MODELING OF VELOCITY TERMS IN 3D EDDY CURRENT PROBLEMS

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Abstract - A 3D eddy current formulation based on constrained eddy currents and direct inclusion of the velocity terms is presented. Velocity terms in all three dimensions can be accommodated although, only two velocity components are normally needed, and often, one velocity term is sufficient. The inclusion of all three terms allows modeling of velocity terms that are not parallel to element sides. The eddy currents, as well as induced currents through velocity are constrained by enforcing the continuity equation. Results for a moving coil over a conductor are presented as an example for the use of this method.

I. INTRODUCTION

Modeling of velocity terms in eddy current problems is an important extension often included in eddy current codes to handle moving parts. The inclusion of velocity leads to nonphysical oscillations in the solution. These oscillation cause loss of accuracy and must be eliminated. The simplest method of eliminating oscillations is to refine the mesh. For nonoscillatory solution, the length of the element in the direction of a velocity term v_a must satisfy the condition $L<2/\mu\sigma v_a$ [1,2]. This condition can be easily satisfied for low velocities or in two dimensional geometries. In three dimensional geometries, especially for high permeability, high conductivity materials, it may not be possible to satisfy this condition with any realistic number of elements. A more efficient approach is to use upwinding of the finite elements, a method that consists of weighting the shape functions proportional to the velocity terms[2,3]. Formulations based on the magnetic vector potential and the electric scalar potential using upwinding [4-6] and nonsymmetric weighting functions [7] have been used for this purpose. The present method uses a constrained eddy current formulation in terms of the magnetic vector potential A and the electric scalar potential V. A is used in nonconducting regions and A and V in conducting regions. The eddy currents are constrained through the use of the continuity equation. Upwinding is used to eliminate oscillations due to velocity. The method presented here includes velocities in all three directions although often, moving bodies can be aligned to move in a preferred direction associated with one of the axes. The inclusion of three terms allows modeling of geometries in which the direction of movement cannot be assumed to be parallel to one of the sides of the finite elements.

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II. FORMULATION

The inclusion of velocity in the eddy current equations is done by adding the velocity induced currents. The modified electric field due to velocity is

$$\mathbf{E} = \mathbf{u}\times\nabla\times\mathbf{A} - j\omega\mathbf{A} - \nabla V \tag{1}$$

where $\mathbf{u}$ is the velocity vector. This electric field is introduced in the general constrained eddy current formulation [8]

The governing equations are:

$$-\frac{1}{\mu}\nabla^2\mathbf{A} = \mathbf{J}_s + \sigma\mathbf{u}\times\nabla\times\mathbf{A} - j\omega\sigma\mathbf{A} - \sigma\nabla V \tag{2a}$$

$$\nabla.\left[\sigma\mathbf{u}\times\nabla\times\mathbf{A} - j\omega\sigma\mathbf{A} - \sigma\nabla V\right] = 0 \tag{2b}$$

where the second equation is the constrained eddy current equation ($\nabla.J_e=0$). The source current is not constrained. The use of the constraint in 2b, together with 2a and the use of Coulomb's gauge guarantee a unique solution[8].

To implement these equations we use $\overline{W}$ and W as weighting functions and $\overline{N}$ and N as shape functions. After introducing the finite element approximation for A and V, and using Galerkin's method we have

$$\begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}\begin{Bmatrix} A \\ V \end{Bmatrix} = \begin{Bmatrix} c_1 \\ 0 \end{Bmatrix} \tag{3}$$

where

$$a_{11}=\int_v\left\{(\nabla\times\overline{W})^T.\frac{1}{\mu}\nabla\times\overline{N}-(\nabla.\overline{W})^T\frac{1}{\mu}\nabla.\overline{N}-\overline{W}^T.\sigma(u\times\nabla\times\overline{N})\right\}dv+$$

$$+\int_v\left\{j\omega\sigma\overline{W}^T.\overline{N}\right\}dv \qquad (4a)$$

$$a_{12}=\int_v\left\{\sigma\overline{W}^T.\nabla N\right\}dv \qquad (4b)$$

$$a_{21}=\int_v\left\{(\nabla W)^T.(\sigma u\times\nabla\times\overline{N})\right\}dv-\int_v\left\{j\omega\sigma(\nabla W)^T.\overline{N}\right\}dv \qquad (4c)$$

$$a_{22}=-\int_v\sigma(\nabla W)^T.\nabla N dv \qquad (4d)$$

$$c_1=\int_v\overline{W}.J dv \qquad (4e)$$

III. UPWIND FINITE ELEMENTS

To ensure solution without oscillations, "upwind" finite elements, similar to those in [2] are used. The finite elements are defined in a local coordinate system (ξ,η,ζ) normalized in the range ±1. The shape functions for eight node hexahedral elements are

$$N_i(\xi,\eta,\zeta)=N_i(\xi)N_i(\eta)N_i(\zeta),\qquad i=1,2,...,8 \qquad (5)$$

where

$$N_i(\xi)=\frac{(1\pm\xi)}{2},\qquad N_i(\eta)=\frac{(1\pm\eta)}{2},\qquad N_i(\zeta)=\frac{(1\pm\zeta)}{2} \qquad (6)$$

The corresponding weighting functions are

$$W_i(\xi,\eta,\zeta)=\left[N_i(\xi)\pm\alpha F(\xi)\right]\left[N_i(\eta)\pm\beta F(\eta)\right]\left[N_i(\zeta)\pm\gamma F(\zeta)\right]\qquad i=1,2,...,8 \qquad (7)$$

where

$$F(\xi)=-3(1+\xi)(1-\xi)/4=-3(1-\xi^2)/4 \qquad (8a)$$

$$F(\eta)=-3(1+\eta)(1-\eta)/4=-3(1-\eta^2)/4 \qquad (8b)$$

$$F(\zeta)=-3(1+\zeta)(1-\zeta)/4=-3(1-\zeta^2)/4 \qquad (8c)$$

and the optimal values of α, β, and γ are related to the finite element mesh, material properties and velocity by:

$$\alpha_{opt.}=\coth\left(\frac{P_x}{2}\right)-\frac{2}{P_x},\qquad P_x=\mu\,\sigma u_x h_x \qquad (9a)$$

$$\beta_{opt.}=\coth\left(\frac{P_y}{2}\right)-\frac{2}{P_y},\qquad P_y=\mu\sigma u_y h_y \qquad (9b)$$

$$\gamma_{opt.}=\coth\left(\frac{P_z}{2}\right)-\frac{2}{P_z},\qquad P_z=\mu\,\sigma u_z h_z \qquad (9c)$$

and h_x, h_y, h_z are the element dimensions in the x, y, and z directions respectively.

These upwind finite elements guarantee non-oscillatory solutions at relatively high velocities, but produce a nonsymmetric system of equations.

IV. RESULTS

As an example to the type of problems solvable by this formulation, consider the geometry in figure 1. It consists of a square coil moving at a velocity v over a conductor. This particular geometry only requires a velocity term in one dimension and is typical of many moving conductor geometries where the direction of movement can be associated with one dimension. However, the currents in the coil are two dimensional and the fields are three dimensional everywhere in space. The results for zero velocity and for a velocity of 10m/sec with a source current at 60Hz are shown in figure 2. Both the real and imaginary parts of A are shown. Figure 3 shows similar results at zero frequency.

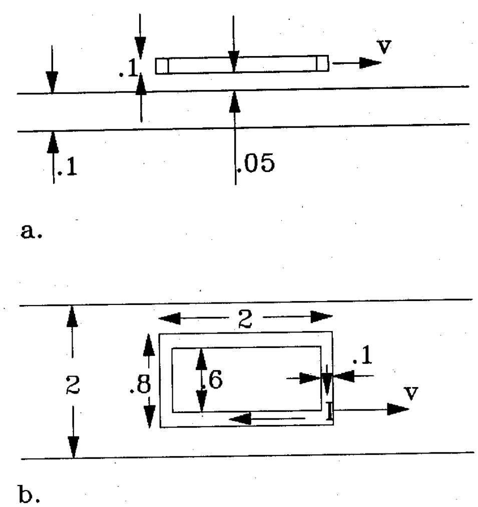
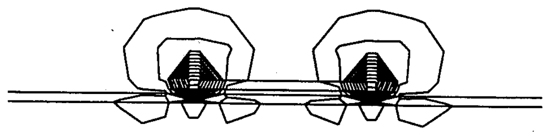
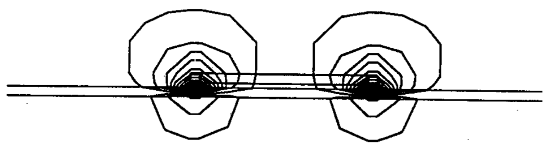


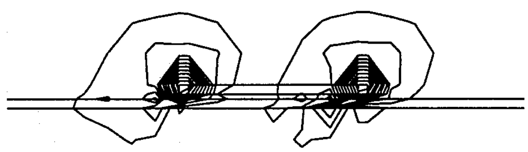
Figure 1.Moving coil over conductor. The moving coil carries a current density of 10^9 Amp/m^2. Conductor properties are μ_r=200, σ=10^6 S/m.
a. side view,
b. top view.



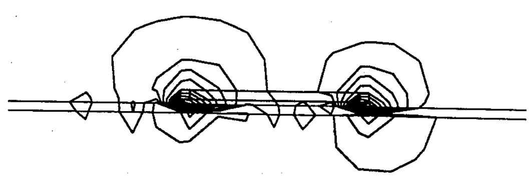
a.



b.

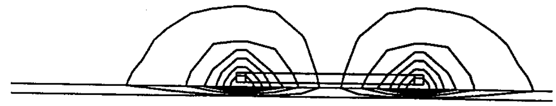


c.



d.

Figure 2 Results for the moving coil with a current at 60Hz.
a. real part of **A** at v=0,
b.Imaginary part of **A** at v=0,
c. real part of **A** at v=10m/sec,
d. imaginary part of **A** at v=10m/sec.



a.

b.

Figure 3. Results for the moving coil with a DC current.
a. at v=0,
b. at v=10m/sec.

V. CONCLUSIONS

The formulation presented here for moving conductors allows computation of fields at relatively high velocities. The formulation is a simple extension of an eddy current formulation. Uniqueness of solution for eddy currents is guaranteed through the use of a constraint equation. Although not shown here, the method allows movement of conductors in arbitrary directions in space.

VI. REFERENCES

[1] N. Ida, "Modeling of velocity effects in eddy current applications," Journal of Applied Physics, Vol. 63, No. 8, April 15, 1988, pp.3007-3009.

[2] J.C. Heinrich, P.S. Huyakorn, O.C. Zienkiewicz and A.R. Mitchell, " An "Upwind" Finite Element Scheme for Two Dimensional Convective Transport Equations", International Journal for Numerical Methods in Engineering, Vol. 10, 1977, pp. 131-143.

[3] T. J. R. Hughes, "A Simple Scheme for Developing 'Upwind' Finite Elements", International Journal for Numerical Methods in Engineering, Vol. 12, 1978, p. 1359-1365.

[4] D. Rodger, T. Karaguler and P. J. Leonard, "A Formulation for 3D Moving Conductor Eddy Current Problems", IEEE Transactions on Magnetics, Vol. 25, No. 5, 1989, pp. 4147-4149.

[5] T. Furukawa, K. Komiya, and I. Muta, "An Upwind Galerkin Finite Element Analysis of Linear Induction Motors", IEEE Transactions on Magnetics, Vol. 26, No. 2, 1990, pp. 662-665.

[6] D. Rodger, P. J. Leonard and T. Karaguler, "An Optimal Formulation for 3D Moving Conductor Eddy Current Problems with Smooth Rotors", IEEE Transactions on Magnetics, Vol. 26, No. 5, 1990, pp. 2359-2363.

[7] Y. Marechal and G. Meunier, "Computation of 2D and 3D Eddy Currents in Moving Conductors of Electromagnetic Retarders", IEEE Transactions on Magnetics, Vol. 26, No. 5, 1990, pp. 2382-2384.

[8] H. Song and N. Ida, "An Eddy Current Constraint Formulation for 3D Electromagnetic Field Calculations", presented at the Fourth Biennial IEEE Conference on Electromagnetic Field Computation, Toronto, Oct. 22-24, 1990, accepted for publication.