DELAY ESTIMATION USING MAXIMUM ENTROPY DERIVED PHASE INFORMATION

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ABSTRACT. The frequencies chosen for ultrasound reflectometry, based on attenuation and resolution constraints, often result in overlapping echo waveforms [1-3]. Phase information [4] can be used to estimate echo contributions in reflectometry using the Maximum Entropy Method [5]. The method of analysis is presented, along with experimental verification for a simple reflecting structure.

1. Introduction

The waveforms used in ultrasound reflectometry are typically several cycles, rapidly damped, at the resonant frequency of the transducer [6]. When low frequencies are chosen to obtain the low attenuation through the medium under test, then the time waveforms of reflections from adjacent obstacles will often overlap. This overlap of echos makes it difficult to interpret where the sources of the echos are located. The phase information, found through the use of the Maximum Entropy Method (MEM), can be used to identify such echos (which we demonstrate for a monolayer, though the method is not restricted to a monolayer).

The information content of the phase component of systems has been recognized [4,7] in different applications. In this work, the phase is obtained from the power spectral estimates of the even and odd constituents of the MEM power spectrum estimate (the terms even and odd originating from the respective time-domain sequences). We shall illustrate this by processing experimental signals obtained from a thin sheet obstacle.

The Maximum Entropy Method [8] is used to estimate the power spectra. The MEM spectral estimate provides a non-zero extension for missing data by making use of only the available data.

We must emphasize here that our aim is not to estimate the

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power spectrum of the output signal nor is it to model the system as an all pole, minimum phase filter (although we, in fact, do both). We are simply utilizing the robustness and accuracy of the MEM power spectral estimation as a tool in our computations. The component of our calculation which makes use of the MEM spectrum estimation (which can be represented as a filter) allows us to consider superposition of signals or echos. Though the MEM uses a non-linear criterion of goodness, the predictive filter which is computed is a linear filter.

To demonstrate our technique we use a thin sheet of aluminum immersed in water; the resulting echoes overlap each other, resulting in a long, ill-defined reflected signal. The phase information obtained from the MEM spectra is further processed to yield an estimation of the time delay between a signal passing straight through the sheet and one reflecting back and forth within the sheet once before exiting (double the time delay to pass through the aluminum sheet).

Two examples are presented. The first example is a direct transmission of a pulse from one 3.5 MHz transducer to a similar receiving transducer (the case of removal of the obstacle from Figure 1a). No delayed pulse is observed, as spurious reflections have been reduced by echo absorbing material. The second example is a reflection from two nearby surfaces which cause overlapping echoes (created by the delay in passing through twice the reflecting sheet thickness). The signal path is shown in Figure 1b. The experimental apparatus corresponds to Figure 1a with the obstacle present.

All systems and processes are assumed real, unless otherwise specified.

2. Method: Obtaining Phase through the Use of Maximum Entropy Filters

The principal of maximum entropy, which involves autoregressive modeling, has been applied successfully to power spectral estimation [9-12]. The significance of the method compared to some traditional techniques [13] is higher frequency resolution, dependence only on the available data, and simple storage requirements owing to the infinite impulse response (IIR) structure of the resulting all pole filter. The all pole filter yields a smoothly changing phase estimate, which does not appear to have the experimental difficulties reported with other methods [14,15]. The model is linear, but overall, the processing is not linear.

The data $x(n)$ are used to compute a predictor polynomial, $H(z)$, which predicts the next data point, $x(n+1)$. The order of the predictive filter chosen will set the number of poles of the predictive filter, Y(z). Y(z) will be a minimum phase filter, where

$$Y(z)=\frac{\sigma}{H(z)} \tag{1}$$

The unit sample response y(n) of Y(z) is not directly related to x(n), however their respective autocorrelation values R.(n) are related by

$$R_{xx}(n)=R_{yy}(n) \qquad n=0,1,\ldots,N \tag{2}$$

where N is the order of the filter. Hence $R_{yy}(n)$, for n>N, provides a non-zero extension to $R_{xx}(n)$. Since the power spectrum $S.(e^{(j\omega)})$ is the Fourier transform of its autocorrelation sequence; then

$$|Y(e^{j\omega})|^2=S_{yy}(e^{j\omega})=\left|\frac{\sigma^2}{H(z)H(1/z)}\right|_{z=e^{j\omega}} \tag{3}$$

is an estimate of the power spectrum of x(n). It is clear that the estimation is independent of the phase of $X(e^{(j\omega)})$. The error noise power of the predictive filter is σ^2.

If the phase of the signal, composed of outgoing and reflected waves, were only known, then the phase would yield an estimate of the delay of the reflected wave compared to the outgoing wave. In a simplistic model, a signal x(t) consisting of an original unit amplitude cosine and a reflected fraction, a_o, delayed by time t_o would be

$$x(t)=\cos\omega t+a_o\cos\omega(t-t_o)$$

This can be written as

$$x(t)=\sqrt{(1+a_o^2+2a_o\cos\omega t_o)}\cos(\omega t-\theta)$$

where

$$\theta=Arctan\left(\frac{a_o\sin\omega t_o}{1+a_o\cos\omega t_o}\right)$$

For small reflection coefficients, a_o, and for $a_o\cos\omega t_o \ll 1$

$$\theta \doteq Arctan(a_o\sin\omega t_o)$$

$$\theta \doteq a_o\sin\omega t_o$$

The periodicity of $\sin\omega t_o$ with respect to ω yields t_o which equals $2d/v$, for a back-and-forth pair of reflections in a slab of thickness d and sound velocity v.

Unfortunately, we do not have the infinite range of ω to allow observing the phase, but only a small window in the frequency domain, due to the transducer bandwidth (and corresponding window in the time domain covering the overlapped pulse and echo).

We wished to avoid the FFT, as the windowed FFT provides the usual windowing problems, though its imaginary and real parts might allow computing phase (with some restrictions). The MEM, in contrast, yields a power spectral estimate which is real and non-negative. Thus it is necessary to use the even and odd components of the data to arrive at a phase estimate, a minor trick.

To estimate the phase of the process x from the known data samples x(0), x(1),..., x(M) and still be consistent with the principles of maximum entropy, we write the time series x(k) as the sum of an even series, e(k), and an odd series o(k), where the point of symmetry is chosen at

$$N_s = M/2 \tag{4}$$

and M, the data length, is an even integer.

The power spectra of these three sequences are related by

$$S_{xx}(e^{j\omega}) = S_{ee}(e^{j\omega}) + S_{oo}(e^{j\omega}) \tag{5}$$

We denote by

$$X(e^{j\omega}), E(e^{j\omega})e^{-j\omega T_s}, jO(e^{j\omega})e^{-j\omega T_s} \tag{6}$$

the Fourier Transforms, respectively, of x(k), e(k), and o(k).

$$E(e^{j\omega}) \text{ and } O(e^{j\omega})$$

are real functions of ω and the factor $e^{-j\omega T_s}$ is due to the fact that the origin of symmetry is $T_s = N_s \Delta t$. N_s is the number of samples to the center of the data block, Δt is the sample spacing. The phase $\theta(\omega)$ is defined as

$$\theta(\omega) = Arctan\frac{O(e^{j\omega})}{E(e^{j\omega})} \tag{7}$$

Use the trigonometric identity

$$\cos[2\theta] = \cos^2\theta - \sin^2\theta \tag{8}$$

and replace $|E|^2$ with S_{ee}, from the Fourier transform property

$$E(e^{j\omega}) = Re\{X(e^{j\omega})\}$$

and $|O|^2$ with S_{oo}, from the corresponding

$$O(e^{j\omega}) = Im\{X(e^{j\omega}\}$$

The geometry is described by

$$\mathrm{Cos}\,\theta = \frac{E(\omega)}{\sqrt{E^2(\omega)+O^2(\omega)}}$$

and

$$\sin\theta = \frac{O(\omega)}{\sqrt{E^2(\omega)+O^2(\omega)}}$$

Use of the $\mathrm{Cos}(2\theta)$ identity and using

$$\mathrm{Cos}\,2\theta = \frac{|E(\omega)|^2 - |O(\omega)|^2}{|E(\omega)|^2 + |O(\omega)|^2}$$

allows us to write

$$\mathrm{Cos}[2\theta(\omega)] = \frac{S_{ee}(e^{j\omega}) - S_{oo}(e^{j\omega})}{S_{ee}(e^{j\omega}) + S_{oo}(e^{j\omega})} \tag{9}$$

Both power spectral terms in (9) can be expressed in the form of (3) as

$$S_{ee}(e^{j\omega}) = \frac{P_e}{H_e(e^{j\omega})H_e(e^{-j\omega})} \tag{10}$$

$$S_{oo}(e^{j\omega}) = \frac{P_o}{H_o(e^{j\omega})H_o(e^{-j\omega})} \tag{11}$$

Substituting (10) and (11) in (9) yields

$$\mathrm{Cos}[2\theta(\omega)] = \frac{P_e H_o(e^{j\omega})H_o(e^{-j\omega}) - P_o H_e(e^{j\omega})H_e(e^{-j\omega})}{P_e H_o(e^{j\omega})H_o(e^{-j\omega}) + P_o H_e(e^{j\omega})H_e(e^{-j\omega})} \tag{12}$$

Since maximum entropy filters are minimum phase, the right hand side of (12) converges for all values of ω. The phase $\theta(\omega)$ can be calculated from (12).

3. Phase Estimate applied to Range Detection

A pulse of energy traveling through a uniform lossless medium is reflected from a boundary and the echo is received at the point of origin of the signal. Let the incident signal be denoted x(t), the total traveled distance 2d, and the velocity of propagation v. The received signal y(t) is a function of the delay time, t_d, where, $t_d = 2d/v$. The aim is to determine the distance d to the boundary by computing the delay t_d. Preprocessing the echo signal according to the method discussed in the previous section yields

$$\mathrm{Cos}\,2\theta(\omega) = \mathrm{Cos}\left[2\omega t_d\right] \tag{13}$$

where the argument is proportional to the delay time of the echo; all data is referenced to the the origin of symmetry (midpoint of the data). The power spectrum of the $\cos 2\theta$ (the phase estimate of our original data) curve exhibits a peak at a time corresponding to t_d. This problem of echo location of the source of echos (while having overlapping echos) is encountered in radar, seismology, and echo cardiography. To illustrate the usefulness of the foregoing technique to this class of problems, a simple geometry will be used as an example.

4. Experimental Confirmation

To test for robustness of the Maximum Entropy Phase Estimation (MEPE) method, we inserted a sheet of aluminum of 0.16 cm thickness into the ultrasound path described in Figure 1. The ultrasound transducers used were 3.5 MHz focused transducers, mounted on opposing sides of a plastic tank. The data were oversampled at 20 M samples/second, then averaged for an effective rate of 10M samples/second. We recognize that the transducer energy is centered around the 3.5 MHz resonance of the transducer; however, the information content in the signal phase (and amplitude) is distributed around the unit circle in the z plane. This is because the poles of the AR predictive filter convey their information as we traverse the unit circle, changing most rapidly in the vicinity of the poles. Though the signal energy is concentrated near 3.5 MHz, the computation of the AR coefficients, and therefore the pole locations, evidence this information on the whole unit circle. To use only the information on the unit circle adjacent to the poles is to window the result, thereby throwing away information.

To obtain a sufficiently large signal, a bijunction transistor was used to discharge a capacitor across the transmitter transducer. The received signal was sufficiently large to be digitized by a Tektronix model 2220 digital oscilloscope without prior amplification.

A signal without multiple reflections is shown in Figure 2a. The amplitude is the digitized value, which may be regarded as a relative value. The evaluation of equation (12), the cosine of twice the phase angle, is shown in Figure 2b. Taking the spectrum of the Figure 2b data yields the echo identification, Figure 2c (We have used the fact that there is no DC term, due to the odd symmetry of phase around the unit circle). A single large return with minimal artifacts is shown in Figure 2c. The relative time of arrival has been shifted to allow a full display in the drawing.

A contrasting signal with multiple reflections is shown in Figure 3a. Here, an extended echo appears without clear delineation between the different components of the multiple echo. The cosine of twice the phase angle is shown in Figure 3b; the oscillatory nature of the curve allows identifying the multiple echoes, shown in Figure 3c. The position of the first peak in Figure 3c corresponds to the time of transit (round trip, i.e. twice the 0.16 cm path) through the aluminum sheet that comprises the test obstacle. The sample calculation is

$$d = \frac{1}{2}[6420m/s][0.5\mu s]\tilde{\ }1.6mm$$

The relative amplitudes of the multiple echoes are consistent, too.

The simple geometry illustrates the efficacy of the algorithm. The driving reason for using it, of course, is the need to identify multiple echoes in systems in which the choice of waveforms (duration and frequency) is constrained by the attenuation (and the limitations in signal sources). In a medical application, Erdol has simulated waveform responses for thin layers of tissue, which encourages the algorithm's application to echocardiography and similar applications [16]. (This paper has extended Erdol's method by using the Maximum Entropy Method for obtaining both the odd and even sequence spectra, as well as the spectra of the resulting phase estimate.)

5. Conclusions

The algorithm proved resilient for the data examined. The method is of more interest than just the data above, in that it attempts to estimate phase using power spectral estimates.

6. Acknowledgements

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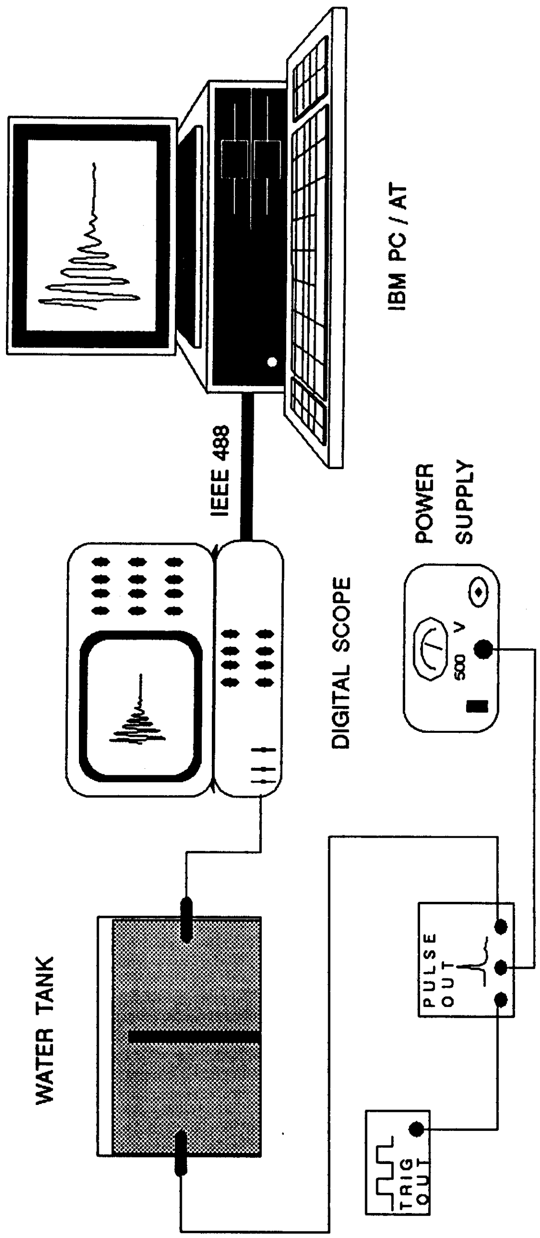


Figure 1a: Apparatus

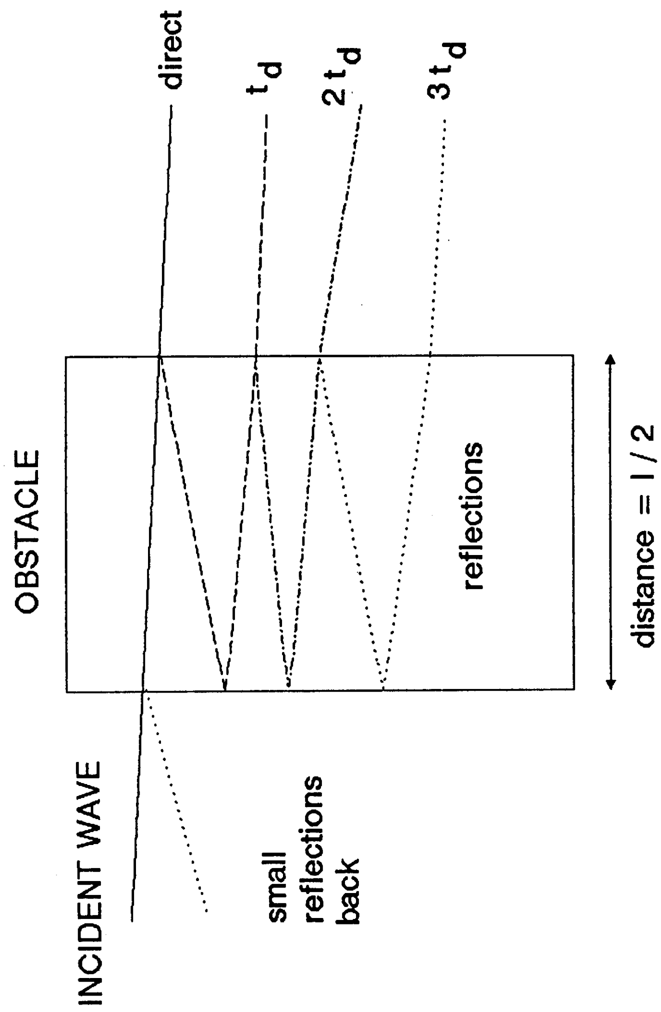


Figure 1b: Signal Path

Figure 2a: Received Signal Without Obstacle

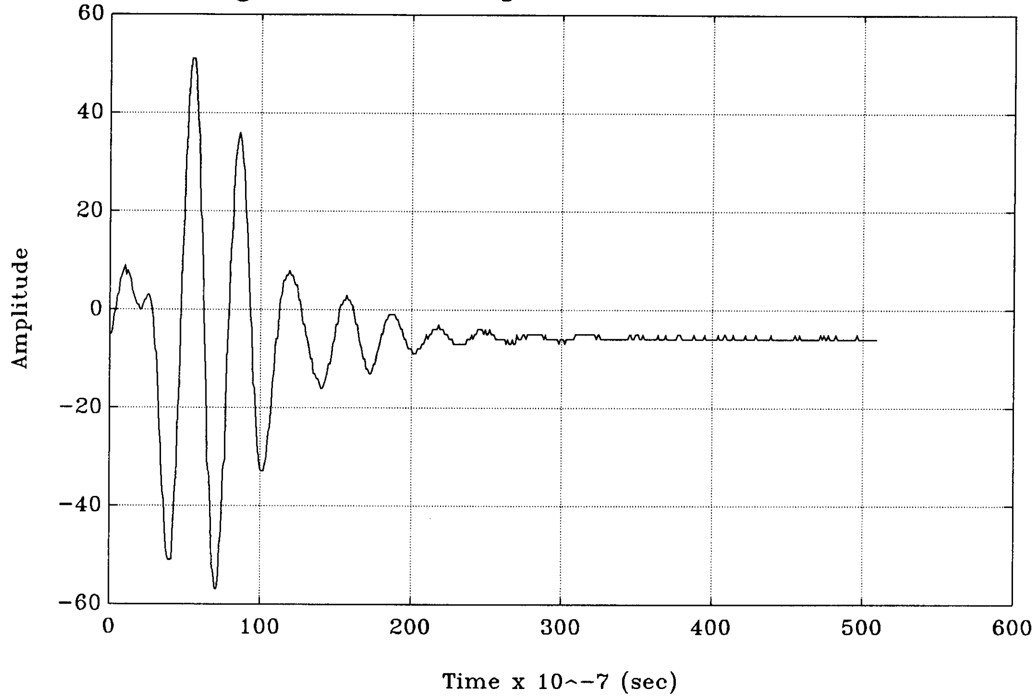


Figure 2b: Cos(2*phase) Without Obstacle

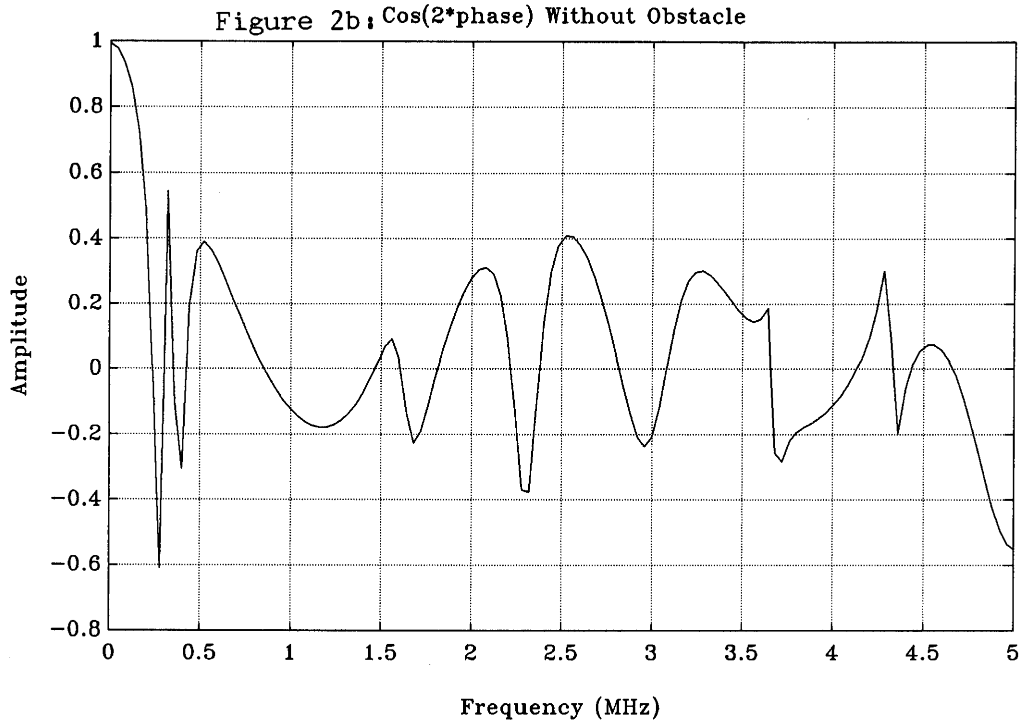


Figure 2c: Time Delay Without Obstacle

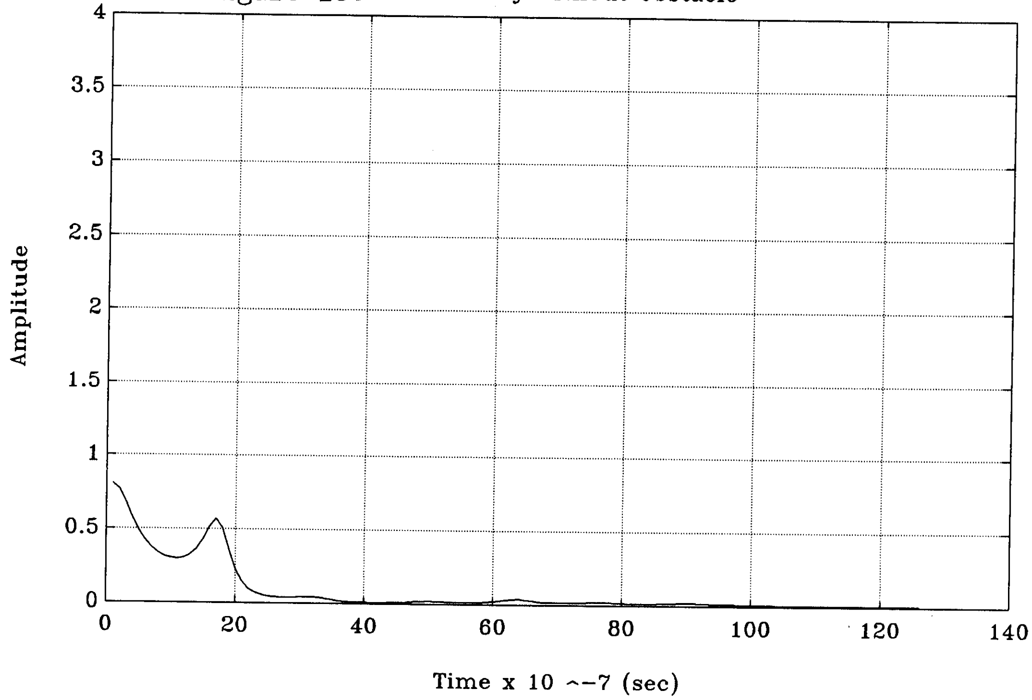
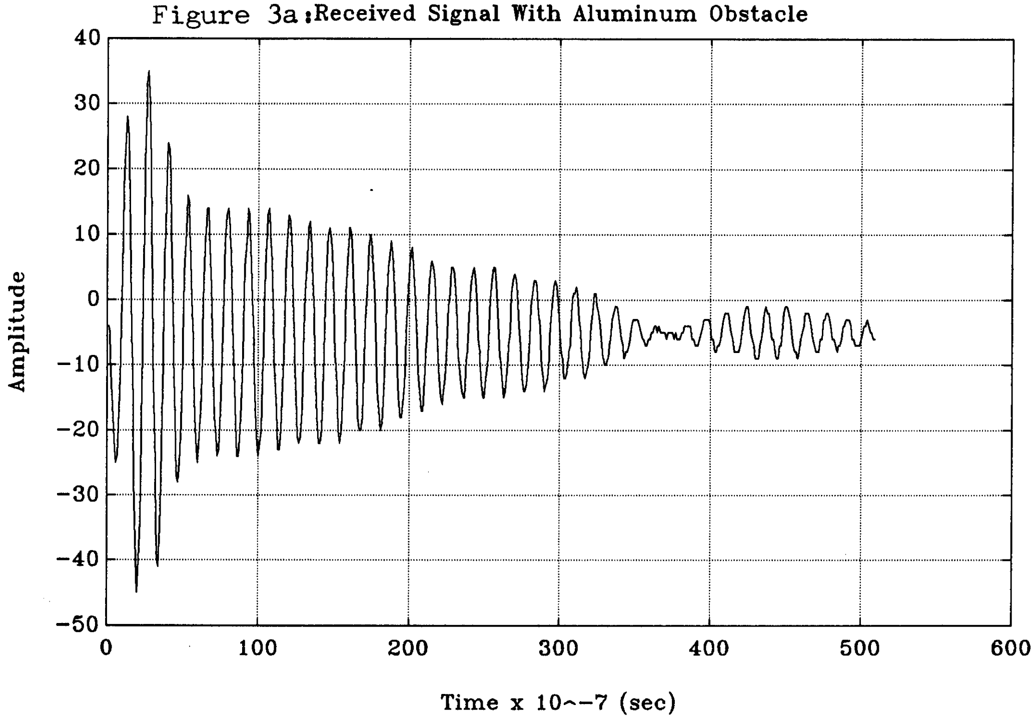


Figure 3a: Received Signal With Aluminum Obstacle



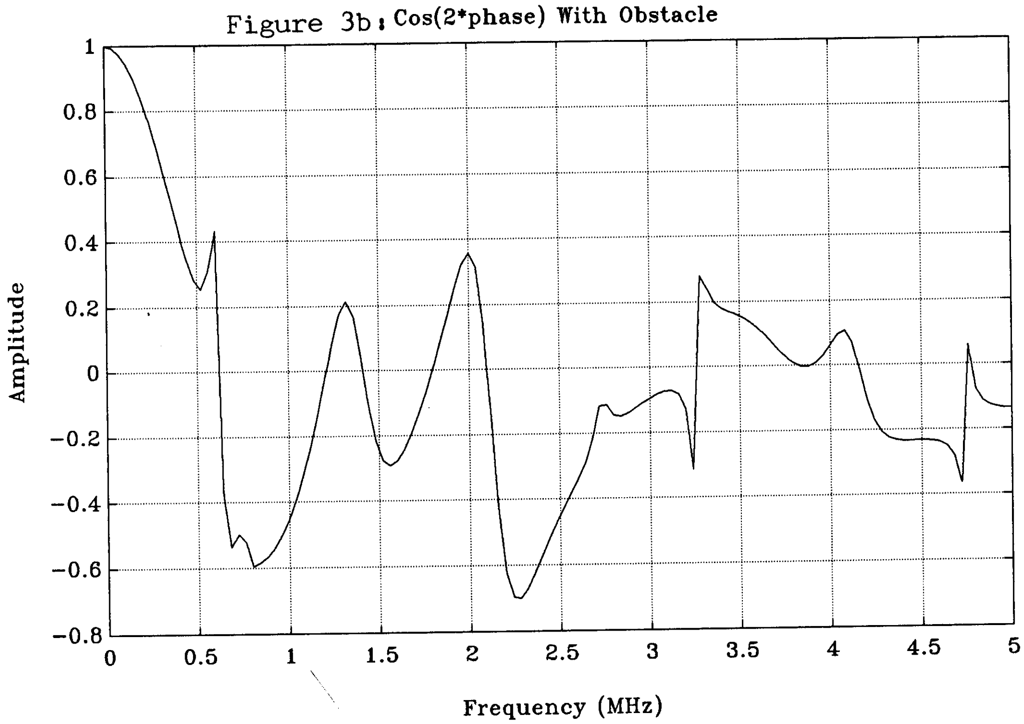


Figure 3b: Cos(2*phase) With Obstacle
Amplitude
1
0.8
0.6
0.4
0.2
0
−0.2
−0.4
−0.6
−0.8
0
0.5
1
1.5
2
2.5
3
3.5
4
4.5
5
Frequency (MHz)

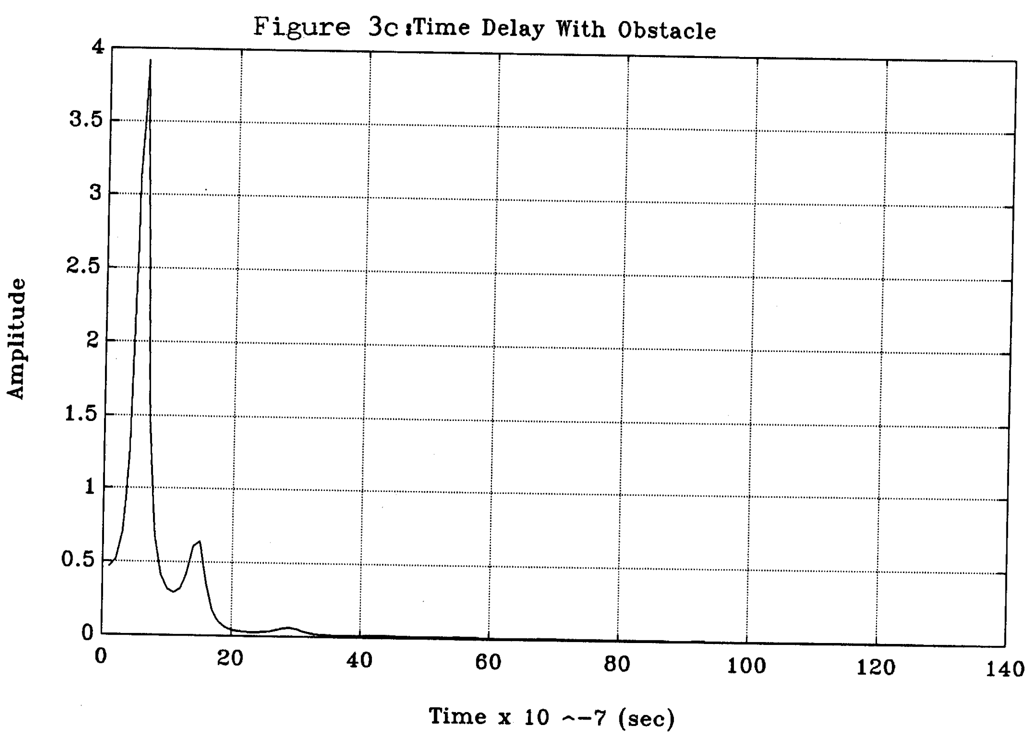


Figure 3c: Time Delay With Obstacle
Amplitude
4
3.5
3
2.5
2
1.5
1
0.5
0
0
20
40
60
80
100
120
140
Time x 10 ^-7 (sec)