Time Domain Surface Impedance Boundary Conditions of High Order of Approximation

Sergei Yuferev * and Nathan Ida **

* Tampere University of Technology, P.O.Box 692, FIN-33101 Tampere, Finland

** The University of Akron, Akron, OH, 44325-3904, USA

Abstract-**The problem of diffusion of transient electromagnetic field into a lossy dielectric homogeneous body is solved by using the perturbations method in the small parameter p, equal to the ratio of the electromagnetic penetration depth and characteristic dimension of the body. Time and frequency domain solutions for the tangential component of the electric field and the normal component of the magnetic field on the smooth curved surface of the body (the surface impedance boundary conditions - SIBCs) are obtained with the accuracy up to $O(p^4)$. It is shown that the proposed SIBCs in the frequency domain generalize well-known Leontovich's and Mitzner's boundary conditions that provide approximation errors $O(p^2)$ and $O(p^3)$, respectively. A numerical example of using the high order SIBCs with the surface integral equations in time domain is considered to illustrate the method.**

Index terms - **Transient scattering, integral equations, time domain analysis, surface impedance boundary condition, skin effect, transient analysis, perturbation methods.**

I. Introduction

There is a class of electromagnetic problems in which the electromagnetic penetration depth in the conducting body is so short that the variation of the field in the direction tangential to the body's surface is much less than the field variation in the normal direction, so that the complete equation of the electromagnetic field diffusion into the body can be replaced by 1-D equation in the direction normal to the surface of the body. The solution of the reduced equation can be then used to derive so-called surface impedance boundary conditions (SIBCs) involving only the external fields imposed on the outer surface to simulate the material properties of the body and to convert thereby a two (or more) media problem into a one media problem.

In practice, the following SIBC is usually used [1-3]

$$\vec{n}\times\vec{E}(t) = -Z_t * \left(\vec{n}\times\vec{H}(t)\right)\times\vec{n} \qquad (1)$$

$$Z_t = \left(\frac{\mu}{\varepsilon}\right)^{1/2}\left\{\delta(t) + \frac{\sigma}{2\varepsilon}\left[I_1\left(\frac{\sigma t}{2\varepsilon}\right) - I_0\left(\frac{\sigma t}{2\varepsilon}\right)\right]\exp\left(-\frac{\sigma t}{2\varepsilon}\right)\right\}$$

where * denotes a time domain convolution product, $I_n(x)$ is the modified Bessel function of the order n and $\delta(t)$ is the unit step function. Note that (1), obtained from well-known frequency domain SIBC for the planar surface (Leontovich's approximation), is the condition of *low* order approximation since it does not take into account the following important factors: the curvature of the body surface and the field diffusion in the direction tangential to the body surface. In frequency domain analysis, SIBCs of *high* order of approximation, allowing for both these factors, were developed by Mitzner [4] and Rytov [5]. Increase of the SIBC approximation order allows extention of the range of problems for which the formulations based on the surface impedance concept can be applicable. Therefore, the objective of this paper is development of the time domain SIBCs of high order of approximation, taking into account both factors.

Manuscript received November 3, 1997.

S. Yuferev, (on leave from Ioffe Physical-Technical Institute, Russian Acad. of Sci.), +358-3-365-2739, fax +358-3-365-2160, yuferev@alpha.cc.tut.fi; N. Ida, +1-330-972-6525, fax +1-330-972-6487, ida@uakron.edu.

II. Statement of the Problem

Consider a homogeneous body of finite conductivity surrounded by the non-conductive medium. Let the time variation of the incident field be such that the penetration depth δ into the body remains small as compared with the characteristic dimension D of the surface of the body.

$$\delta = \sqrt{\tau/\sigma\mu_0} << D \qquad (2)$$

where τ is the incident pulse duration. The electric $\vec{E}$ and magnetic $\vec{H}$ fields inside the body can be described by the Maxwell equations in the following form

$$\nabla\times\vec{H} = \sigma\vec{E} + \varepsilon_R\varepsilon_0\,\partial\vec{E}/\partial t \qquad (3)$$

$$\nabla\times\vec{E} = -\mu_0\,\partial\vec{H}/\partial t \quad (4); \qquad \nabla\cdot\vec{H} = 0 \quad (5)$$

Presence of the condition (2) allows transformation of equations (3)-(5) by using asymptotic expansion techniques with the purpose of deriving the normal component of the magnetic field and the tangential component of the electric field at the body's surface in explicit form.

III. Local Coordinates

Following Mitzner's approach of deriving the SIBCs allowing for the curvature of the body surface, we re-write (3)-(5) in the local quasi-spherical orthogonal curvilinear system (x_1, x_2, x_3), related to the body surface (Fig. 1):

$$\frac{\partial(e_3 H_{x_3})}{\partial x_2} - \frac{\partial(e_2 H_{x_2})}{\partial x_3} = e_2 e_3\left(E_{x_1} + \varepsilon_r\varepsilon_0\frac{\partial E_{x_1}}{\partial t}\right) \qquad (6a)$$

$$\frac{\partial(e_3 E_{x_3})}{\partial x_2} - \frac{\partial(e_2 E_{x_2})}{\partial x_3} = -\mu_0 e_2 e_3\frac{\partial H_{x_1}}{\partial t} \qquad (7a)$$

$$\frac{\partial}{\partial x_1}\left(e_2 e_3 H_{x_1}\right)+\frac{\partial}{\partial x_2}\left(e_3 e_1 H_{x_2}\right)+\frac{\partial}{\partial x_3}\left(e_1 e_2 H_{x_3}\right)=0 \qquad (8)$$

where e_1, e_2 and e_3 are the Lame coefficients. Equations (6b)-(6c) and (7b)-(7c) can be obtained by cyclic permutation of indexes. The coordinates x_1 and x_2 are defined as angles and the coordinate x_3 is defined as "length" coordinate directed from the surface inside the body, therefore the Lame coefficients are written as follows:

$$e_1 = d_1 - x_3; \qquad e_2 = d_2 - x_3; \qquad e_3 = 1 \qquad (9)$$

where d_k, $k=1,2$, are the local radii of curvature of the corresponding coordinate line.

Now we replace the system (x_1,x_2,x_3), containing angle coordinates, by the system (ξ_1,ξ_2,η) in which all coordinates are linear (Fig.1). The characteristic lengths associated with coordinates ξ_k and η are D and δ, respectively. For the smooth surface $D=\min(d_1,d_2)$. Both coordinate systems are related as follows:

$$\xi_1 = d_1 x_1; \qquad \xi_2 = d_2 x_2 \qquad \eta = x_3 \qquad (10)$$

Equations (6)-(8) with the coordinates (10) are written in the form ($k=1,2$):

$$\frac{\partial H_{\xi_k}}{\partial \eta}-\frac{d_k}{d_k-\eta}\frac{\partial H_\eta}{\partial \xi_k}-\frac{H_{\xi_k}}{d_k-\eta}=(-1)^{3-k}\left(\sigma E_{\xi_{3-k}}+\varepsilon_r\varepsilon_0\frac{\partial E_{\xi_{3-k}}}{\partial t}\right)$$

$$\frac{d_1}{d_1-\eta}\frac{\partial H_{\xi_2}}{\partial \xi_1}-\frac{d_2}{d_2-\eta}\frac{\partial H_{\xi_1}}{\partial \xi_2}=\sigma E_\eta+\varepsilon_r\varepsilon_0\frac{\partial E_\eta}{\partial t} \qquad (11b)$$

$$\frac{\partial E_{\xi_k}}{\partial \eta}-\frac{d_k}{d_k-\eta}\frac{\partial E_\eta}{\partial \xi_k}-\frac{E_{\xi_k}}{d_k-\eta}=(-1)^k\mu_0\frac{\partial H_{\xi_{3-k}}}{\partial t} \qquad (12a)$$

$$\frac{d_2}{d_2-\eta}\frac{\partial E_{\xi_2}}{\partial \xi_1}-\frac{d_1}{d_1-\eta}\frac{\partial E_{\xi_1}}{\partial \xi_2}=-\mu_0\frac{\partial H_\eta}{\partial t} \qquad (12b)$$

$$\frac{\partial H_\eta}{\partial \eta}+\frac{d_1}{d_1-\eta}\frac{\partial H_{\xi_1}}{\partial \xi_1}+\frac{d_2}{d_2-\eta}\frac{\partial H_{\xi_2}}{\partial \xi_2}=H_\eta\left(\frac{1}{d_1-\eta}+\frac{1}{d_2-\eta}\right) \qquad (13)$$

IV. Expansions in the Small Parameter

Let us transform to dimensionless variables by choosing appropriate scale factors. We introduce the basic scale factors

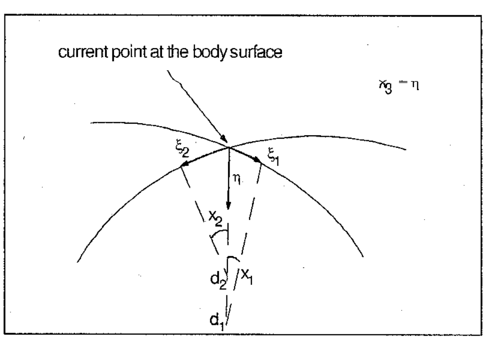

Fig. 1. Local orthogonal curvilinear coordinate systems related to the surface.

I, D and τ for the current, surface coordinates ξ_k and time, respectively. As the latter it is natural to choose the duration of the incident pulse. The scale factors for other values can be expressed as combinations of the basic scale factors:

$$[\eta]=pD; \quad [H]=\frac{I}{D}; \quad [E]=\frac{\mu_0 I}{\tau}; \quad p=\sqrt{\frac{\tau}{\sigma\mu_0 D^2}} \qquad (14)$$

Here square brackets denote a scale factor for the corresponding value. The quantity p in (14) is a small parameter since it is proportional to δ/D.

With the dimensionless variables, (11)-(12) are written in the form ($k=1,2$):

$$p\frac{\partial \tilde{H}_{\xi_k}}{\partial \tilde{\eta}}-\frac{p^2\tilde{H}_{\xi_k}}{\tilde{d}_k-p\tilde{\eta}}-\frac{p^2\tilde{d}_k}{\tilde{d}_k-p\tilde{\eta}}\frac{\partial \tilde{H}_\eta}{\partial \tilde{\xi}_k}=(-1)^{3-k}\left(\tilde{E}_{\xi_{3-k}}+a\frac{\partial \tilde{E}_{\xi_{3-k}}}{\partial \tilde{t}}\right)$$

$$p^2\left(\frac{\tilde{d}_1}{\tilde{d}_1-p\tilde{\eta}}\frac{\partial \tilde{H}_{\xi_2}}{\partial \tilde{\xi}_1}-\frac{\tilde{d}_2}{\tilde{d}_2-p\tilde{\eta}}\frac{\partial \tilde{H}_{\xi_1}}{\partial \tilde{\xi}_2}\right)=\tilde{E}_\eta+a\frac{\partial \tilde{E}_\eta}{\partial \tilde{t}} \qquad (15b)$$

$$\frac{\partial \tilde{E}_{\xi_k}}{\partial \tilde{\eta}}-\frac{p\tilde{E}_{\xi_k}}{\tilde{d}_k-p\tilde{\eta}}-\frac{p\tilde{d}_k}{\tilde{d}_k-p\tilde{\eta}}\frac{\partial \tilde{E}_\eta}{\partial \tilde{\xi}_k}=(-1)^k p\frac{\partial \tilde{H}_{\xi_{3-k}}}{\partial \tilde{t}} \qquad (16a)$$

$$\frac{\tilde{d}_1}{\tilde{d}_1-p\tilde{\eta}}\frac{\partial \tilde{E}_{\xi_2}}{\partial \tilde{\xi}_1}-\frac{\tilde{d}_2}{\tilde{d}_2-p\tilde{\eta}}\frac{\partial \tilde{E}_{\xi_1}}{\partial \tilde{\xi}_2}=-\frac{\partial \tilde{H}_\eta}{\partial \tilde{t}} \qquad (16b)$$

$$\frac{\partial \tilde{H}_\eta}{\partial \tilde{\eta}}-p\tilde{H}_\eta\sum_{i=1}^{2}\frac{1}{\tilde{d}_i-p\tilde{\eta}}=-p\sum_{i=1}^{2}\frac{\tilde{d}_i}{\tilde{d}_i-p\tilde{\eta}}\frac{\partial \tilde{H}_{\xi_i}}{\partial \tilde{\xi}_i} \qquad (17)$$

where $a=\varepsilon_0\,\varepsilon_R/(\sigma\tau)$ and $\tilde{d}_k=d_k/D$, $k=1,2$. Sign "~" denotes dimensionless value. By introducing the scale factors (14), the small parameter p appears in the field equation inside the body.

Now we can represent the functions $\tilde{\vec{E}}$ and $\tilde{\vec{H}}$ in the form of the asymptotic expansions in the small parameter p:

$$\tilde{\vec{H}}=\sum_{m=0}^{\infty}p^m\tilde{\vec{H}}_m \qquad \tilde{\vec{E}}=\sum_{m=0}^{\infty}p^m\tilde{\vec{E}}_m \qquad (18)$$

By substituting the expansions (18) into (15)-(17) and equating the coefficients of equal powers of p, the following equations for the expansion coefficients are obtained and written in the Laplace domain as follows:

$m=0$: (19)

$$(\tilde{\tilde{E}}_0)_{\xi_1}=(\tilde{\tilde{E}}_0)_{\xi_2}=(\tilde{\tilde{E}}_0)_\eta=(\tilde{\tilde{H}}_0)_\eta=0$$

$m=1$: (20)

$$\partial(\tilde{\tilde{E}}_1)_{\xi_k}\Big/\partial\tilde{\eta}=(-1)^k s(\tilde{\tilde{H}}_0)_{\xi_{3-k}}$$

$$(1+sa)(\tilde{\tilde{E}}_1)_{\xi_k}=(-1)^k\,\partial(\tilde{\tilde{H}}_0)_{\xi_{3-k}}\Big/\partial\tilde{\eta}$$

$$\sum_{i=1}^{2}(-1)^i\,\partial(\tilde{\tilde{E}}_1)_{\xi_{3-i}}\Big/\partial\tilde{\xi}_i=s(\tilde{\tilde{H}}_1)_\eta$$

$$\partial(\tilde{\tilde{H}}_1)_\eta\Big/\partial\tilde{\eta}=-\sum_{i=1}^{2}\partial(\tilde{\tilde{H}}_0)_{\xi_i}\Big/\partial\tilde{\xi}_i$$

$m=2$: (21)

$$\partial(\tilde{\tilde{E}}_2)_{\xi_k}\Big/\partial\tilde{\eta}=(\tilde{\tilde{E}}_1)_{\xi_k}\Big/\tilde{d}_k+(-1)^k s(\tilde{\tilde{H}}_1)_{\xi_{3-k}}$$

$$(1+sa)(\tilde{\bar{\bar{E}}}_2)_{\xi_k} = (-1)^k\left[\partial(\tilde{\bar{\bar{H}}}_1)_{\xi_{3-k}}\big/\partial\eta - (\tilde{\bar{\bar{H}}}_0)_{\xi_{3-k}}\big/\tilde{d}_{3-k}\right]$$

$$\sum_{i=1}^{2}(-1)^i\left[\partial(\tilde{\bar{\bar{E}}}_2)_{\xi_{3-i}}\big/\partial\tilde{\xi}_i + \tilde{\eta}\,\partial(\tilde{\bar{\bar{E}}}_1)_{\xi_{3-i}}\big/(\tilde{d}_i\,\partial\tilde{\xi}_i)\right] = s(\tilde{\bar{\bar{H}}}_2)_\eta$$

$$(1+sa)(\tilde{\bar{\bar{E}}}_2)_\eta = \sum_{i=1}^{2}(-1)^{3-i}\,\partial(\tilde{\bar{\bar{H}}}_0)_{\xi_{3-i}}\big/\partial\tilde{\xi}_i$$

$$\frac{\partial(\tilde{\bar{\bar{H}}}_2)_\eta}{\partial\tilde{\eta}} = (\tilde{\bar{\bar{H}}}_1)_\eta\sum_{i=1}^{2}\tilde{d}_i^{-1} - \sum_{i=1}^{2}\left[\frac{\partial(\tilde{\bar{\bar{H}}}_1)_{\xi_i}}{\partial\tilde{\xi}_i} + \frac{\tilde{\eta}}{\tilde{d}_i}\frac{\partial(\tilde{\bar{\bar{H}}}_0)_{\xi_i}}{\partial\tilde{\xi}_i}\right]$$

$m=3$: (22)

$$\frac{\partial(\tilde{\bar{\bar{E}}}_3)_{\xi_k}}{\partial\tilde{\eta}} = \frac{(\tilde{\bar{\bar{E}}}_2)_{\xi_k}}{\tilde{d}_k} + \tilde{\eta}\frac{(\tilde{\bar{\bar{E}}}_1)_{\xi_k}}{\tilde{d}_k^2} + \frac{\partial(\tilde{\bar{\bar{E}}}_2)_\eta}{\partial\tilde{\xi}_k} + (-1)^k s(\tilde{\bar{\bar{H}}}_2)_{\xi_{3-k}}$$

$$(1+sa)(\tilde{\bar{\bar{E}}}_3)_{\xi_k} = (-1)^k\Big[\partial(\tilde{\bar{\bar{H}}}_2)_{\xi_{3-k}}\big/\partial\tilde{\eta} - (\tilde{\bar{\bar{H}}}_1)_{\xi_{3-k}}\big/\tilde{d}_{3-k} - \tilde{\eta}(\tilde{\bar{\bar{H}}}_0)_{\xi_{3-k}}\big/\tilde{d}_{3-k}^2 - \partial(\tilde{\bar{\bar{H}}}_1)_\eta\big/\partial\tilde{\xi}_{3-k}\Big]$$

$$\sum_{i=1}^{2}(-1)^i\left[\frac{\partial(\tilde{\bar{\bar{E}}}_3)_{\xi_{3-i}}}{\partial\tilde{\xi}_i} + \frac{\tilde{\eta}}{\tilde{d}_i}\frac{\partial(\tilde{\bar{\bar{E}}}_2)_{\xi_{3-i}}}{\partial\tilde{\xi}_i} - \frac{\tilde{\eta}^2}{2\tilde{d}_i^2}\frac{\partial(\tilde{\bar{\bar{E}}}_1)_{\xi_{3-i}}}{\partial\tilde{\xi}_i}\right] = s(\tilde{\bar{\bar{H}}}_3)_\eta$$

$$(1+sa)(\tilde{\bar{\bar{E}}}_3)_\eta = \sum_{i=1}^{2}(-1)^{3-i}\left[\frac{\partial(\tilde{\bar{\bar{H}}}_1)_{\xi_{3-i}}}{\partial\tilde{\xi}_i} + \frac{\tilde{\eta}}{\tilde{d}_i}\frac{\partial(\tilde{\bar{\bar{H}}}_0)_{\xi_{3-i}}}{\partial\tilde{\xi}_i}\right]$$

$$\partial(\tilde{\bar{\bar{H}}}_3)_\eta\big/\partial\tilde{\eta} = (\tilde{\bar{\bar{H}}}_2)_\eta\sum_{i=1}^{2}\tilde{d}_i^{-1} + \tilde{\eta}(\tilde{\bar{\bar{H}}}_1)_\eta\sum_{i=1}^{2}\tilde{d}_i^{-2} - \sum_{i=1}^{2}\left[\frac{\partial(\tilde{\bar{\bar{H}}}_2)_{\xi_i}}{\partial\tilde{\xi}_i} + \frac{\tilde{\eta}}{\tilde{d}_i}\frac{\partial(\tilde{\bar{\bar{H}}}_1)_{\xi_i}}{\partial\tilde{\xi}_i} - \frac{\tilde{\eta}^2}{2\tilde{d}_i^2}\frac{\partial(\tilde{\bar{\bar{H}}}_0)_{\xi_i}}{\partial\tilde{\xi}_i}\right]$$

The representation (18) has clear physical meaning, namely:

a. The zero-order terms of the expansions (18) give the solution of the problem in the so-called perfect electrical conductor (PEC) limit, in which the magnetic field diffusion into the body is neglected.
b. The first-order terms describe the diffusion in the well-known Leontovich's approximation, in which the body's surface is considered as a plane and the field is assumed to be penetrating into the body only in the direction normal to the body's surface.
c. The second-order terms yield the correction by taking into account the curvature of the body's surface, but the diffusion is assumed to be only in the direction normal to the surface as in Leontovich's approximation. This is Mitzner's approximation.
d. The third-order terms and higher allow for the magnetic field diffusion in the directions tangential to the body's surface. This approximation can be called Rytov's approximation.

The problems (20)-(22) were solved in succession. The results substituted in the expansions (18) are represented in the form

$$\tilde{\bar{E}}^o_{\xi_k} = (-1)^{3-k}s\bar{F}_{3-k} \quad (23); \qquad \tilde{\bar{H}}^o_\eta = \sum_{i=1}^{2}\partial\bar{F}_i/\partial\tilde{\xi}_i \quad (24)$$

where superscript "o" denotes the values at the body's surface. The Laplace domain functions $\bar{F}_k = \bar{F}_k(\xi_1,\xi_1,s)$, $k=1,2$, have been introduced as follows

$$\bar{F}_{3-k} = \frac{p}{\sqrt{s+s^2a}}\tilde{\bar{H}}^o_{\xi_{3-k}} + \frac{p^2}{s+s^2a}\frac{\tilde{d}_k-\tilde{d}_{3-k}}{2\tilde{d}_k\tilde{d}_{3-k}}\tilde{\bar{H}}^o_{\xi_{3-k}} + \frac{p^3}{s+s^2a}\frac{3\tilde{d}_k^2-\tilde{d}_{3-k}^2-2\tilde{d}_k\tilde{d}_{3-k}}{8\tilde{d}_k^2\tilde{d}_{3-k}^2}\tilde{\bar{H}}^o_{\xi_{3-k}} + \frac{p^3}{2\left(s+s^2a\right)^{3/2}}\left(-\frac{\partial^2\tilde{\bar{H}}^o_{\xi_{3-k}}}{\partial\tilde{\xi}_k^2} + \frac{\partial^2\tilde{\bar{H}}^o_{\xi_{3-k}}}{\partial\tilde{\xi}_{3-k}^2} + 2\frac{\partial^2\tilde{\bar{H}}^o_{\xi_k}}{\partial\tilde{\xi}_k\partial\tilde{\xi}_{3-k}}\right) + O(p^4)$$

The functions F_k describe the perturbation of the external field surrounding the body due to the field diffusion into the body and dissipation of the energy by the body.

Finally, by using the inverse Laplace transform and returning to the dimensional variables, the proposed SIBCs can be written in the following form ($k=1,2$):

$$E^o_{\xi_k} = (-1)^{3-k}\partial F_{3-k}/\partial t \quad (26) \qquad H^o_\eta = \sum_{i=1}^{2}\partial F_i/\partial\xi_i \quad (27)$$

$$F_{3-k} = T_1 * H^o_{\xi_{3-k}} + \frac{d_k-d_{3-k}}{2d_kd_{3-k}}T_2 * H^o_{\xi_{3-k}} + \frac{3d_k^2-d_{3-k}^2-2d_kd_{3-k}}{8d_k^2d_{3-k}^2}T_3 * H^o_{\xi_{3-k}} + \frac{1}{2}T_3 * \left(-\frac{\partial^2 H^o_{\xi_{3-k}}}{\partial\xi_k^2} + \frac{\partial^2 H^o_{\xi_{3-k}}}{\partial\xi_{3-k}^2} + 2\frac{\partial^2 H^o_{\xi_k}}{\partial\xi_k\partial\xi_{3-k}}\right) \quad (28)$$

Here T_m, $m=1,2,3$, are the time-functions defined as follows

$$T_1(t) = (\varepsilon\mu_0)^{-1/2}I_0(\sigma t/(2\varepsilon))\exp(-\sigma t/(2\varepsilon))$$

$$T_2(t) = (\sigma\mu_0)^{-1}\left[1-\exp(-\sigma t/\varepsilon)\right]$$

$$T_3(t) = 2t(\sigma\mu_0)^{-1}(\varepsilon\mu_0)^{-1/2}I_1(\sigma t/(2\varepsilon))\exp(-\sigma t/(2\varepsilon))$$

V. Notes on the Applicability of the Method

a. The approach developed is based on the assumption that the variation of the components of the electromagnetic field in the directions ξ_1, ξ_2 is small compared to the variation in the normal direction η. This approximation leads to the following basic restrictions:
   - the penetration depth should be much less than the characteristic dimension of the body (see (2));
   - the characteristic dimension of the field variation along the body's surface must not exceed the characteristic dimension of the body by more than one order of magnitude. This restriction can be written in the following form:

   $$O(c\tau) \geq O(D)$$

   where c is velocity of light in vacuum.
b. The boundary conditions (26)-(28) have been initially formulated in the form of asymptotic expansions in the

small parameter, therefore the following conditions should be necessarily satisfied:

$$\tilde{H}_m >> p\tilde{H}_{m+1}; \qquad \tilde{E}_m >> p\tilde{E}_{m+1}; \qquad m=0,1,2..$$

Note that, as a rule the terms of the asymptotic expansions rise with the order of approximation so that the expansions are nonconvergent. However, the calculatations demonstrate that our terms are of the same order of magnitude. Therefore, expansions (18) can have the radius of convergence as for normal power series.

c. Because in the expansions (18) the terms of order $O(p^4)$ have been neglected, the SIBCs (26)-(28) are valid only for the penetration depths for which

$$p^4 << 1$$

In practice it means that if the ratio of the penetration depth and body thickness is 1/3, then the approximation error due to neglecting the terms $m>3$ is about 0.8%.

VI. Numerical Example

The SIBCs (26)-(28) have been coupled with time-domain surface integral equations for the magnetic and electric field using the approach, described in [3], and then solved by the marching-in-time technique. To illustrate the theory, the problem of transient scattering from infinitely long straight cylinder of round cross section illuminated by a Gaussian pulse (TE case) has been solved. From Fig. 2 it follows that $d_1 = \infty$ and $d_2 = D$ where D is the radius of the cylinder. Under these conditions only the circumferential ξ_2-component of the equivalent surface electric current $\tilde{\vec{J}}^s = \vec{n}' \times \tilde{\vec{H}}^o$ is non-zero. The components of the expansion of $\tilde{\vec{J}}^s$ are related to $\tilde{\vec{H}}^o_m$ by

$$(\tilde{\vec{J}}_m)_{\xi_2} = -(\tilde{\vec{H}}^o_m)_{\xi_1}$$

The scale factor $J^{inc}_{\max} D$ was used as l; here $J^{inc}_{\max}$ is the maximum of J^{inc}. The width of the incident pulse was taken to be D, i.e. $q=1$. The time $\tilde{t} = 0$ corresponds to the time that the pulse reaches the body.

Figure 3 shows the time-distributions of the terms $\tilde{J}_0$ (solid line), $\tilde{J}_1$ (dashed line), $\tilde{J}_2$ (short line) and $\tilde{J}_3$ (dotted line) at point c. Final time-distributions of the surface electric current obtained using the PEC-limit ($\tilde{J}^s = \tilde{J}_0$), Leontovich's approximation ($\tilde{J}^s = \tilde{J}_0 + p\tilde{J}_1$) and Rytov's high order approximation ($\tilde{J}^s = \tilde{J}_0 + p\tilde{J}_1 + p^2\tilde{J}_2 + p^3\tilde{J}_3$) at point C are shown in Fig. 4. The parameter p is equal 1/3. From these figures it follows that the proposed time-domain SIBCs of high order of approximation improves the accuracy of the calculations by about 10-20% compared with solving the problem using standard time-domain SIBC (1) of the order of the Leontovich's approximation. Thus the range of problems for which the approach based on the surface impedance concept is applicable, is extended.

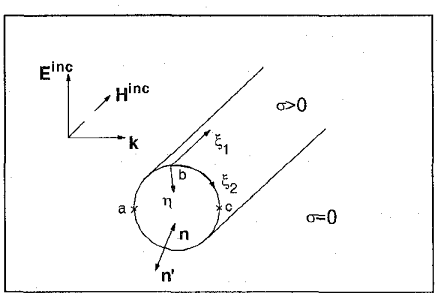

Fig.2. The problem of transient scattering from infinitely long cylinder

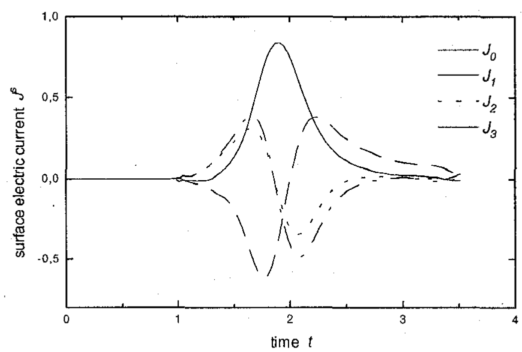

Fig. 3. The time-distributions of the coefficients of the expansions of the surface electric current at point C on the contour of the cylinder cross section

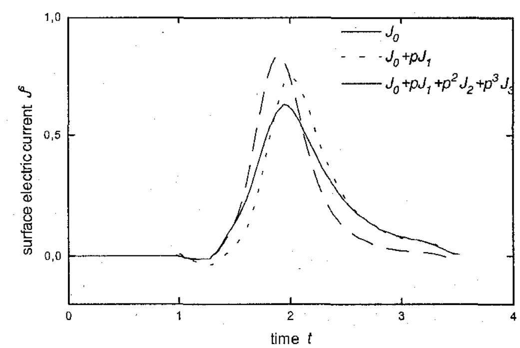

Fig. 4. The time-distributions of the surface electric current, calculated in the PEC-limit (dotted line), in the Leontovich's approximation (dashed line) and the proposed high order approximation (solid line), at point c of the surface.

References

[1] F.M.Tesche, "On the inclusion of loss in time-domain solutions of electromagnetic interation problems", *IEEE Trans. Electromag. Compat.*, Vol. 31, No. 1, pp. 1-4, January 1990.

[2] S.Celozzi and M.Feliziani, "Transient scattering problems solution by surface equaivalent sources", *IEEE Trans. Magnetics*, Vol. 30, No. 5, pp. 3148-3151, September 1994.

[3] N.Ida and S.Yuferev, "Impedance boundary conditions for transient scattering problems", *IEEE Trans. Magnetics*, Vol. 33, No. 2, pp. 1444-1447, March 1997.

[4] K.M.Mitzner, "An integral equation approach to scattering from a body of finite conductivity", *Radio Science*, Vol. 2, pp. 1459-1470, 1967.

[5] S.M.Rytov, "Calculation of skin effect by perturbation method", *Zhurnal Experimental'noi I Teoreticheskoi Fiziki*, Vol. 10, pp.180-189, 1940.