Electromagnetic Forces and Applications
J. Tani and T. Takagi (eds.)

ELECTROMAGNETIC FIELD MODELING FOR NONDESTRUCTIVE TESTING OF COMPOSITE MATERIALS

Nathan Ida

Department of Electrical Engineering, The University of Akron. Akron, OH. 44325-3904, U.S.A.

Abstract

The use of electromagnetic fields for testing of composites takes a variety of methods but, most methods use high frequency methods because of the properties of composites. Eddy currents at high frequencies can also be used for conducting composites. Some of the methods applicable to modeling of testing phenomena at high frequencies are presented here. Examples from testing in microwave cavities and scattering from lossy dielectrics are given.

1. EDDY CURRENT FORMULATIONS

Eddy current formulations, can be modified to allow computation of electromagnetic fields at high frequencies (such as in resonant cavities) by adding the coupling between the electric and magnetic fields. The effect of displacement currents can be added by including the electric field directly in the formulation. A general formulation, based on the magnetic vector potential and the electric scalar potential can be written as[1]:

$$\nabla^2\mathbf{A} = \mu\mathbf{J}_s - \omega^2\mu\varepsilon\mathbf{A} - \sigma\mu(j\omega\mathbf{A}+\nabla\psi) \quad , \qquad \nabla.\,(j\sigma\omega\mathbf{A} + \sigma\nabla\psi) = 0 \tag{1}$$

The second equation in (1) is used to constrain the eddy currents in the solution domain. The continuity equation is used for this purpose. The only assumption implicit here is Coulomb's gauge. The solution of these equations leads to the correct field quantities based on the general field representation at any frequency. The magnetic flux density is calculated from ∇X**A** and the electric field intensity from $\mathbf{E}=-(j\omega\mathbf{A}+\nabla\psi)$. While **B** and **E** are important quantities, for the purpose of detecting resonance or shift in resonant frequencies of cavities, it is more convenient to look at the total stored energy in the cavity. A peak in the stored energy indicates resonance. This choice is convenient in that the actual energy in the system is not important, only the relative values. In addition, the Q-factor of a cavity can be calculated as:

$$Q = \omega\frac{W_s}{P_d + P_m} \tag{2}$$

where W_s is the stored energy in the cavity, P_d are the dielectric losses and P_m are the losses in the metallic walls of the cavity. These are given by:

$$W_s=\frac{1}{2}\int_v \varepsilon\mathbf{E}.\mathbf{E}^*dV+\frac{1}{2}\int_v \mu\mathbf{H}.\mathbf{H}^*dV\,, \qquad P_d=\frac{1}{2}\int_v \sigma\mathbf{E}.\mathbf{E}^*dV\,, \qquad P_m=\frac{1}{2}\int_s R_s\mathbf{H}.\mathbf{H}^*dV \tag{3}$$

where R_S is the surface resistivity of the walls ($R_S=1/\sigma\delta$), σ is the conductivity of the cavity walls and δ the skin depth. In geometries where the penetration might be deep, surface resistivity is not defined and P_m is calculated directly from **B** and **E**.

2. CHARACTERIZATION OF SPECIMENS IN A MICROWAVE CAVITY

A second method of calculating fields at high frequency is based on the solution of either of the following equations[2,3]:

$$\nabla\times\frac{1}{\varepsilon}\nabla\times\mathbf{H} - k^2\mu\mathbf{H} = 0 \quad \text{or} \quad \nabla\times\frac{1}{\mu}\nabla\times\mathbf{E} - k^2\varepsilon\mathbf{E} = 0 \tag{4}$$

where: $k = L\omega\sqrt{\mu_0\varepsilon_0}$ $\quad \varepsilon=\varepsilon_r\left(1+j\frac{\sigma\sqrt{\mu_0/\varepsilon_0}}{k\varepsilon_r}\right)$ and L a spatial scaling factor

Although not limited to NDT applications, our interest is in finding the resonant frequencies and the corresponding field distributions in a microwave cavity containing a test specimen. The cavity wall is assumed to have finite conductivity and any materials within the cavity are characterized by $\{\mu_0\mu_r, \varepsilon_0(\varepsilon_r-j\varepsilon''_r),\sigma\}$. Spatial dependence for ε and σ is implied. On material interfaces, **E** and **H** must be tangentially continuous. Boundary conditions are specified as **nXE**=0 on an electric wall or **nXH**=0 on a magnetic wall.

The weak form of equation (1) for the magnetic field is:

$$\int_v\left(\frac{1}{\varepsilon}\nabla\times\mathbf{H}\right).(\nabla\times\mathbf{w}_m dv - k^2\int_v \mu\mathbf{H}.\mathbf{w}_m dV = -\frac{jk}{\eta_0}\int_s (\mathbf{n}\times\mathbf{E}).\mathbf{w}_m dS \tag{5}$$

where $\mathbf{w_m}$ are any set of real, vector weighting functions. The use of vector or edge finite elements is chosen here to ensure elimination of nonphysical modes in the solution. For a tetrahedral element, the six edge shape functions and the finite element approximation are:

$$\mathbf{w}_n = \mathrm{sgn}(n)\frac{l_n}{\delta}(\mathbf{p}_{7\text{-}n,1}\times\mathbf{p}_{7\text{-}n,2} + \mathbf{e}_{7\text{-}n}\times\mathbf{r}) \qquad \mathbf{H} = \sum_{n=1}^{M} H_n\mathbf{w}_n \tag{6}$$

where H_n are the tangential components of **H** along edges. **H** is tangentially continuous on material interfaces. With vector weighting functions chosen to be the same as the shape functions, (6), Eq. (5) reduces to an eigenvalue problem. For the lossless case this reduces to the generalized algebraic eigenvalue problem:

$$[A]\{H\} = k^2[B]\{H\} \tag{7}$$

For loaded cavities, (including lossy boundary walls), a complex eigenvalue problem must be solved

$$\left([A(k)] - k^2[B] + jk[G]\right)\{H\} = 0 \tag{8}$$

[A(k)] denotes that the matrix [A] depends on k. Similar forms may be written for frequency dependent dielectric loading[4].

3. FINITE DIFFERENCE TIME DOMAIN (FDTD) METHODS

A third method, well suited to calculations related to NDT is the FDTD method. The main attraction of this method is in its ability to solve time dependent problems including transient applications. While the method can be easily used in three dimensional applications, we present here an axisymmetric formulation in terms of vector potentials as an example[5]. This has been found very useful in scattering problems, at microwave frequencies as well as for low frequency eddy current applications[6]. The equation to solve is:

$$\frac{\partial^2 A}{\partial z^2} + \frac{\partial^2 A}{\partial \rho^2} + \frac{1}{\rho}\frac{\partial A}{\partial \rho} - \frac{A}{\rho^2} = \mu\sigma\frac{\partial A}{\partial t} + \mu\sigma\frac{\partial^2 A}{\partial t^2} \tag{9}$$

To ensure a finite solution domain as well as to avoid reflections from artificial boundaries, radiation boundary conditions must be used. These boundary conditions are[5,6]:

$$\frac{\partial A_0}{\partial r} + \mu_a\varepsilon_a\frac{\partial A_0}{\partial t} + \frac{1}{r}A_0 = 0 \quad \text{in air,} \qquad \frac{\partial A_0}{\partial r} + \mu_m\varepsilon_m\frac{\partial A_0}{\partial t} + \left(\frac{\sigma_m}{2}\sqrt{\frac{\mu_m}{\varepsilon_m}} + \frac{1}{r}\right)A_0 = 0 \quad \text{in dielectrics} \tag{10}$$

The more common condition $A_0=0$ is used for good conductors. To solve this problem, it is assumed that A is known. This is normally found from independent calculations.

4. RESULTS

The application of the eddy current formulation to the calculation of resonant modes in a cavity in the presence of a piece of lossy dielectrics is shown first. A cubic cavity, 28.96cm on the side is lined with alumina ($\varepsilon_r=6$, $\sigma=10^{-4}$ S/m), to create a cubic inner space, 20.32cm on the side. A piece of carbon composite, (5.334x5.334x4.826cm, $\varepsilon_r=9$, $\sigma=10^4$ S/m) is located at the bottom of the cavity. With the alumina lining the measured resonant frequency is 554 mHz. The results of the finite element calculation are shown in figure 1, where the resonant frequencies with and without the composite are shown. As a reference, the resonant frequency for a dielectric test sample is also shown. Figure 2 shows some of the higher resonant modes for the same carbon composite. This is done by scanning the required frequency range, solving the problem for each frequency and plotting the absolute value of the stored potential energy.

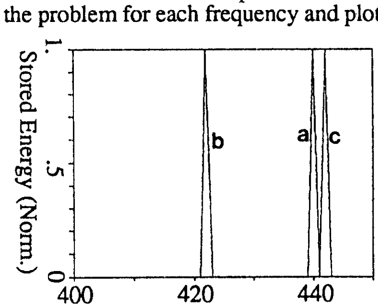

Figure 1. First resonant mode in the cavity. a. Empty cavity, b. with dielectric, c. with carbon composite.

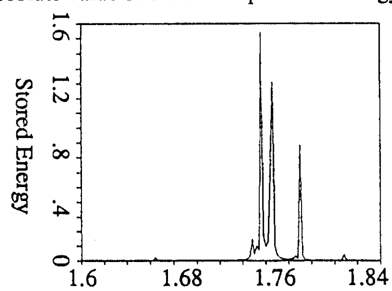

Figure 2. Higher order modes. Resonant peaks are at 1. gHz, 1. gHz and 1. gHz

To demonstrate the use of the eigenvalue method for cavities, the first few modes in a spherical cavity, concentrically loaded with a dielectric, spherical sample of radius 0.1m are calculated. In this solution, the radius of the cavity is varied to obtain the mode curves. The results are shown in figure 3 where the resonant frequency is shown as k. The same problem can be calculated with a lossy sample. In figure 4, the radius of the cavity is 0.5m and the conductivity is varied. Results are shown for ε_r=1,2,3 and 4 for TM_{21} and TM_{11} modes.

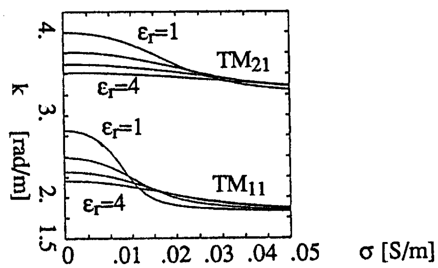

Figure 3. Four modes of a dielectric sphere in a spherical cavity of radius b.

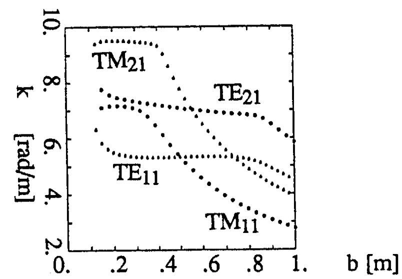

Figure 4. Modes of a lossy sphere in a spherical cavity.

An example to the use of the FDTD method for NDT is the use of a small loop to generate a field in a dielectric or lossy dielectric. The scattered or transmitted field can be then used for analysis purposes. A small loop, with a diameter of 0.8mm is used to generate a wave over a flat sample of a dielectric material. The field of the loop is calculated analytically[5] and used as input to the FDTD program. Figure 5 shows the wave generated in a flat piece of dielectric (ε_r=6), as well as it's propagating properties. The small blank area is the loop area. This is not shown because of the very high line density. Figure 6 shows propagation of the waves into sea water (ε_r=80, σ=4S/m)

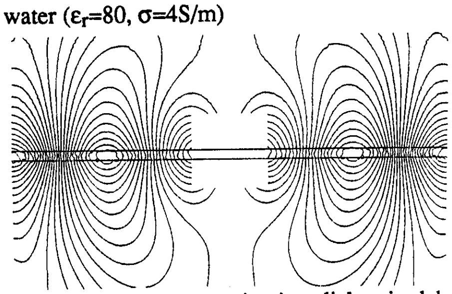

Figure 5. Wave propagation in a dielectric slab

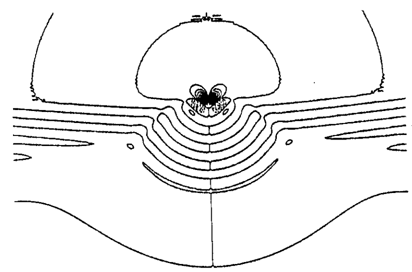

Figure 6. Wave propagation in sea water

* Supported in part by the National Science Foundation under grant EET8714628, and in part by The Ohio Board of Regents Academic Challenge Program. Computation facilities were provided by the Ohio Supercomputer Center.

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Elsevier Studies in
Applied Electromagnetics in Materials, 1

ELECTROMAGNETIC FORCES AND APPLICATIONS

Proceedings of the Third International ISEM Symposium on the Application of Electromagnetic Forces
Sendai, Japan, 28-30 January, 1991

Supplement to Volume 2 of the International Journal of Applied Electromagnetics in Materials

Editors:
J. TANI AND T. TAGAKI
The Institute of Fluid Science
Tohoku University
Sendai 980, Japan

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