Efficient Implementation of the Time Domain Surface Impedance Boundary Conditions for the Boundary Element Method

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Abstract - **Efficient implementation of the time domain surface impedance boundary conditions on the surface of a homogeneous lossy dielectric body for the space-time domain surface integral equations for the electric and magnetic fields is proposed. The governing equations are transformed by using the perturbation techniques in the small parameter, equal to the ratio of the penetration depth and body's characteristic dimension. As a result, the terms, containing time-convolution integrals, are moved from the left-hand side to the right-hand side of the integral equations, so that the computer resources required for numerical realisation of the formulations are greatly reduced. A numerical example is included to illustrate the theory.**

Index terms - **Transient scattering, integral equations, time domain analysis, surface impedance boundary condition, skin effect, boundary element methods, transient analysis.**

I. INTRODUCTION

For many engineering problems, the transient response from a lossy dielectric body, excited as electromagnetic scatterer, need be calculated. With the advent of fast computers, the time domain integral equation techniques are gaining acceptance. However, until now, they remain computationally expensive in most cases. The problem can be essentially simplified, if the electromagnetic penetration depth into the body is small when compared with the thickness of the body. Then the conducting region may be replaced by appropriate boundary relations (surface impedance boundary conditions - SIBCs) and eliminated from the numerical procedure.

Recently, the time domain SIBCs were successfully implemented into the finite-difference time domain method [1] and the finite element method [2]. Although it is natural to couple the time domain SIBCs with the time domain surface integral equations (SIEs), surprisingly, only a limited amount of work has been reported [3-5]. The reason in probably that Teshce, who first formulated a time domain integral equation enforcing the time domain SIBC, pointed out that direct coupling SIBC and SIE in the time domain is impractical due to large computation time and storage required. Indeed, the time domain SIBC necessarily includes the time-convolution integral that should be calculated at every time step together with the integral equation. However, the computational expense could be greatly reduced if the time-convolution integrals are moved from the left- hand side of the SIE to the right-hand side. In this case the time-convolution integrals can be calculated and tabulated in advance for some time-points and then the result for the required instant can be obtained by the interpolation. The objective of this paper is development of the time domain surface integral equation formulations of the problem in which the approach, described above, is implemented.

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II. STATEMENT OF THE PROBLEM AND GOVERNING EQUATIONS

Consider a homogeneous lossy dielectric body surrounded by the non-conductive medium. Let the time variation of the incident field be such that the penetration depth δ into the body remains small as compared with the characteristic dimension D of the body surface

$$\delta = \sqrt{\tau/\sigma\mu_0} << D \qquad (1)$$

where τ is the incident pulse duration. The distribution of the electric $\vec{E}$ and magnetic $\vec{H}$ fields can be described by Maxwell's equations in the differential and integral form:

Inside the body:

$$\nabla \times \vec{H}^i = \sigma \vec{E}^i + \varepsilon \partial \vec{E}^i / \partial t \ (2); \qquad \nabla \times \vec{E}^i = -\mu_0 \partial \vec{H}^i / \partial t \ (3)$$

Outside the body:

$$\vec{n} \times \vec{H}^e - \frac{\vec{n}\times}{2\pi} \iint_s \left\{ \frac{\varepsilon_0}{R} \frac{\partial}{\partial t'} \left(\vec{n} \times \vec{E}^e \right) + L\left[\vec{n} \cdot \vec{H}^e\right] \frac{\vec{R}}{R} + \right.$$
$$\left. + L[\vec{n} \times \vec{H}^e] \times \frac{\vec{R}}{R} \right\}_{t'=t-R/c} ds = 2\vec{n} \times \vec{H}^{inc} \qquad (4)$$

$$\vec{n} \times \vec{E}^e + \frac{\vec{n}\times}{2\pi} \iint_s \left\{ \frac{\mu_0}{R} \frac{\partial}{\partial t'} \left(\vec{n} \times \vec{H}^e \right) - L\left[\vec{n} \cdot \vec{E}^e\right] \frac{\vec{R}}{R} - \right.$$
$$\left. - L[\vec{n} \times \vec{E}^e] \times \frac{\vec{R}}{R} \right\}_{t'=t-R/c} ds = 2\vec{n} \times \vec{E}^{inc} \qquad (5)$$

where c is the velocity of light, S is the surface of the body the operator L is defined as follows

$$L[f] = fR^{-2} + (cR)^{-1} \partial f / \partial t$$

Boundary relations:

$$\left(\vec{n} \cdot \vec{H}^e\right)^s = \left(\vec{n} \cdot \vec{H}^i\right)^s; \qquad \vec{n} \times \left(\vec{n} \times \vec{E}^e\right)^s = \vec{n} \times \left(\vec{n} \times \vec{E}^i\right)^s \qquad (6)$$

where superscripts "*e*", "*i*" and "*s*" denote the field outside the body, inside the body and at the body's surface, respectively. The superscripts "*e*" and "*i*" will be omitted below to simplify the description of the transformations.

III. Time Domain Surface Impedance Boundary Conditions

Condition (1) enables consideration of the problem (2)-(3) by using the surface impedance concept. As a result, the following expressions for the tangential component of the electric field and the normal component of the magnetic field on the body's surface were obtained [5,7]:

$$E^s_{\xi_k} = (-1)^{3-k} H^s_{\xi_{3-k}} * \left(T_0' + (d_{3-k}^{-1} - d_k^{-1})T_1'\right), \quad k=1,2; \qquad (7)$$

$$H^s_\eta = \sum_{i=1}^{2} \left(\partial H^s_{\xi_i} / \partial \xi_i\right) * \left(T_0 + (d_i^{-1} - d_{3-i}^{-1})T_1\right) \qquad (8)$$

where * denotes a time convolution product, d_k, $k=1,2$, are the local radii of curvature, while (ξ_1, ξ_2, η) are the principal curvature coordinates defined as

$$\vec{e}_{\xi_1} \times \vec{e}_{\xi_2} = \vec{e}_\eta = -\vec{n}$$

where $\vec{e}_{\xi_1}, \vec{e}_{\xi_2}, \vec{e}_\eta$ are the basis unit vectors of the system and the unit normal vector $\vec{n}$ is directed outside the body (Fig.1). The time functions T_m, T_m', $m=0,1$, are defined as

$$T_0(t) = (\varepsilon\mu_0)^{-1/2} I_0(\sigma t/(2\varepsilon)) \exp(-\sigma t/(2\varepsilon))$$

$$T_1(t) = (\sigma\mu_0)^{-1}\left[1 - \exp(-\sigma t/\varepsilon)\right]$$

$$T_0'(t) = \left(\frac{\mu_0}{\varepsilon}\right)^{1/2} \left\{\delta(t) + \frac{\sigma}{2\varepsilon}\left[I_1\left(\frac{\sigma t}{2\varepsilon}\right) - I_0\left(\frac{\sigma t}{2\varepsilon}\right)\right]\exp\left(-\frac{\sigma t}{2\varepsilon}\right)\right\}$$

$$T_1'(t) = \varepsilon^{-1}\exp(-\sigma t/\varepsilon)$$

Here $I_n(x)$ is the modified Bessel function of order n, and $\delta(t)$ is the unit step function.

Note that the time domain SIBCs are related to the frequency domain SIBCs and are often obtained directly from the latter by using the inverse Laplace transform. In particular, (7) can be obtained from the following well-known frequency domain SIBC named after Mitzner [6]:

$$\dot{E}^s_{\xi_k} = (-1)^{3-k} \frac{1+j}{2\sqrt{1+j\chi}} \mu_0 \omega \delta \left[1 + \frac{1-j}{4\sqrt{1+j\chi}} \delta \frac{d_{3-k} - d_k}{d_{3-k} d_k}\right] \dot{H}^s_{\xi_{3-k}}$$

$$\delta = \sqrt{2/(\omega\sigma\mu_0)}; \qquad \chi = \varepsilon\omega/\sigma$$

The SIBCs (7)-(8) are substituted into (4) and (5) with the purpose to decouple the integral equations. However, direct coupling leads to the formulations in which the terms containing the time-convolution integrals are on the left hand side of the equations. To avoid this, we use one of the basic properties of the SIBCs, namely: that $E^s_{\xi_k}$ and H^s_η on the body surface are less than $H^s_{\xi_k}$ by the order of magnitude of the skin layer thickness. It should be noted that only the tangential component of the electric field and the normal component of the magnetic field contain the time-convolution integrals.

IV. Dimensionless Variables

Condition (1) means that the variation of the electromagnetic field in the tangential directions ξ_1, ξ_2 is small compared to the variation in the normal direction η. The characteristic lengths associated with these variables are D and δ, respectively. Because δ/D<<1, we can apply the perturbation technique to transform the integral equations and the SIBCs.

As a first step we switch to dimensionless variables, choosing the following scale factors:

$$\left.\begin{array}{l}[\xi_1, \xi_2] = D; \quad [\eta] = (\delta/D)D = pD; \quad p = \sqrt{\tau^*/(\sigma\mu_0 D^2)} \\ \qquad [H] = I^*/D; \quad [E] = \mu_0 I^*/\tau^* \end{array}\right\} \qquad (9)$$

where square brackets denote a scale factor for the corresponding value; I^* and τ^* are the scale factors for the current and time, respectively. As the latter it is natural to choose the duration of the incident pulse.

With the dimensionless variables, the SIBCs and the integral equations are written in the following form:

Surface Impedance Boundary Conditions:

$$\tilde{E}^s_{\xi_k} = (-1)^{3-k} p \tilde{H}^s_{\xi_{3-k}} * \left(\tilde{T}_0' + p\frac{\tilde{d}_k - \tilde{d}_{3-k}}{2\tilde{d}_k \tilde{d}_{3-k}} \tilde{T}_1'\right), \quad k=1,2; \qquad (10)$$

$$\tilde{H}^s_\eta = p\sum_{i=1}^{2} \frac{\partial \tilde{H}^s_{\xi_{3-k}}}{\partial \tilde{\xi}_i} * \left(\tilde{T}_0 + p\frac{\tilde{d}_{3-i} - \tilde{d}_i}{2\tilde{d}_i \tilde{d}_{3-i}} \tilde{T}_1\right) \qquad (11)$$

$$\tilde{T}_0(t) = a^{-1/2} I_0(\tilde{t}/(2a)) \exp(-\tilde{t}/(2a))$$

$$\tilde{T}_1(t) = 1 - \exp(-\tilde{t}/a)$$

$$\tilde{T}_0'(t) = \frac{1}{2a^{3/2}}\left[I_1\left(\frac{\tilde{t}}{2a}\right) - I_0\left(\frac{\tilde{t}}{2a}\right)\right]\exp\left(-\frac{\tilde{t}}{2a}\right) + \delta(\tilde{t})a^{-1/2}$$

$$\tilde{T}_1'(t) = a^{-1}\exp(-\tilde{t}/a)$$

where $\tilde{d}_k = d_k/D$, $a = \varepsilon_0 \varepsilon_r/(\sigma\tau)$ and the sign "~" denotes dimensionless value. Note that functions $\tilde{T}_m'$ are related to the functions $\tilde{T}_m$ by Duhamel's theorem

$$\tilde{T}_m'(\tilde{t}) * \tilde{f}(\tilde{t}) = \frac{d\tilde{T}_m(\tilde{t})}{d\tilde{t}} * \tilde{f}(t) + \tilde{T}_m(0)\tilde{f}(\tilde{t}) = \frac{\partial}{\partial \tilde{t}}\left(\tilde{T}_m(\tilde{t}) * \tilde{f}(\tilde{t})\right)$$

By introducing the scale factors (9), the small parameter p appears in the SIBCs.

Surface Integral Equations:

$$\vec{n} \times \tilde{\vec{H}} - \frac{\vec{n}\times}{2\pi} \iint_s \left\{ \frac{q^2}{\tilde{R}} \frac{\partial}{\partial \tilde{t}'}\left(\vec{n} \times \tilde{\vec{E}}\right) + L_1\left[\vec{n} \cdot \tilde{\vec{H}}\right]\frac{\tilde{\vec{R}}}{\tilde{R}} + \right.$$

$$\left. + L_1[\vec{n} \times \tilde{\vec{H}}] \times \frac{\tilde{\vec{R}}}{\tilde{R}} \right\}_{\tilde{t}' = \tilde{t} - q\tilde{R}} d\tilde{s} = 2\vec{n} \times \tilde{\vec{H}}^{inc} \qquad (12)$$

$$\vec{n} \times \tilde{\vec{E}} + \frac{\vec{n}\times}{2\pi} \iint_s \left\{ \frac{1}{\tilde{R}} \frac{\partial}{\partial \tilde{t}'}\left(\vec{n} \times \tilde{\vec{H}}\right) - L_1\left[\vec{n} \cdot \tilde{\vec{E}}\right]\frac{\tilde{\vec{R}}}{\tilde{R}} - \right.$$

$$\left. - L_1[\vec{n} \times \tilde{\vec{E}}] \times \frac{\tilde{\vec{R}}}{\tilde{R}} \right\}_{\tilde{t}' = \tilde{t} - q\tilde{R}} d\tilde{s} = 2\vec{n} \times \tilde{\vec{E}}^{inc} \qquad (13)$$

where $q = D/(c\tau)$ and the operator L_1 is defined as follows

$$L_1\left[\tilde{f}\right] = \tilde{f}\tilde{R}^{-2} + q\tilde{R}^{-1}\partial\tilde{f}/\partial\tilde{t}$$

From Maxwell's equations for free space

$$\vec{n}\cdot\tilde{\vec{E}}^s = q^{-2}\int_0^{\tilde{t}} \vec{n}\cdot(\nabla\times\tilde{\vec{H}}^s)d\tilde{t}' = q^{-2}\int_0^{\tilde{t}} Div(\vec{n}\times\tilde{\vec{H}}^s)d\tilde{t}' \quad (14)$$

By substituting (14) into (13) and introducing the equivalent surface electric current $\tilde{\vec{J}}^s = \vec{n}\times\tilde{\vec{H}}^s$, the equations (12)-(13) are written in the form:

$$\tilde{\vec{J}}^s + \frac{\vec{n}\times}{2\pi}\iint_s \left\{ \frac{\tilde{\vec{R}}}{\tilde{R}}\times L_1\left[\tilde{\vec{J}}^s\right] \right\}_{\tilde{t}'=\tilde{t}-q\tilde{R}} d\tilde{s} = 2\vec{n}\times\tilde{\vec{H}}^{inc} +$$

$$+\frac{\vec{n}\times}{2\pi}\iint_s \left\{ \frac{q^2}{\tilde{R}}\frac{\partial\left(\vec{n}\times\tilde{\vec{E}}^s\right)}{\partial\tilde{t}'} + \frac{\tilde{\vec{R}}}{\tilde{R}}L_1\left[\vec{n}\cdot\tilde{\vec{H}}^s\right] \right\}_{\tilde{t}'=\tilde{t}-q\tilde{R}} d\tilde{s} \quad (15)$$

$$\frac{\vec{n}\times}{2\pi}\iint_s \left\{ \frac{1}{\tilde{R}}\frac{\partial\tilde{\vec{J}}^s}{\partial\tilde{t}'} - \frac{\tilde{\vec{R}}}{q^2\tilde{R}}L_1\left[\int_0^{\tilde{t}'} Div\tilde{\vec{J}}^s d\tilde{t}''\right] \right\}_{\tilde{t}'=\tilde{t}-q\tilde{R}} d\tilde{s} = 2\vec{n}\times\tilde{\vec{E}}^{inc} -$$

$$-\vec{n}\times\tilde{\vec{E}}^s - \frac{\vec{n}\times}{2\pi}\iint_s \left\{ \frac{\tilde{\vec{R}}}{\tilde{R}}\times L_1\left[\vec{n}\times\tilde{\vec{E}}^s\right] \right\}_{\tilde{t}'=\tilde{t}-q\tilde{R}} d\tilde{s} \quad (16)$$

According to the vector identities, one writes

$$\vec{n}\times(\tilde{\vec{R}}\times\tilde{\vec{J}}^s) = -\tilde{\vec{J}}^s(\vec{n}\cdot\tilde{\vec{R}}) \qquad (\text{since } \vec{n}\perp\tilde{\vec{J}}^s) \quad (17)$$

$$\vec{n}\times\left(\tilde{\vec{R}}\times L_1\left[\vec{n}\times\tilde{\vec{E}}^s\right]\right) = -L_1\left[\vec{n}\times\tilde{\vec{E}}^s\right](\vec{n}\cdot\tilde{\vec{R}}) \quad (18)$$

Substituting (17)-(18) into (16)-(17) and switching to the internal normal vector $\vec{n}' = -\vec{n}$, we obtain

$$\tilde{J}^s_{\xi_k} + \frac{1}{2\pi}\iint_s \left\{ \frac{(\vec{n}'\cdot\tilde{\vec{R}})}{\tilde{R}}L_1\left[\tilde{J}^s_{\xi_k}\right] \right\}_{\tilde{t}'=\tilde{t}-q\tilde{R}} d\tilde{s} = (-1)^{3-k}2\tilde{H}^{inc}_{\xi_{3-k}} -$$

$$-\frac{1}{2\pi}\iint_s \left\{ \frac{q^2}{\tilde{R}}\frac{\partial\tilde{E}^s_{\xi_k}}{\partial\tilde{t}'} + \frac{(\vec{n}'\times\tilde{\vec{R}})_{\xi_k}}{\tilde{R}}L_1\left[\vec{n}'\cdot\tilde{\vec{H}}^s\right] \right\}_{\tilde{t}'=\tilde{t}-q\tilde{R}} d\tilde{s} \quad (19)$$

$$\frac{1}{2\pi}\iint_s \left\{ \frac{1}{\tilde{R}}\frac{\partial\tilde{J}^s_{\xi_k}}{\partial\tilde{t}'} + \frac{\tilde{R}_{\xi_k}}{q^2\tilde{R}}L_1\left[\int_0^{\tilde{t}'} Div\tilde{\vec{J}}^s d\tilde{t}''\right] \right\}_{\tilde{t}'=\tilde{t}-q\tilde{R}} d\tilde{s} =$$

$$= 2\tilde{E}^{inc}_{\xi_k} - \tilde{E}^s_{\xi_k} + \frac{1}{2\pi}\iint_s \left\{ \frac{(\vec{n}'\cdot\tilde{\vec{R}})}{\tilde{R}}L_1\left[\tilde{E}^s_{\xi_k}\right] \right\}_{\tilde{t}'=\tilde{t}-q\tilde{R}} d\tilde{s} \quad (20)$$

Note that the integral equations (19) and (20) involve the terms of different order of magnitude, namely: the terms containing $\tilde{E}^s_{\xi_k}$ or $\tilde{H}^s_\eta$ are of the orders of magnitude $O(p)$ whereas the terms containing $\tilde{H}^s_{\xi_k}$ are $O(1)$. Therefore, it is possible to transform (19)-(20) by using the perturbation techniques in the small parameter p.

V. Perturbation Technique

We can represent the electric and magnetic fields in the form of the asymptotic expansions in the small parameter p:

$$\tilde{\vec{E}} = \sum_{m=1}^{\infty} p^m\tilde{\vec{E}}_m; \qquad \tilde{\vec{H}} = \sum_{m=1}^{\infty} p^m\tilde{\vec{H}}_m; \qquad \tilde{\vec{J}}^s = \sum_{m=1}^{\infty} p^m\tilde{\vec{J}}^s_m \quad (21)$$

By substituting the expansion (21) into the SIBCs (10)-(11), the SIEs (19)-(20) and equating the coefficients of equal powers of p, the following time domain surface integral equations for the terms of expansions are obtained:

Magnetic field surface integral equation formulation:

$$(\tilde{\vec{J}}^s_m)_{\xi_k} + \frac{1}{2\pi}\iint_s \left\{ \frac{(\vec{n}'\cdot\tilde{\vec{R}})}{\tilde{R}}L_1\left[(\tilde{\vec{J}}^s_m)_{\xi_k}\right] \right\}_{\tilde{t}'=\tilde{t}-q\tilde{R}} d\tilde{s} = (\tilde{\vec{F}}^{magn}_m)_{\xi_k} \quad (22)$$

m=0,1,2, $\qquad$ *k=1,2*

where the right-hand sides $\tilde{\vec{F}}^{magn}_m$ are written in the form:

$$(\tilde{\vec{F}}^{magn}_0)_{\xi_k} = (-1)^{3-k}2\tilde{H}^{inc}_{\xi_{3-k}} \quad (23a)$$

$$(\tilde{\vec{F}}^{magn}_1)_{\xi_k} = -\frac{1}{2\pi}\iint_s \left\{ \frac{q^2}{\tilde{R}}\frac{\partial}{\partial\tilde{t}'}\left(\tilde{T}_0' * (\tilde{\vec{J}}^s_0)_{\xi_k}\right) + \right.$$

$$\left. +(-1)^k\frac{\tilde{R}_{\xi_{3-k}}}{\tilde{R}}L_1\left[\tilde{T}_0 * \sum_{i=1}^{2}(-1)^i\frac{\partial}{\partial\tilde{\xi}_i}(\tilde{\vec{J}}^s_0)_{\xi_{3-i}}\right] \right\}_{\tilde{t}'=\tilde{t}-q\tilde{R}} d\tilde{s} \quad (23b)$$

$$(\tilde{\vec{F}}^{magn}_2)_{\xi_k} = -\frac{1}{2\pi}\iint_s \left\{ \frac{q^2}{\tilde{R}}\frac{\partial}{\partial\tilde{t}'}\left(\tilde{T}_0' * (\tilde{\vec{J}}^s_1)_{\xi_k}\right) + \right.$$

$$+\frac{q^2}{\tilde{R}}\frac{\tilde{d}_k - \tilde{d}_{3-k}}{2\tilde{d}_k\tilde{d}_{3-k}}\frac{\partial}{\partial\tilde{t}'}\left(\tilde{T}_1' * (\tilde{\vec{J}}^s_0)_{\xi_k}\right) +$$

$$+\frac{\tilde{R}_{\xi_{3-k}}}{\tilde{R}}L_1\left[\tilde{T}_1 * \sum_{i=1}^{2}(-1)^{k+i}\frac{\tilde{d}_{3-i} - \tilde{d}_i}{2\tilde{d}_i\tilde{d}_{3-i}}\frac{\partial}{\partial\tilde{\xi}_i}(\tilde{\vec{J}}^s_0)_{\xi_{3-i}}\right] +$$

$$\left. +\frac{\tilde{R}_{\xi_{3-k}}}{\tilde{R}}L_1\left[\tilde{T}_0 * \sum_{i=1}^{2}(-1)^{k+i}\frac{\partial}{\partial\tilde{\xi}_i}(\tilde{\vec{J}}^s_1)_{\xi_{3-i}}\right] \right\}_{\tilde{t}'=\tilde{t}-q\tilde{R}} d\tilde{s} \quad (23c)$$

Electric field surface integral equation formulation

$$\frac{1}{2\pi}\iint_s \left\{ \frac{1}{\tilde{R}}\frac{\partial(\tilde{\vec{J}}^s_m)_{\xi_k}}{\partial\tilde{t}'} + \frac{\tilde{R}_{\xi_k}}{q^2\tilde{R}}L_1\left[\int_0^{\tilde{t}'} Div(\tilde{\vec{J}}^s_m)_{\xi_k}d\tilde{t}''\right] \right\}_{\tilde{t}'=\tilde{t}-q\tilde{R}} d\tilde{s} =$$

$$= (\tilde{\vec{F}}^{elec}_0)_{\xi_k}; \qquad m=0,1,2, \qquad k=1,2 \quad (24)$$

where the right-hand sides $\tilde{\vec{F}}^{elec}_m$ are written in the form:

$$(\tilde{\vec{F}}^{elec}_0)_{\xi_k} = 2\tilde{E}^{inc}_{\xi_k} \quad (25a)$$

$$(\tilde{\vec{F}}^{elec}_1)_{\xi_k} = -\tilde{T}_0' * (\tilde{\vec{J}}^s_0)_{\xi_k} +$$

$$+\frac{1}{2\pi}\iint_s \left\{ \frac{(\vec{n}'\cdot\tilde{\vec{R}})}{\tilde{R}}L_1\left[\tilde{T}_0' * (\tilde{\vec{J}}^s_0)_{\xi_k}\right] \right\}_{\tilde{t}'=\tilde{t}-q\tilde{R}} d\tilde{s} \quad (25b)$$

$$(\tilde{\vec{F}}^{elec}_2)_{\xi_k} = -\tilde{T}_0' * (\tilde{\vec{J}}^s_1)_{\xi_k} - \frac{\tilde{d}_k - \tilde{d}_{3-k}}{2\tilde{d}_k\tilde{d}_{3-k}}\tilde{T}_1' * (\tilde{\vec{J}}^s_0)_{\xi_k} +$$

$$+\frac{1}{2\pi}\iint_s \left\{ \frac{(\vec{n}'\cdot\tilde{\vec{R}})}{\tilde{R}}L_1\left[\tilde{T}_0' * (\tilde{\vec{J}}^s_1)_{\xi_k}\right] + \right.$$

$$+\frac{\tilde{d}_k-\tilde{d}_{3-k}}{2\tilde{d}_k\tilde{d}_{3-k}}\frac{(\vec{n}'\cdot\vec{\tilde{R}})}{\tilde{R}}L_1\left[\tilde{T}_1'*(\vec{\tilde{J}}_0^s)_{\xi_k}\right]\Bigg\}_{\tilde{t}'=\tilde{t}-q\tilde{R}} d\tilde{s} \qquad (25c)$$

In (23) and (25) we used the following relation:

$$\tilde{J}^s_{\xi_k}=(-1)^{3-k}\tilde{H}^s_{\xi_{3-k}}, \qquad k=1,2$$

The formulations developed have the following basic advantages:

1. The integral equations for the terms of the expansions differ only in the form of the right-hand side and can be solved by the same solving procedures, therefore, new computational complications do not arise as compared with solving the problem using the well-known perfect electrical conductor (PEC) limit ($m=0$);
2. The convolution integrals in the integral equations are on the right-hand sides only and can be calculated and tabulated before solving the appropriate integral equation so that the computer resources required for the computations are greatly reduced as compared with the formulation developed by Tesche.

VI. Numerical Example

We consider the problem of transient scattering from an infinitely long straight cylinder of round cross section illuminated by a Gaussian pulse (TE case) by using the magnetic field integral equation formulation (22)-(23). As shown in Fig. 1, the coordinate ξ_1 is directed along the cylinder so $d_1=\infty$. Since the radius d of the cylinder cross section is constant, then $d_2=D$ and $\tilde{d}_2=1$. Under these conditions only the circumferential ξ_2-component of the surface electric current is non-zero and the vector can be treated as scalar. The scale factor $J^{inc}_{\max}D$ is used as I^*; here $J^{inc}_{\max}$ is the maximum of J^{inc}. The width of the incident pulse was taken to be D, i.e. $q=1$. The parameter a was taken equal to unity. The time $\tilde{t}=0$ corresponds to the time that the pulse reaches the body.

Figure 2 shows the distributions of the functions $\tilde{J}^s_0=\tilde{J}^s_0(\tilde{\xi}_2)$, $\tilde{J}^s_1=\tilde{J}^s_1(\tilde{\xi}_2)$ and $\tilde{J}^s_2=\tilde{J}^s_2(\tilde{\xi}_2)$ at time $\tilde{t}=1$. The coordinate $\tilde{\xi}_2$ has its origin at point A and proceeds around the contour ending at the point B (Fig. 1). The function $\tilde{J}^s_0$ is the solution of the problem in the perfect electrical conductor limit. In this limit, the magnetic field diffusion into the body is neglected and the current is assumed to be flowing only on the body surface. Therefore, by keeping only one term in the expansions (21) it is impossible to describe the redistribution of the current *along* the body surface induced by the field diffusion into the body. The effect of this process is to smooth out nonuniformities in the surface current distribution. This effect is of the order of magnitude $O(p)$ and can be taken into account by the next terms of expansions. From Fig. 2 it follows that the surface electric current will increase in the regions where the function $\tilde{J}^s_1$ is positive and decrease where the function $\tilde{J}^s_1$ is negative. The second-order term $\tilde{J}^s_2$ gives the further correction by taking into account the principal curvature of the body surface.

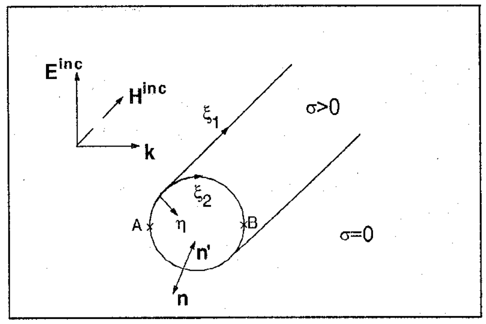

Fig. 1. The problem of transient scattering from infinitely long cylinder

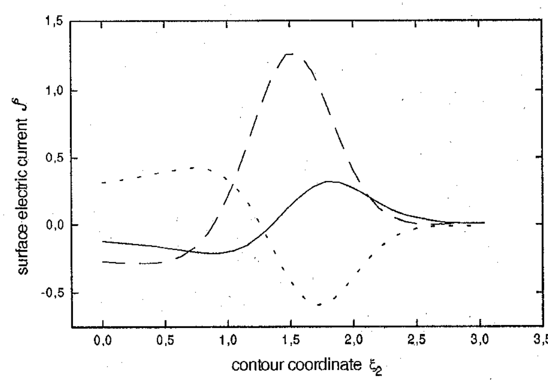

Fig.2. The distributions of the terms $\tilde{J}^s_2$ (solid line), $\tilde{J}^s_1$ (dotted line) and $\tilde{J}^s_0$ (dashed line) of the expansion of the surface electric current over one half of the cross section of the cylinder

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