tested by Capillary Flow Porometer and Pycnometer.

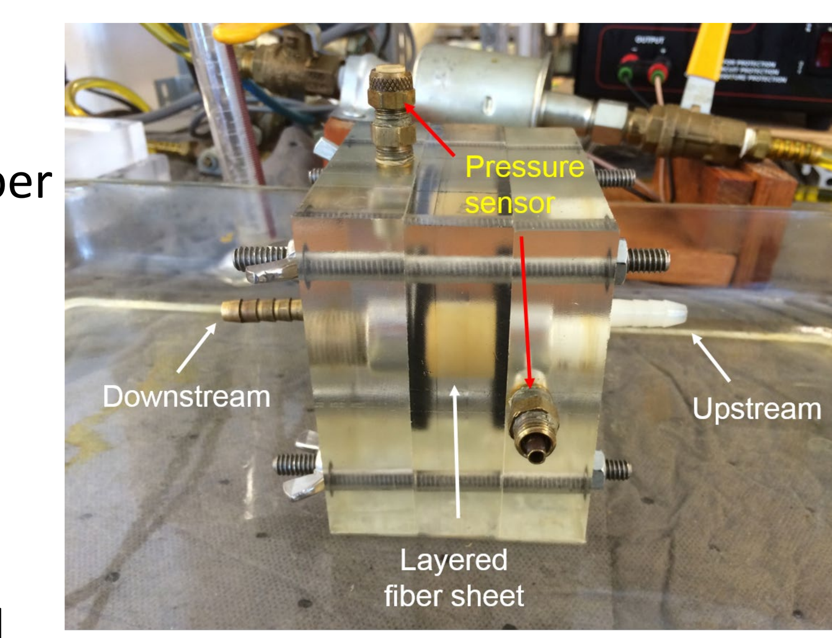

Fig.4 Experimental setup

Results and discussion

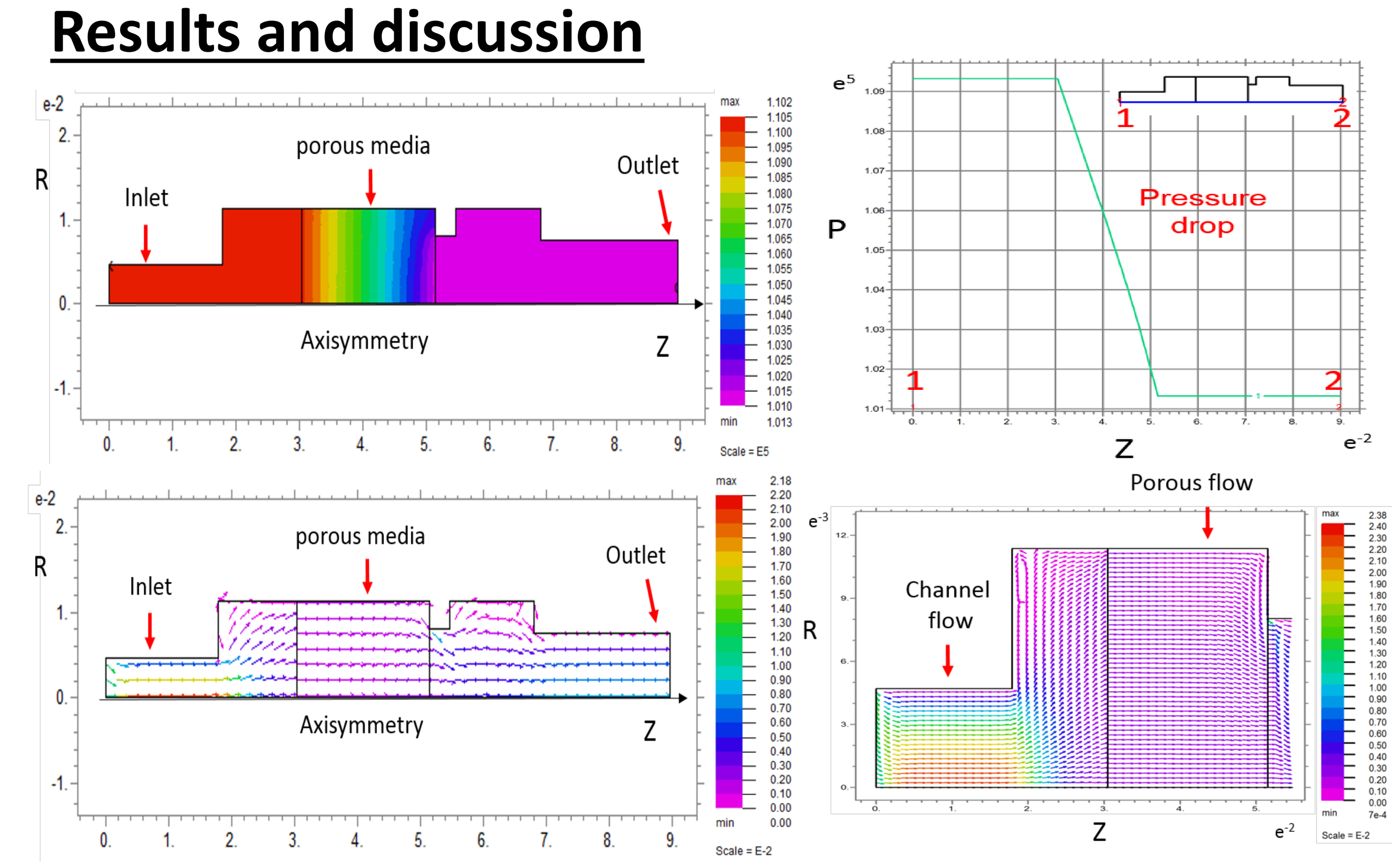

Fig.5 Velocity and pressure profile in the flow domain
The right bottom image shows a zoom-in view of flow field

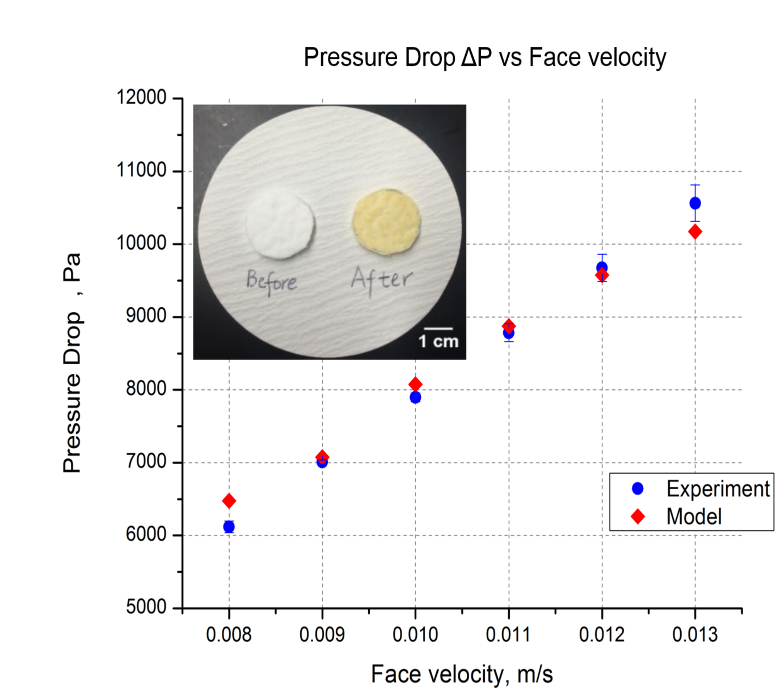

Fig.6 Measured pressure drop vs simulated result for glass fiber sheet. The inset photograph shows an images of the sheets before and after the experiments

- Mass balance is evaluated throughout the whole flow domain by calculating the volumetric flow rates at different cross-sections, the maximum discrepancy does not exceed 1%.

Future research

- Perform modeling prediction and experimental validation of flow through layered structure porous media with different porosities and permeabilities.
- Set up mass, momentum and species balance equations for gas and liquid phase and necessary constitutive correlations. Determine the saturation profile within the filter media for different stages during gas-liquid filtration.

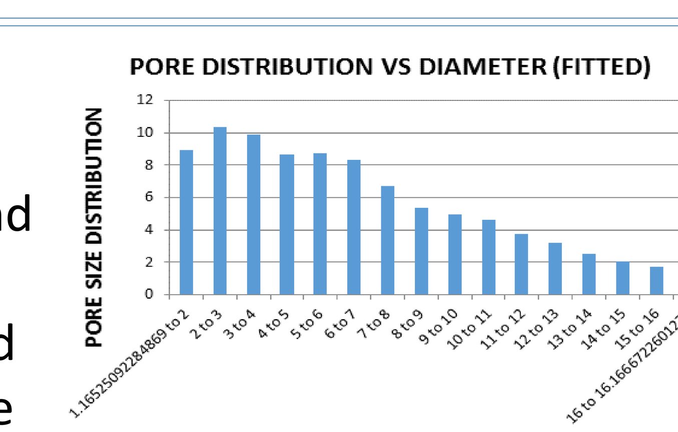

Fig.7 Pore size distribution of fiber sheet