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3D FINITE ELEMENT CALCULATION OF HARMONIC ELECTROMAGNETIC FIELDS

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Abstract - A procedure is implemented using edge-type elements for the solution of frequency-domain electromagnetic field problems. It consists of a coupled simple-layer integral equation procedure for an interior-exterior problem where a homogeneous object is under external excitation. In addition, a hybrid finite element procedure using $\vec{\mathbf{E}}$ or $\vec{\mathbf{H}}$ as the state variable for efficiently handling interior inhomogeneities is given.

INTRODUCTION

The problem under consideration is to solve an interface type electromagnteic field problem where a three-dimensional structure is under a given external field excitation. Numerical modeling of such a situation usually involves a large solution domain or inverting a very large matrix, as a direct consequence of the following two features. One is the unboundedness of the problem, because it is assumed that the incident wave field is emitted from and the measurement is performed in an inifinite background medium. In finite difference or finite element modeling, an artificial boundary condition (more or less, frequency dependent) is located far away from the scatterer in order to simulate the nonreflecting nature of the scattered wave field. Another feature is the vector nature of the electromagnetic waves. Thus, the field variable at each node in the solution domain has three components. To get rid of the interface condition, finite element modeling usually reformulates the problem by introducing scalar and vector potentials, which means more unknowns per node and even larger matrices. The purpose here is to review and propose a class of edge-based methods with minimal solution domains, fewer indepenedent unknowns and higher accuracy. These methods use a new class of elements having unknows on the edges (tangential component of field along the edge) instead of the nodes. By using vector basis functions, each edge has only one degree of freedom (first order model) [1].

This paper is organized as follows. First, the physical model and governing equations are introduced. Then, the solution of surface integral equations based on vector boundary elements (BEM) is considerd. Coupled finite element method - boundary element method (FEM-BEM) using scalar and vector finite elements are given next. Finally, some representative results and conclusions are provided.

PHYSICAL MODEL AND BASIC EQUATIONS

Consider the problem of obtaining a numerical solution to the time - harmonic Maxwell's equations in a source - free domain. This physical configuration is shown in Fig. 1. Let the space $\Re^3$ be divided into two parts by a closed surface S (see figure 1). The exterior domain Ω_1 is characterized by $(\mu_1, \epsilon_1, \sigma_1 = 0)$, and the interior domain Ω_b by $(\mu_2, \epsilon_2(\vec{\mathbf{r}}), \sigma_2(\vec{\mathbf{r}}))$, where the $\vec{\mathbf{r}}$ dependence of conductivity and permittivity reflects inhomogeneities in the structure. Let the source $(\vec{\mathbf{E}}^{inc}, \vec{\mathbf{H}}^{inc})$ be emitted from domain Ω_1 and excite the object Ω_2 externaly. The purpose here is to compute the out-going wave fields $(\vec{\mathbf{E}}_1^s, \vec{\mathbf{H}}_1^s)$ and the penetrating wave fields $(\vec{\mathbf{E}}_2^s, \vec{\mathbf{H}}_2^s)$. Then the total wave fields are gorverned by the following basic equations

$$\begin{cases} \nabla \times \vec{\mathbf{E}}_1 = -j\omega\mu_1\vec{\mathbf{H}}_1, & \nabla \times \vec{\mathbf{H}}_1 = j\omega\epsilon_1'\vec{\mathbf{E}}_1 \quad \text{in } \Omega_1 \\ \nabla \times \vec{\mathbf{E}}_2 = -j\omega\mu_2\vec{\mathbf{H}}_2, & \nabla \times \vec{\mathbf{H}}_2 = j\omega\epsilon_2'(\vec{\mathbf{r}})\vec{\mathbf{E}}_2 \quad \text{in } \Omega_2 \\ (\vec{n} \times \vec{\mathbf{E}})^+ = (\vec{n} \times \vec{\mathbf{E}})^-, & (\vec{n} \times \vec{\mathbf{H}})^+ = (\vec{n} \times \vec{\mathbf{H}})^- \quad \text{on } S \end{cases} \quad (1)$$

where

$$\epsilon_1' = \epsilon_1 + \frac{\sigma_1}{j\omega}, \quad \epsilon_2' = \epsilon_2 + \frac{\sigma_2}{j\omega}. \quad (2)$$

It is possible to eliminate one field variable and obtain a *curlcurl* equation in terms of the other field variable, for example, in Ω_2,

$$\nabla \times \frac{1}{j\omega\mu_2}\nabla \times \vec{\mathbf{E}} + j\omega\epsilon_2'(\vec{\mathbf{r}})\vec{\mathbf{E}} = \vec{\mathbf{0}}, \quad (3)$$

$$\nabla \times \frac{1}{j\omega\epsilon_2'(\vec{\mathbf{r}})}\nabla \times \vec{\mathbf{H}} + j\omega\mu_2\vec{\mathbf{H}} = \vec{\mathbf{0}}. \quad (4)$$

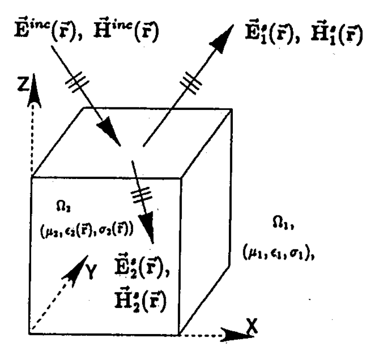

Fig. 1 Physical Configuration.

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A COUPLED INTEGRAL PROCEDURE

In this section, an edge-based boundary element model is outlined for electromagnetic field interation with homogeneous material. By introducing vector simple-layer potentials and scalar simple-layer potentials, the total electric and magnetic fields in Ω_1 and Ω_2 can be expressed as [2]

$$\vec{\mathbf{E}}_1(\vec{\mathbf{r}}) = \vec{\mathbf{E}}^{inc} - j\omega\vec{\mathbf{A}}_1(\vec{\mathbf{r}}) - \nabla\mathbf{V}_1(\vec{\mathbf{r}}) \tag{5}$$

$$\vec{\mathbf{H}}_1^s(\vec{\mathbf{r}}) = \vec{\mathbf{H}}^{inc}(\vec{\mathbf{r}}) + \frac{1}{\mu_1}\nabla\times\vec{\mathbf{A}}_1(\vec{\mathbf{r}}) \tag{6}$$

for $\vec{\mathbf{r}}$ on or outside S and

$$\vec{\mathbf{E}}_2(\vec{\mathbf{r}}) = -j\omega\vec{\mathbf{A}}_2(\vec{\mathbf{r}}) - \nabla\mathbf{V}_2(\vec{\mathbf{r}}) \tag{7}$$

$$\vec{\mathbf{H}}_2(\vec{\mathbf{r}}) = \frac{1}{\mu_2}\nabla\times\vec{\mathbf{A}}_2(\vec{\mathbf{r}}) \tag{8}$$

for $\vec{\mathbf{r}}$ on or inside S. The various potential functions are

$$\vec{\mathbf{A}}_1(\vec{\mathbf{r}}) = \frac{\mu_1}{4\pi}\iint_S \vec{\mathbf{J}}(\vec{\mathbf{r}}')\mathbf{G}_1(\vec{\mathbf{r}},\vec{\mathbf{r}}')dS(\vec{\mathbf{r}}') \tag{9}$$

$$\vec{\mathbf{A}}_2(\vec{\mathbf{r}}) = \frac{\mu_2}{4\pi}\iint_S \vec{\mathbf{M}}(\vec{\mathbf{r}}')\mathbf{G}_2(\vec{\mathbf{r}},\vec{\mathbf{r}}')dS(\vec{\mathbf{r}}') \tag{10}$$

$$\mathbf{V}_1(\vec{\mathbf{r}}) = \frac{1}{4\pi\epsilon_1'}\iint_S \rho^e(\vec{\mathbf{r}}')\mathbf{G}_1(\vec{\mathbf{r}},\vec{\mathbf{r}}')dS(\vec{\mathbf{r}}') \tag{11}$$

$$\mathbf{V}_2(\vec{\mathbf{r}}) = \frac{1}{4\pi\epsilon_2'}\iint_S \rho^m(\vec{\mathbf{r}}')\mathbf{G}_2(\vec{\mathbf{r}},\vec{\mathbf{r}}')dS(\vec{\mathbf{r}}') \tag{12}$$

where the Green's functions are defined by

$$\mathbf{G}_i(\vec{\mathbf{r}},\vec{\mathbf{r}}') = \frac{e^{-jk_iR}}{R}, \quad R = |\vec{\mathbf{r}} - \vec{\mathbf{r}}'|. \tag{13}$$

Except for a multiplying constant, these functions possess the standard properties of simple-layer potential [3]. In the constructions (6-9), the megnetic field $\vec{\mathbf{H}}$ is automatically divergence free. By demanding the divergence free condition for the electric field $\vec{\mathbf{E}}$, one can see that the scalar layer densities are related to the vector layer densities by the continuity relations

$$\rho^e(\vec{\mathbf{r}}') = \frac{-1}{j\omega}\nabla_S'\bullet\vec{\mathbf{J}}(\vec{\mathbf{r}}') \quad \rho^m(\vec{\mathbf{r}}') = \frac{-1}{j\omega}\nabla_S'\bullet\vec{\mathbf{M}}(\vec{\mathbf{r}}') \tag{14}$$

The coupled integral equation formualtion is obtained by requiring the continuity of tangential components of the electric and magnetic fields on the interface S:

$$\vec{\mathbf{E}}^{inc}(\vec{\mathbf{r}})\,|_{\tan} = \left\{j\omega\vec{\mathbf{A}}_1(\vec{\mathbf{r}}) + \nabla\mathbf{V}_1(\vec{\mathbf{r}}) - j\omega\vec{\mathbf{A}}_2 - \mathbf{V}_2(\vec{\mathbf{r}})\right\}|_{\tan} \quad \vec{\mathbf{r}}\in S \tag{15}$$

$$-\vec{\mathbf{H}}^{inc}(\vec{\mathbf{r}})\,|_{\tan} = \left\{\frac{1}{\mu_1}\nabla\times\vec{\mathbf{A}}_1(\vec{\mathbf{r}}) - \frac{1}{\mu_2}\nabla\times\vec{\mathbf{A}}_2(\vec{\mathbf{r}})\right\}|_{\tan} \quad \vec{\mathbf{r}}\in S \tag{16}$$

A solution to the coupled integral equations is achieved by dividing the surface S into triangular elements. A vector basis function is defined as [4]

$$\vec{\mathbf{f}}_n(\vec{\mathbf{r}}') = \begin{cases} \frac{l_n}{2A_n^+}\vec{\rho}_n^{\,+}, & \vec{\mathbf{r}} \text{ in } T_n^+ \\ \frac{l_n}{2A_n^-}\vec{\rho}_n^{\,-}, & \vec{\mathbf{r}} \text{ in } T_n^- \\ \vec{\mathbf{0}}, & \text{otherwise} \end{cases} \tag{17}$$

The surface divergence of the basis function is given by

$$\nabla_S\bullet\vec{\mathbf{f}}_n(\vec{\mathbf{r}}') = \begin{cases} \frac{l_n}{A_n^+}, & \vec{\mathbf{r}} \text{ in } T_n^+ \\ -\frac{l_n}{A_n^-}, & \vec{\mathbf{r}} \text{ in } T_n^- \\ 0, & \text{otherwise} \end{cases} \tag{18}$$

where T_n^+ and T_n^- are two adjacent triangles with the nth edge common, l_n is the length of the nth common edge, $A_n^{\pm}$ the area of triangle $T_n^{\pm}$, and $\vec{\rho}_n^{\,\pm}$ are the approriate position vectors in $T_n^{\pm}$. Let there be N common edges in the surface triangulation. Now the surface outer layer-density and inner layer-density are expanded in terms of the basis function,

$$\vec{\mathbf{J}}(\vec{\mathbf{r}}') = \sum_{n=1}^{N} I_n\vec{\mathbf{f}}_n(\vec{\mathbf{r}}'), \quad \vec{\mathbf{M}}(\vec{\mathbf{r}}') = \sum_{n=1}^{N} M_n\vec{\mathbf{f}}_n(\vec{\mathbf{r}}') \tag{19}$$

where the coefficients I_n and M_n stand for the perpendicular components of the outer layer-density and inner layer-density flowing past the nth common edge. To determine these coefficients, the coupled integral equations are tested against the basis functions in the sense of the following symmetrical product

$$<\vec{\mathbf{f}},\vec{\mathbf{g}}> = \iint_S \vec{\mathbf{f}}\bullet\vec{\mathbf{g}}dS \tag{20}$$

to get

$$<j\omega\vec{\mathbf{A}}_1,\ \vec{\mathbf{f}}_m> + <\nabla\mathbf{V}_1,\ \vec{\mathbf{f}}_m> - <j\omega\vec{\mathbf{A}}_2,\ \vec{\mathbf{f}}_m> - <\nabla\mathbf{V}_2,\ \vec{\mathbf{f}}_m> = <\vec{\mathbf{E}}^{inc},\ \vec{\mathbf{f}}_m>, \quad \vec{\mathbf{r}}\in S \tag{21}$$

$$\frac{1}{2}<\vec{\mathbf{J}},\ \vec{\mathbf{f}}_m> + <\vec{\mathbf{K}}_1(\vec{\mathbf{J}}),\ \vec{\mathbf{f}}_m> + \frac{1}{2}<\vec{\mathbf{M}},\ \vec{\mathbf{f}}_m> - <\vec{\mathbf{K}}_2(\vec{\mathbf{M}}),\ \vec{\mathbf{f}}_m> = -<\vec{\mathbf{H}}^{inc},\ \vec{\mathbf{f}}_m>, \quad \vec{\mathbf{r}}\in S \tag{22}$$

where the principal value integral operators are given by

$$\vec{\mathbf{K}}_1(\vec{\mathbf{J}}) = \frac{1}{4\pi}\iint_S \vec{\mathbf{J}}(\vec{\mathbf{r}}')\times\nabla'\mathbf{G}_1(\vec{\mathbf{r}},\vec{\mathbf{r}}')dS(\vec{\mathbf{r}}'), \tag{23}$$

$$\vec{\mathbf{K}}_2(\vec{\mathbf{M}}) = \frac{1}{4\pi}\iint_S \vec{\mathbf{M}}(\vec{\mathbf{r}}')\times\nabla'\mathbf{G}_2(\vec{\mathbf{r}},\vec{\mathbf{r}}')dS(\vec{\mathbf{r}}'). \tag{24}$$

The two symmetrical product terms can be integrated exactly, the terms invloving principal value integrals are integrated numerically using the Galerkin procedure (double surface integrals) to take into account the rapid change of $1/R^3$, while the remaining terms may be simplified to simple surface integrals by testing at the centroid of the triangle. Careful calculation yields a partitioned matrix equation

$$\begin{bmatrix} \{Z_{mn}^{JJ}\} & \{Z_{mn}^{JM}\} \\ \{Z_{mn}^{MJ}\} & \{Z_{mn}^{MM}\} \end{bmatrix} \begin{bmatrix} \{I_n\} \\ \{M_n\} \end{bmatrix} = \begin{bmatrix} \{E_n^{inc}\} \\ \{H_n^{inc}\} \end{bmatrix}. \tag{25}$$

Detailed expressions for submatrices in (25) are omited for brevity.

A HYBRID FEM PROCEDURE

Now consider the situation in which Ω_2 is a nonhomogeneous medium. Since the parameter ϵ_2' depends on position vector $\vec{\mathbf{r}}$, no Green function can be found for this case. Surface type integral equations discussed in the previous section can not be applied, and instead, a volume type intgral equation or a finite element procedure has to be considered. A volume type intgral formulation uses the concept of volume polarization current. A detailed account of the method is not included here. We focus on implementation of coupled FEM-BEM approaches. The traditional node-based method has a disadvantage, in that the inter-element conditions and the surface conditions are rather tedious [5]. The method proposed next uses edge-based elements with vector basis functions. The resulting matrix equation has fewer unknowns, and is more sparse. For brevity, only the solution procedure using $\vec{\mathbf{E}}$ as state variable is discussed. The dual procedure, using $\vec{\mathbf{H}}$ as state variable, can be obtained analogously.

The weighted integral form of (3) is [6]

$$\int_{\Omega_2} \nabla \times \frac{1}{j\omega\mu_2} \nabla \times \vec{\mathbf{E}} \cdot \vec{\mathbf{w}}_m d\Omega + \int_{\Omega_2} j\omega\epsilon_2'(\vec{\mathbf{r}})\vec{\mathbf{E}} \cdot \vec{\mathbf{w}}_m d\Omega = 0, \quad (26)$$

where $\vec{\mathbf{w}}_m$ are any set of real vector weighting functions. Using the integral relation

$$\int_{\Omega} \left(\nabla \times \vec{\mathbf{a}} \cdot \nabla \times \vec{\mathbf{b}} - \vec{\mathbf{a}} \cdot \nabla \times \nabla \times \vec{\mathbf{b}}\right) d\Omega = \oint_S \left[\vec{\mathbf{a}} \times (\nabla \times \vec{\mathbf{b}})\right] \cdot \hat{\mathbf{n}} dS$$

yields the weak form

$$\int_{\Omega_2} \left(\frac{1}{j\omega\mu_2} \nabla \times \vec{\mathbf{E}}\right) \cdot (\nabla \times \vec{\mathbf{w}}_m) d\Omega + \int_{\Omega_2} j\omega\epsilon_2'(\vec{\mathbf{r}})\vec{\mathbf{E}} \cdot \vec{\mathbf{w}}_m d\Omega = \oint_S \left(\hat{\mathbf{n}} \times \vec{\mathbf{H}}\right) \cdot \vec{\mathbf{w}}_m dS. \quad (27)$$

Next consider the Galerkin procedure for discretizing the weak form. $\vec{\mathbf{E}}$ and $\vec{n} \times \vec{\mathbf{H}}$ are expanded by

$$\vec{\mathbf{E}} = \sum_{n=1}^{M_e} E_n \vec{\mathbf{w}}_n(\vec{\mathbf{r}}), \quad \hat{\mathbf{n}} \times \vec{\mathbf{H}} = \sum_{n=1}^{M_e} F_n \vec{\mathbf{w}}_n(\vec{\mathbf{r}}) \quad (28)$$

where M_e is the total number of edges in the finite element mesh, and $\vec{\mathbf{w}}_n$ is the vector basis function associated with the nth edge. These vector basis functions are constructed such that the expansion coefficients in (41) are tangential components of corresponding field variables along that edge. Because tangentially continuous finite elements are used, the inter-element conditions are automatically guaranteed. The discretized version of (27) is

$$[A]\{\mathcal{E}\} = [B]\{\mathcal{F}\}, \quad (29)$$

where the coefficients are

$$a_{mn} = \int_{\Omega_2} \left[\frac{1}{j\omega\mu_2} (\nabla \times \vec{\mathbf{w}}_n) \cdot (\nabla \times \vec{\mathbf{w}}_m) + j\omega\epsilon_2'(\vec{\mathbf{r}})\vec{\mathbf{w}}_n \cdot \vec{\mathbf{w}}_m\right] d\Omega, \quad (30)$$

$$b_{mn} = \oint_S \vec{\mathbf{w}}_n \cdot \vec{\mathbf{w}}_m dS, \quad m,\ n = 1, 2, \cdots, M_e. \quad (31)$$

Equation (29) cannot be solved because one does not know $\hat{\mathbf{n}} \times \vec{\mathbf{H}}$ on S *a priori*. This information is obtained by considering the integral formulation (6) and (7). Similar to the finite element formulation, the surface S is modeled by triangular elements (consistent with a tetrahedral FEM model) or quadrilateral elements (consistent with a hexahedral FEM model). Let the position vector be on the surface S. $\vec{\mathbf{E}}$ and $\vec{\mathbf{J}}$ are expanded by a set of vector basis functions $\vec{\mathbf{f}}_n$:

$$\vec{\mathbf{E}} = \sum_{n=1}^{N_e} E_n \vec{\mathbf{f}}_n(\vec{\mathbf{r}}), \quad \vec{\mathbf{J}} = \sum_{n=1}^{N_e} I_n \vec{\mathbf{f}}_n(\vec{\mathbf{r}}) \quad (32)$$

where N_e is the total number of edges in the boundary element mesh. In a conformal case, $\vec{\mathbf{f}}_n$ should be $\vec{\mathbf{w}}_n$ acting only on S. However, in a nonconformal case, $\vec{\mathbf{f}}_n$ can be any set of vector functions as long as the coefficients in the expansion have the same physical meaning as that in (28). Testing (6) in the sense of symmetric product (20)

$$< \vec{\mathbf{E}}(\vec{\mathbf{r}}),\ \vec{\mathbf{f}}_m > = < \vec{\mathbf{E}}^{inc}(\vec{\mathbf{r}}),\ \vec{\mathbf{f}}_m > - < j\omega\vec{\mathbf{A}}_1(\vec{\mathbf{r}}),\ \vec{\mathbf{f}}_m > - < \nabla \mathbf{V}_1(\vec{\mathbf{r}}),\ \vec{\mathbf{f}}_m > \quad \vec{\mathbf{r}} \in S \quad (33)$$

This yields a mtrix equation relating the vector layer $\vec{\mathbf{J}}$ and electric field $\vec{\mathbf{E}}$

$$[C]\{\mathcal{I}\} = -\{\mathcal{E}^{inc}\} + [B]\{\mathcal{E}\}, \quad (34)$$

where b_{mn} is the same as that in (31) if $\vec{\mathbf{f}}_n$ is $\vec{\mathbf{w}}_n$ acting only on S, and

$$c_{mn} = \iint_S \left[\frac{1}{4\pi} \iint_S (-jk_1\eta_1)\vec{\mathbf{f}}_n \mathbf{G} dS\right] \bullet \vec{\mathbf{f}}_m dS + \iint_S \left[\frac{1}{4\pi} \iint_S \left(-\frac{\eta_1}{jk_1}\right) \nabla_S \bullet \vec{\mathbf{f}}_n \mathbf{G} dS\right] \nabla_S \bullet \vec{\mathbf{f}}_m dS, \quad (35)$$

$$e_m^{inc} = \iint_S \vec{\mathbf{E}}^{inc} \bullet \vec{\mathbf{f}}_m dS, \quad m, n = 1, 2, \cdots, N_e. \quad (36)$$

Next, testing $\hat{\mathbf{n}} \times \vec{\mathbf{H}}$ based on (7)

$$< \hat{\mathbf{n}} \times \vec{\mathbf{H}},\ \vec{\mathbf{f}}_m > = < \hat{\mathbf{n}} \times \vec{\mathbf{H}}^{inc},\ \vec{\mathbf{f}}_m > + \frac{1}{2} < \vec{\mathbf{J}},\ \vec{\mathbf{f}}_m > + < \hat{\mathbf{n}} \times \vec{\mathbf{K}}_1(\vec{\mathbf{J}}),\ \vec{\mathbf{f}}_m >, \quad \vec{\mathbf{r}} \in S \quad (37)$$

yields another matrix equation relating $\hat{\mathbf{n}} \times \vec{\mathbf{H}}$ to the vector layer $\vec{\mathbf{J}}$:

$$[B]\{\mathcal{F}\} = \{\mathcal{F}^{inc}\} + [D]\{\mathcal{I}\}, \quad (38)$$

where the coefficients are

$$d_{mn} = \frac{1}{2} \iint_S \vec{\mathbf{f}}_n \bullet \vec{\mathbf{f}}_m dS + \iint_S \left[\frac{1}{4\pi} \iint_S (\hat{\mathbf{n}} \times \vec{\mathbf{f}}_n) \times \nabla' \mathbf{G} dS\right] \bullet \vec{\mathbf{f}}_m dS \quad (39)$$

$$f_m^{inc} = \iint_S \hat{\mathbf{n}} \times \vec{\mathbf{H}}^{inc} \bullet \vec{\mathbf{f}}_m dS, \quad m, n = 1, 2, \cdots, N_e. \quad (40)$$

Since both $\vec{\mathbf{E}}$ and $\hat{\mathbf{n}} \times \vec{\mathbf{H}}$ are tangentially continous, the surface conditions are automatically guaranteed. The three matrix eqautions, (29), (34) and (38), can be linked together to get

$$\left([A] - [D][C]^{-1}[B]\right)\{\mathcal{E}\} = \{\mathcal{F}^{inc}\} - [D][C]^{-1}\{\mathcal{E}^{inc}\}. \quad (41)$$

The solution to (41) yields the tangential components of $\vec{\mathbf{E}}$ along all edges in the finite element mesh. The interior field can be interpolated via the vector basis fucntions. To calculate exterior field, the tangential components of vector layer $\vec{\mathbf{J}}$ along all edges in the boundary element mesh are obtained through (34) or (38), and a numerical integration is performed based on the construction of exterior fields.

REPRESENTATIVE RESULTS

The computer implementation of these methods starts with the numerical implementation of edge-based integral procedures. As an example, consider a conducting sphere under plane wave illumination. The sphere has a radius $a = 1m$. The incident plane wave has its magnetic field polarized along the $+z$ direction, and propagates in the $+x$ direction. The surface of the sphere is spanned by 60 triangular elements. This model requires filling and inverting a 180×180 matrix. Since the incident frequency is not very high, the numerical results can be verified with respect to the exact solution for a conducting sphere in a uniform alternating field. Fig. 2 shows a comparison of the BEM solution with the exact solution of the B_z component along the z axis. The incident frequency is $10kHz$ and the conductivity is $10^3\mho/m$. Fig.3

shows another situtation where the incident frequency is $10kHz$ and the conductivity is $10^4 \mho/m$. Both pictures show good agreement for interior and exterior fields, while some error exists near the interface. This may be due to the following reasons. First, relatively fewer boundary elements were used. Second, the curl of a vector simple-layer potential has a limiting term $\mp 1/2\hat{n} \times \vec{J}$ when approaching the inteface from either side, and the kernel $1/R^3$ is quite sensitive when R is small. Numerical difficulty is expected near the surface.

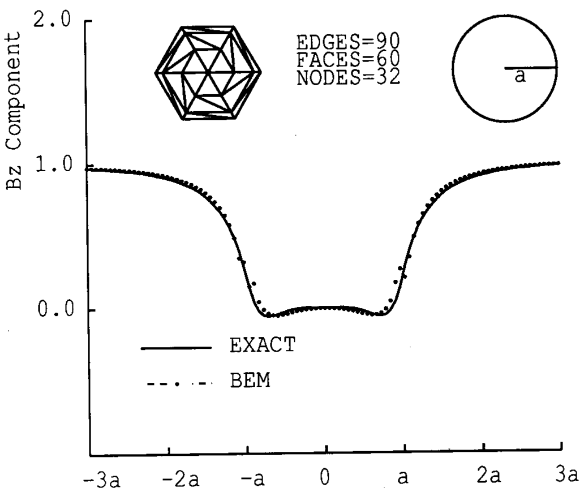

Fig. 2 A Conducting Sphere in Alternating Field for $f = 10kHz$, $\sigma = 10^3 \mho/m$.

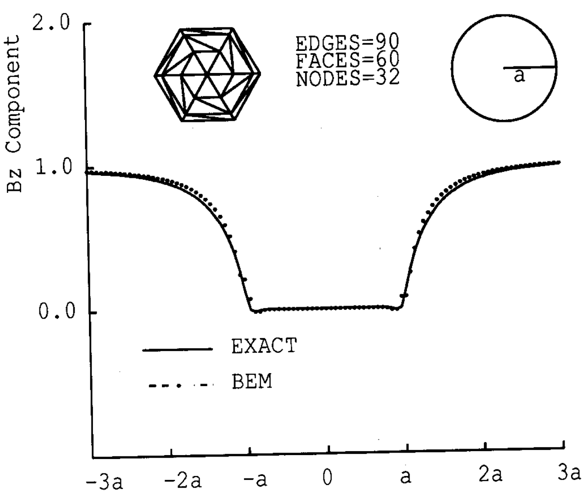

Fig. 3 A Conducting Sphere in Alternating Field for $f = 10kHz$, $\sigma = 10^4 \mho/m$.

Substantial work is under way to build a computer code using a coupled finite element - boundary element method in conjunction with edge type elements, proposed in the previous section. Two kinds of vector elements are reported in the literature. One is a tetrahedral element with six degrees of freedom [7], another is a hexahedral element with twelve degrees of freedom [8]. The latter is preferable due to the fact that hexahedral elements are easy to generate and visualize. However, with tetrahedral elements, the self-term integrals appeared in the boundary integral formulation can be easily worked out, while certain difficulty exists with hexahedral elements. These and other aspects of implementation are currently being examined.

CONCLUSIONS

A coupled simple-layer integral procedure is efficiently implemented for eddy current calculation using triangular surface edge elements. The solution is further facilitated by elminating the scalar layer-densities in the interior and exterior field representations through the continuity requirement without increasing the differentiation requirement of the vector basis function. Good agreement with exact solution is obtained. A rigorous hybrid finite element procedure is proposed using edge finite and boundary elements. Numerical implementation of the procedure is under development.

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