IMPEDANCE BOUNDARY CONDITIONS OF HIGH ORDER APPROXIMATION FOR ELECTROMAGNETIC TRANSIENT SCATTERING PROBLEMS

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Abstract - The problem of the diffusion of transient electromagnetic field into a lossy dielectric body was solved by using the method of perturbations in the small parameter *p* that is equal to the ratio of the electromagnetic penetration depth and characteristic dimension of the body. The time- and frequency domain solutions for the tangential component of the electric field and the normal component of the magnetic field on the smooth curved surface of the body (the surface impedance boundary conditions - SIBC) have been obtained accurate to $O(p^4)$. It was shown that the proposed SIBC in frequency domain generalize well-known Leontovich's and Mitzner's boundary conditions that provides the approximation accuracy within the errors $O(p^2)$ and $O(p^3)$, respectively.

INTRODUCTION

The transient analysis of the electromagnetic scattering is of great interest for practice. With the advent of the fast computers, among the various solution techniques for obtaining transient responses, the time domain integral equation techniques are gaining acceptance. However, even at the present time, they remain computationally expensive in most cases. The problem is simplified if the electromagnetic penetration depth in the conducting body is so short that the variation of the field in the direction tangential to the body's surface is much less than the field variation in the normal direction, so that the complete equation of the electromagnetic field diffusion into the body can be replaced by 1-D equation in the direction normal to the surface of the body. The solution of the reduced equation can be then used to derive so-called surface impedance boundary conditions (SIBC) involving only the external fields imposed at the outer surface to simulate the material properties of the body and to convert thereby a two (or more) media problem into a one medium one.

Originally the surface impedance concept in the electromagnetic field theory arose through time harmonic problems. The classical SIBC for planar surface, frequently called as Leontovich's condition (1), is written in the form

$$\vec{n}'\times\vec{E}(\omega) = Z_\omega \vec{n}'\times\left(\vec{n}'\times\vec{H}(\omega)\right); \quad Z_\omega = \sqrt{\frac{j\omega\mu}{\sigma + j\omega\varepsilon}} \qquad (1)$$

where ω is the angular frequency of the field source, σ and μ are, respectively, the electrical conductivity and magnetic permeability of the body. The Leontovich's SIBC were then corrected by Mitzner (2), who developed the conditions, now known by his name, for any smooth surface of a conducting body. The Mitzner's SIBC are written in the form:

$$E_{\vartheta_k} = \frac{1+j}{2}\mu\omega\delta\left((-1)^k - \delta\theta\right)H_{\vartheta_{3-k}}; \qquad k=1,2 \quad (2)$$

$$\theta = (1-j)\left(C_{\vartheta_2} - C_{\vartheta_1}\right)/4; \qquad \delta = \sqrt{2/(\omega\sigma\mu)}$$

where C_{ϑ_k}, *k=1,2*, are the principal curvatures, while ϑ_k are the principal curvature coordinates; δ is the electromagnetic penetration depth. In the recent years, use of the SIBC with the surface integral equations becames very popular in the high-frequency time harmonic low penetration electromagnetic problems.

Note that the condition (2) includes the term containing δ^2 whereas the condition (1) contains δ only. It is natural to expect that the SIBC of the approximation order exceeding the order of the Mitzner's approximation should include the terms containing δ^3 and higher. The way to obtain these terms was suggested by Rytov (3) more than fifty years ago. He applied the perturbation method and used the time-harmonic solution of 1-D equation of the magnetic field diffusion into perfect conductor as an initial approximation. As a result, Rytov derived the boundary conditions at the planar surface of a highly conducting body in the following form of asymptotic expansions in the skin depth taken as a small parameter

$$E_{\vartheta_k} = (-1)^k \frac{1+j}{2}\mu\omega\delta\left[H_{\vartheta_{3-k}} + \frac{\delta^2}{4j}\left(-\frac{\partial^2 H_{\vartheta_{3-k}}}{\partial\vartheta_k^2} + \frac{\partial^2 H_{\vartheta_{3-k}}}{\partial\vartheta_{3-k}^2} + 2\frac{\partial^2 H_{\vartheta_k}}{\partial\vartheta_k\partial\vartheta_{3-k}}\right) + \ldots\right]; \qquad k=1,2 \qquad (3)$$

The main advantage of the expression in (3) is that the nonuniformity of the magnetic field in the direction *tangential* to the body surface is taken into account under the concept of the surface impedance based on the solution of reduced 1-D problem in the direction *normal* to the body surface. The generality of the condition (3) is not appreciated since only the SIBC of low order of approximation were used until recently.

The SIBC concepts can be also used in transient probelms, when, for instance, the duration of the incident pulse is so short that the field has no time to diffuse deeply into the body and remains concentrated in the thin layer near the body surface. The simplest SIBC in time domain is directly obtained from (1) by using inverse Laplace transformation and written in the form of the convolution with respect to time:

$$\vec{n}'\times\vec{E}(t) = -Z_t * \left(\vec{n}'\times\vec{H}(t)\right)\times\vec{n}' \qquad (4)$$

$$Z_t = \left(\frac{\mu}{\varepsilon}\right)^{1/2}\left\{\delta(t) + \frac{\sigma}{2\varepsilon}\left[I_1\left(\frac{\sigma t}{2\varepsilon}\right) - I_0\left(\frac{\sigma t}{2\varepsilon}\right)\right]\exp\left(-\frac{\sigma t}{2\varepsilon}\right)\right\}$$

where * denotes a time domain convolution product product, $I_n(x)$ is the modified Bessel function of order n and $\delta(t)$ is the unit step function. The condition (4) is successfully implemented into the finite-difference time domain (FDTD) method and the finite element method. Although it is natural to couple the time domain SIBC with the time domain surface integral equations (TDSIE), surprisingly, only a limited amount of work has been reported (Teshce (4), Yuferev (5)).

It should be pointed out that in the cited literature the time domain SIBC (4) of lowest order of approximation have been used. It is rational for the FDTD in its original form, where modelling complex electromagnetic problems with curved surfaces cause difficulties. But for the TDSIE it is preferable to use the SIBC of higher order of approximation as it is usually done in the frequency domain coupled SIBC-SIE techniques (in this connection it is remarkable that Mitzner was also a pioneer in coupling the SIBC with the integral equations). Therefore, the main objective of this work is derivation of the time domain SIBC of high order of approximation for the tangential component of the electric field and the normal component of the magnetic field on the smooth curved surface of lossy dielectric body. In other words, we have to obtain the time domain SIBC which should be of high order of approximation as Rytov's condition, and suitable for smooth curved surfaces as Mitzner's condition.

METHOD

Statement of the Problem

Consider a homogeneous body of finite conductivity ($\varepsilon_1 = \varepsilon_r\varepsilon_0,\ \mu_1 = \mu_0,\ \sigma_1 = \sigma$) surrounded by the non-conductive medium ($\varepsilon_2 = \varepsilon_0,\ \mu_2 = \mu_0,\ \sigma_2 = 0$). Let the time variation of the incident field be such that the penetration depth δ into the body remains small as compared with the characteristic dimension D of the surface of the body

$$\delta = \sqrt{\tau/\sigma\mu_0} << D \qquad (5)$$

where τ is the incident pulse duration. The electric $\vec{E}$ and magnetic $\vec{H}$ fields inside the body can be described by Maxwell's equations in the following form

$$\nabla\times\vec{H} = \sigma\vec{E} + \varepsilon_R\varepsilon_0\,\partial\vec{E}/\partial t \qquad (6)$$

$$\nabla\times\vec{E} = -\mu_0\,\partial\vec{H}/\partial t \quad (7); \qquad \nabla\cdot\vec{H} = 0 \quad (8)$$

Here ε_R is relative electric permeability of the body.

Dimensionless Variables

We introduce a local orthogonal curvilinear coordinate system ξ_1,ξ_2,η related to the surface of the body. We direct the coordinates ξ_1,ξ_2 along the surface and the coordinate η normal to it inside the body. The characteristic lengths associated with coordinates ξ_k and η are D and δ, respectively. In the case of smooth surface the scale factor D can be defined as $D = \min(d_1, d_2)$ where d_k is the principal curvature of the coordinate line ξ_k. The Lame's coefficients for the coordinates introduced are written in the following form:

$$e_k = d_k - \eta; \qquad e_3 = 1; \qquad k=1,2 \qquad (9)$$

Following the theory of the perturbation methods, we now transform to dimensionless variables of the order O(1) by choosing the following scale factors:

$$\left.\begin{aligned}&[\xi_1,\xi_2] = D;\ \ [\eta] = pD;\ \ [H] = \frac{I}{D};\ \ [E] = \frac{\mu_0 I}{\tau};\\ &p = \left(\tau/(\sigma\mu_0 D^2)\right)^{1/2};\ \ q = D/(c\tau)\end{aligned}\right\} \qquad (10)$$

where c is the velocity of light in vacuum; I is the scale for the current. The quantity p in (16) is a small parameter since it is the ratio of the electromagnetic penetration depth δ and the characteristic dimension D of the body. The parameter q is the *electric dimension* of the body in transient case. The scale factors in (10) are suitable for time harmonic problems too, but in this case the ratio $2/\omega$ should be chosen as a time scale factor.

With the variables (9)-(10) the equations (6)-(8) are written in the form:

$$\frac{p\tilde{d}_2}{\tilde{d}_2 - p\eta}\frac{\partial E_\eta}{\partial \xi_2} - \frac{\partial E_{\xi_2}}{\partial \eta} + \frac{pE_{\xi_2}}{\tilde{d}_2 - p\eta} = -p\frac{\partial H_{\xi_1}}{\partial t} \qquad (11a)$$

$$\frac{\partial E_{\xi_1}}{\partial \eta} - \frac{pE_{\xi_1}}{\tilde{d}_1 - p\eta} - \frac{p\tilde{d}_1}{\tilde{d}_1 - p\eta}\frac{\partial E_\eta}{\partial \xi_1} = -p\frac{\partial H_{\xi_2}}{\partial t} \qquad (11b)$$

$$\frac{\tilde{d}_1}{\tilde{d}_1 - p\eta}\frac{\partial E_{\xi_2}}{\partial \xi_1} - \frac{\tilde{d}_2}{\tilde{d}_2 - p\eta}\frac{\partial E_{\xi_1}}{\partial \xi_2} = -\frac{\partial H_\eta}{\partial t} \qquad (11c)$$

$$\frac{p^2\tilde{d}_2}{\tilde{d}_2 - p\eta}\frac{\partial H_\eta}{\partial \xi_2} - p\frac{\partial H_{\xi_2}}{\partial \eta} + \frac{p^2 H_{\xi_2}}{\tilde{d}_2 - p\eta} = E_{\xi_1} + a\frac{\partial E_{\xi_1}}{\partial t} \qquad (12a)$$

$$p\frac{\partial H_{\xi_1}}{\partial \eta} - \frac{p^2 H_{\xi_1}}{\tilde{d}_1 - p\eta} - \frac{p^2\tilde{d}_1}{\tilde{d}_1 - p\eta}\frac{\partial H_\eta}{\partial \xi_1} = E_{\xi_2} + a\frac{\partial E_{\xi_2}}{\partial t} \qquad (12b)$$

$$p^2\left(\frac{\tilde{d}_1}{\tilde{d}_1 - p\eta}\frac{\partial H_{\xi_2}}{\partial \xi_1} - \frac{\tilde{d}_2}{\tilde{d}_2 - p\eta}\frac{\partial H_{\xi_1}}{\partial \xi_2}\right) = E_\eta + a\frac{\partial E_\eta}{\partial t} \qquad (12c)$$

$$\frac{\partial H_\eta}{\partial \eta} - pH_\eta\sum_{i=1}^{2}\frac{1}{\tilde{d}_i - p\eta} = -p\sum_{i=1}^{2}\frac{\tilde{d}_i}{\tilde{d}_i - p\eta}\frac{\partial H_{\xi_i}}{\partial \xi_i} \qquad (13)$$

where $a = p^2 q^2 \varepsilon_R = \varepsilon_0 \varepsilon_R/(\sigma\tau)$ and $\tilde{d}_k = d_k/D$; *k=1,2*. The value a^{-1} can be treated as the loss tangent in transient case.

As a result of the introducion of the dimensionless variables (10), the small parameter *p* appears in the field equations inside the body.

Expansions in the Small Parameter

Since the parameter *p* is small, we represent the functions, for which the solutions are sought, in the form of the asymptotic expansions in the parameter *p*:

$$\vec{H} = \sum_{m=0}^{\infty} p^m \vec{H}_m \qquad \vec{E} = \sum_{m=0}^{\infty} p^m \vec{E}_m \qquad (14)$$

The following function can be also represented as the expansions in the small parameter *p*:

$$\tilde{d}_k/(\tilde{d}_k - p\eta) = 1 + p\eta/\tilde{d}_k - p^2\eta^2/(2\tilde{d}_k^2) + O(p^3) \quad (15)$$

By substituting the expansions (14)-(15) into (11)-(13) and equating the coefficients of equal powers of *p*, the equations for expansions coefficients are obtained and written in the Laplace domain as follows:

m=0:

$$(\bar{\vec{E}}_0)_{\xi_1} = (\bar{\vec{E}}_0)_{\xi_2} = (\bar{\vec{E}}_0)_\eta = (\bar{\vec{H}}_0)_\eta = 0 \qquad (16)$$

m=1: (17)

$$\partial(\bar{\vec{E}}_1)_{\xi_k}/\partial\eta = (-1)^k s(\bar{\vec{H}}_0)_{\xi_{3-k}} \qquad k=1,2$$

$$(1+sa)(\bar{\vec{E}}_1)_{\xi_k} = (-1)^k \partial(\bar{\vec{H}}_0)_{\xi_{3-k}}/\partial\eta$$

$$\sum_{i=1}^{2}(-1)^i \partial(\bar{\vec{E}}_1)_{\xi_{3-i}}/\partial\xi_i = s(\bar{\vec{H}}_1)_\eta$$

$$\partial(\bar{\vec{H}}_1)_\eta/\partial\eta = -\sum_{i=1}^{2}\partial(\bar{\vec{H}}_0)_{\xi_i}/\partial\xi_i\,; \qquad (\bar{\vec{E}}_1)_\eta = 0$$

m=2: (18)

$$\partial(\bar{\vec{E}}_2)_{\xi_k}/\partial\eta = (\bar{\vec{E}}_1)_{\xi_k}/\tilde{d}_k + (-1)^k s(\bar{\vec{H}}_1)_{\xi_{3-k}}$$

$$(1+sa)(\bar{\vec{E}}_2)_{\xi_k} = (-1)^k\left[\partial(\bar{\vec{H}}_1)_{\xi_{3-k}}/\partial\eta - (\bar{\vec{H}}_0)_{\xi_{3-k}}/\tilde{d}_{3-k}\right]$$

$$\sum_{i=1}^{2}(-1)^i\left[\partial(\bar{\vec{E}}_2)_{\xi_{3-i}}/\partial\xi_i + \eta\partial(\bar{\vec{E}}_1)_{\xi_{3-i}}/(\tilde{d}_i\partial\xi_i)\right] = s(\bar{\vec{H}}_2)_\eta$$

$$(1+sa)(\bar{\vec{E}}_2)_\eta = \sum_{i=1}^{2}(-1)^{3-i}\partial(\bar{\vec{H}}_0)_{\xi_{3-i}}/\partial\xi_i$$

$$\frac{\partial(\bar{\vec{H}}_2)_\eta}{\partial\eta} = (\bar{\vec{H}}_1)_\eta\sum_{i=1}^{2}\tilde{d}_i^{-1} - \sum_{i=1}^{2}\left[\frac{\partial(\bar{\vec{H}}_1)_{\xi_i}}{\partial\xi_i} + \frac{\eta}{\tilde{d}_i}\frac{\partial(\bar{\vec{H}}_0)_{\xi_i}}{\partial\xi_i}\right]$$

m=3: (19)

$$\frac{\partial(\bar{\vec{E}}_3)_{\xi_k}}{\partial\eta} = \frac{(\bar{\vec{E}}_2)_{\xi_k}}{\tilde{d}_k} + \eta\frac{(\bar{\vec{E}}_1)_{\xi_k}}{\tilde{d}_k^2} + \frac{\partial(\bar{\vec{E}}_2)_\eta}{\partial\xi_k} + (-1)^k s(\bar{\vec{H}}_2)_{\xi_{3-k}}$$

$$(1+sa)(\bar{\vec{E}}_3)_{\xi_k} = (-1)^k\Big[\partial(\bar{\vec{H}}_2)_{\xi_{3-k}}/\partial\eta - (\bar{\vec{H}}_1)_{\xi_{3-k}}/\tilde{d}_{3-k} - $$
$$-\eta(\bar{\vec{H}}_0)_{\xi_{3-k}}/\tilde{d}_{3-k}^2 - \partial(\bar{\vec{H}}_1)_\eta/\partial\xi_{3-k}\Big]$$

$$\sum_{i=1}^{2}(-1)^i\left[\frac{\partial(\bar{\vec{E}}_3)_{\xi_{3-i}}}{\partial\xi_i} + \frac{\eta}{\tilde{d}_i}\frac{\partial(\bar{\vec{E}}_2)_{\xi_{3-i}}}{\partial\xi_i} - \frac{\eta^2}{2\tilde{d}_i^2}\frac{\partial(\bar{\vec{E}}_1)_{\xi_{3-i}}}{\partial\xi_i}\right] = s(\bar{\vec{H}}_3)_\eta$$

$$(1+sa)(\bar{\vec{E}}_3)_\eta = \sum_{i=1}^{2}(-1)^{3-i}\left[\frac{\partial(\bar{\vec{H}}_1)_{\xi_{3-i}}}{\partial\xi_i} + \frac{\eta}{\tilde{d}_i}\frac{\partial(\bar{\vec{H}}_0)_{\xi_{3-i}}}{\partial\xi_i}\right]$$

$$\partial(\bar{\vec{H}}_3)_\eta/\partial\eta = (\bar{\vec{H}}_2)_\eta\sum_{i=1}^{2}\tilde{d}_i^{-1} + \eta(\bar{\vec{H}}_1)_\eta\sum_{i=1}^{2}\tilde{d}_i^{-2} -$$
$$-\sum_{i=1}^{2}\left[\frac{\partial(\bar{\vec{H}}_2)_{\xi_i}}{\partial\xi_i} + \frac{\eta}{\tilde{d}_i}\frac{\partial(\bar{\vec{H}}_1)_{\xi_i}}{\partial\xi_i} - \frac{\eta^2}{2\tilde{d}_i^2}\frac{\partial(\bar{\vec{H}}_0)_{\xi_i}}{\partial\xi_i}\right]$$

where *s* is the variable of the Laplace transform

$$\bar{f}(s) = \int_0^\infty f(t)\exp(-st)dt$$

where f is an arbitrary function.

The procedure, described above, can be continued and the equations for the following terms of expansions can be derived. However, in the present paper we restrict ourselve by four terms of the expansions (m=3) and neglect the terms of the order $O(p^4)$. For example, if the ratio of the penetration depth and body thickness is 1/3, then the approximation error due to neglecting the terms *m>3* is about 0.8%.

The representation (14) has clear physical meaning, namely:

a. The zero-order terms of the expansions (14) give the solution of the problem in so-called perfect electrical conductor (PEC) limit, in which the magnetic field diffusion into the body is neglected.
b. The first-order terms describe the diffusion in the well-known Leontovich's approximation, in which the body's surface is considered as a plane and the field is assumed to be penetrating into the body only in the direction normal to the body's surface.
c. The second-order terms yield the correction by taking into account the curvature of the body's surface, but the diffusion is assumed only in the direction normal to the surface as in the Leontovich's approximation. This is Mitzner's approximation.
d. The third-order terms and higher allow for the magnetic field diffusion in the directions tangential to the body's surface. This approximation can be called Rytov's approximation.

The problems (17)-(19) have been solved sequently to derive the first-, second- and third-order terms in explicit form. By using the inverse Laplace transform, the time domain SIBC of high order approximation are written in the following form:

$$E^s_{\xi_k} = (-1)^{3-k}\frac{\partial F_{3-k}}{\partial t} \quad (20a) \qquad H^s_\eta = \sum_{i=1}^{2}\frac{\partial F_i}{\partial\xi_i} \quad (20b)$$

where the superscript "s" denote the values on the body's surface and the functions $F_k = F_k(\xi_1, \xi_1, t)$, *k=1,2*, have been introduced as follows

$$F_{3-k} = pT_1 * H^s_{\xi_{3-k}} + p^2 \frac{\tilde{d}_k - \tilde{d}_{3-k}}{2\tilde{d}_k \tilde{d}_{3-k}} T_2 * H^s_{\xi_{3-k}} +$$
$$+p^3 \frac{3\tilde{d}_k^2 - \tilde{d}_{3-k}^2 - 2\tilde{d}_k \tilde{d}_{3-k}}{8\tilde{d}_k^2 \tilde{d}_{3-k}^2} T_3 * H^s_{\xi_{3-k}} +$$
$$+\frac{p^3}{2} T_3 * \left(-\frac{\partial^2 H^s_{\xi_{3-k}}}{\partial \xi_k^2} + \frac{\partial^2 H^s_{\xi_{3-k}}}{\partial \xi_{3-k}^2} + 2\frac{\partial^2 H^s_{\xi_k}}{\partial \xi_k \partial \xi_{3-k}} \right) + O(p^4)$$

Here T_m are the time-functions defined as follows

$$T_1(t) = a^{-1/2} I_0(t/(2a)) \exp(-t/(2a))$$
$$T_2(t) = 1 - \exp(-t/a)$$
$$T_3(t) = 2a^{-1/2} t I_1(t/(2a)) \exp(-t/(2a))$$

Note that the functions F_k describe the perturbation of the external field surrounding the body due to the field diffusion into the body and dissipation of the energy by the body.

By remembering that the ratio $2/\omega$ is the time scale factor in the time harmonic case and replacing the tranform variable s by $2j$ in (17)-(19), the condition (20a) is written in the frequency domain as follows:

$$\dot{E}^s_{\xi_k} = (-1)^{3-k} p(1+j) \left\{ \left[1 + p\frac{1-j}{4}\left(\tilde{d}^{-1}_{3-k} - \tilde{d}^{-1}_k\right) + \right.\right.$$
$$\left. + \frac{p^2}{2j} \frac{3\tilde{d}_k^2 - \tilde{d}_{3-k}^2 - 2\tilde{d}_k \tilde{d}_{3-k}}{8\tilde{d}_k^2 \tilde{d}_{3-k}^2} \right] \dot{H}^s_{\xi_{3-k}} +$$
$$\left. + \frac{p^3}{2j} \left(-\frac{\partial^2 \dot{H}^s_{\xi_{3-k}}}{\partial \xi_k^2} + \frac{\partial^2 \dot{H}^s_{\xi_{3-k}}}{\partial \xi_{3-k}^2} + 2\frac{\partial^2 \dot{H}^s_{\xi_k}}{\partial \xi_k \partial \xi_{3-k}} \right) \right\} + O(p^4)$$

By neglecting the terms of the order $O(p^3)$, this formula reduces to the Mitzner's condition (2). By setting $d_1 \to \infty$, $d_2 \to \infty$ and $a \to 0$, the Rytov's condition (3) is directly obtained.

NUMERICAL EXAMPLE

To illustrate the theory, we consider the problem of transient scattering form infinitely long straight cylinder of round cross section illuminated by incident Gaussian pulse, TE case. The width of the incident pulse was taken to be radius of the cylinder, i.e. $q=1$. The coordinate ξ_1 is directed along the cylinder so $d_1 = \infty$. Since the radius d of the cylinder cross section is constant, then $d_2 = D$ and $\tilde{d}_2 = 1$ (Fig. 1). Under these conditions, only circumferential ξ_2-component of the equivalent surface current $\vec{J}^s = \vec{n} \times \vec{H}^s$ is non-zero.

Figure 2 shows the time-distributions of the surface electric current obtained in the PEC-limit, in the Leontovich's approximation and in the proposed high order approximation at the point C. From this figure it follows that the proposed time domain SIBC of the high order of approximation allow the accuracy of the calculations to be about 10-20% higher as compared with the solving the problem using standard time domain SIBC (4) of order of the Leontovich's approximation. Therefore, the range of the problems, for which the approach based on the surface impedance concept can be applicable, is extended.

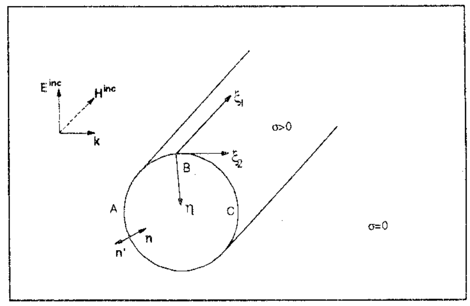

Figure 1: The problem of scattering from the cylinder

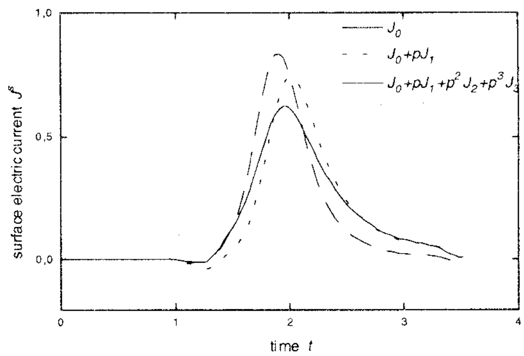

Figure 2: The time-distributions of the surface electric current, calculated in the perfect conductor limit (dotted line), in the Leontovich's approximation (dashed line) and proposed high order approximation (solid line), at the point C (see Fig. 1)

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