Modeling of velocity effects in eddy current applications

Nathan Ida
Electrical Engineering Department, The University of Akron, Akron, Ohio 44325

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The effects of velocity on moving sources are encountered in many practical eddy current applications. In many instances these effects are ignored either because velocities are relatively low, because of our inability to quantify these effects, or for purposes of simplifying the solution. There are, however, a number of important applications in which this cannot be done and full account of velocity must be taken. Some obvious applications are magnetic recording, magnetic braking, and nondestructive testing. This work presents a finite element formulation for eddy current problems that takes into account the relative movement of sources. Results are presented indicating that velocity effects are significant at high velocities, and are important for correct signal interpretation. The effect of velocity on nondestructive testing signals is investigated and shown to display significant deviation from static behavior. Because of the form of the governing equations, spurious, nonphysical solutions may be generated. These are eliminated by two separate methods. One involves refinement of the finite element mesh and the second, upwinding of the finite elements.

INTRODUCTION

A number of eddy current applications involve relative movement of sources or of some other material in the vicinity of the sources. In such problems, an additional current due to the movement is generated. An area of particular concern in this respect is that of nondestructive testing. The interaction between the moving probe's field and material discontinuities constitute the signals that one is interested in. Reference will be made in this work to specific nondestructive test (NDT) applications. The results and the governing equations, however, are completely general. The effect of movement is customarily neglected in eddy current testing, in the interpretation of eddy current signals, and indeed, in modeling of general eddy current problems. This approach is based on the fact that the effects due to velocity are negligible in many practical situations, especially at low velocities. There are, however, a number of very important testing problems where failure to correctly account for the currents induced through motion may introduce significant errors in the test signal.

The work presented here describes a finite element formulation in axisymmetric geometries that includes velocity effects. The formulation is based on quadrilateral elements and takes into account velocity by including in the field equation, an induced current density due to relative movement. Because of the convective term in the field equation, the solution may, under certain conditions contain nonphysical oscillations, leading to loss of accuracy.[1-3] This aspect is well known in flow problems and is related to the Reynolds number.[1] A similar approach is taken here, where the existence of oscillations in the solution is related to the magnetic Reynolds number[4] of the media involved. These oscillations are eliminated either by refinement of the finite element mesh or by upwinding the finite elements.

FIELD EQUATIONS

The axisymmetric eddy current field equation to be solved is

$$\nu\left(\frac{\partial^2 A}{\partial r^2} + \frac{1}{r}\frac{\partial A}{r\partial r} + \frac{\partial^2 A}{\partial z^2} - \frac{A}{r^2}\right) + J_s - j\omega\sigma A - \sigma v\frac{\partial A}{\partial z} = 0, \quad (1)$$

where $\nu = 1/\mu$, A is the magnetic vector potential, J_s is the applied current density, and v is the velocity of the source. A similar equation applies to other 2D geometries. The equation is written in its linear form. This is appropriate for a variety of eddy current problems especially at low excitation levels. It is certainly the case in most practical eddy current testing applications.

In the general 3D case, one would have to allow for movement in all three directions. In two dimensions, it is possible to have movement in an arbitrary direction in a plane. More often, however, the movement is in a specified direction and, by properly choosing the coordinate system, only one velocity component need be considered. In axisymmetric geometries the movement can only be considered along the axis of the geometry (z direction) as in Eq. (1).

FINITE ELEMENT APPROXIMATION

For inclusion of velocity effects in eddy current problems, the appropriate form is the Galerkin (or weak) formulation[5-7] of Eq. (1). With standard shape functions, N_i, for a finite element (quadrilateral, in this case), the weak form can be written as

$$\int_\Omega N_i\left[\nu\left(\frac{\partial^2 A}{\partial r^2} + \frac{1}{r}\frac{\partial A}{\partial r} + \frac{\partial^2 A}{\partial z^2} - \frac{A}{r^2}\right) + J_s - j\omega\sigma A - \sigma v\frac{\partial A}{\partial z}\right]d\Omega = 0. \quad (2)$$

This is now integrated by parts and the finite element approximation for the magnetic vector potential is substituted. Surface integration is eliminated by assuming zero vector potential on the boundaries. With these conditions, the elemental contribution to the global matrix is

$$[[K] + [S] + j[R]]\{A\} = \{Q\}. \quad (3)$$

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The elemental matrices $[K]$, $[R]$ and the vector $\{Q\}$ are identical to those obtained in any axisymmetric finite element formulation. The matrix $[S]$ is due to the velocity term alone and unlike $[K]$ and $[R]$, is nonsymmetric.

The global matrix assembled by summation of the individual elemental contributions is therefore asymmetric because of the introduction of the velocity term.

MESH REFINEMENT AND UPWINDING

The inclusion of the convection term in Eq. (1) is known to generate oscillatory results that are due to the ratio between the magnitude of the first and second derivative terms. Thus, the larger the velocity, the larger this ratio and the more severe the loss of accuracy due to this effect. For convenience, and in order to quantify this aspect, the condition for a stable solution is related to the so-called magnetic Reynolds number. This is defined as the product of velocity, permeability, conductivity and a characteristic length[4] L,

$$R_m = \mu\sigma v L\,, \tag{4}$$

where the characteristic length is taken as the length of the element in the direction of the motion. Using this notation, a condition for a stable solution is obtained as[3]

$$L < 2/\mu\sigma v\,. \tag{5}$$

For low velocity applications, particularly with low permeability materials, this condition is easily satisfied. For high velocity, high permeability materials, there are two approaches that can be taken. One uses a mesh refining scheme that guarantees that the relation in Eq. (5) is satisfied in conducting media. This is a simple approach that works very well at relatively low velocities. At high velocities, the refinement necessary may be excessive and may require an unusually large mesh with the associated expense in computer resources. A second approach is to use upwinding of the elements. Although many methods for upwinding finite elements are available,[1,3] the method used here is that of Hughes[3] and relies on modification of the shape functions for the convective term. This is done by evaluating the shape functions at an adjustable point within the element. The location of this point within the element determines the degree of biasing with limiting cases that are equivalent to forward or backward finite difference schemes.[3]

RESULTS

In order to investigate the effects velocities have on NDT measurements, a geometry consisting of a simple circular coil with square cross section, 0.5 in. wide inside a tube, 6 in. in diameter, and 0.25 in. wall thickness was used. The tube is either magnetic or nonmagnetic. The quantity of interest in NDT is the impedance of the coil. Variations in the impedance indicate changes in permeability and conductivity due to anomalies in the material being tested. The effects of velocities on the impedance of the probe are important because movement of the probe changes the impedance of the coil and, if not taken into account, may be interpreted as an anomaly. For correct interpretation of results and comparison of numerical calculations and experimental data, the correct computation of the impedance, including velocity effects, is vital.

To evaluate the formulation presented here, the impedance of a coil at various frequencies and velocities in magnetic and nonmagnetic tubes was calculated. Table I summarizes these results as variations from the impedance at rest. The variations are given for the real (R) and imaginary (X) parts of the impedance separately. All results were computed without upwinding or mesh refinement, then through mesh refinement, and then by upwinding. The results given in brackets are only those that varied by more than 1% from the results without either upwinding or mesh refinement. In most cases, the difference between the calculation with upwinding and mesh refinement is less than 0.5%. Where the variations are larger, it is because the element size was not small enough. It is clear that the variations in impedance are quite large and ignoring the coil velocity will result in large errors. At the same time, for nonmagnetic materials, at practical testing velocities, these effects can be neglected. In most practical cases, the variations between the computation with upwinding or mesh refinement and that without is not large. Upwinding or mesh refinement becomes necessary only at very high velocities. Part of the reason for this is that the

TABLE I. Comparison of coil impedances at various velocities and frequencies for magnetic and nonmagnetic materials. Conductivity is $1.0E+06$ for the nonmagnetic and magnetic materials.

Velocity	40 Hz	($\mu_r = 1000$)	2 kHz	($\mu_r = 1000$)
(m/s)	X	R	X	R
10	3.6%	49.0% (52.1)	13.2%	15.0% (16.0)
50	16.0% (17.1)	60.0% (63.2)	17.8% (19.4)	21.0% (22.9)
100	22.1% (23.7)	69.6% (72.1)	26.2% (27.8)	25.8% (27.3)
1000	19.6% (21.1)	94.3% (95.8)	41.1% (43.2)	67.0% (69.4)
Velocity	2 kHz	($\mu_r = 1$)	20 kHz	($\mu_r = 1$)
(m/s)	X	R	X	R
10	0.9%	1.9%	0.07%	2.0%
50	4.6%	8.5%	0.4 %	11.7%
100	9.2%	14.0%	0.8 %	24.2%
1000	38.4% (39.7)	48.4% (51.0)	22.1 % (23.3)	48.6% (49.9)

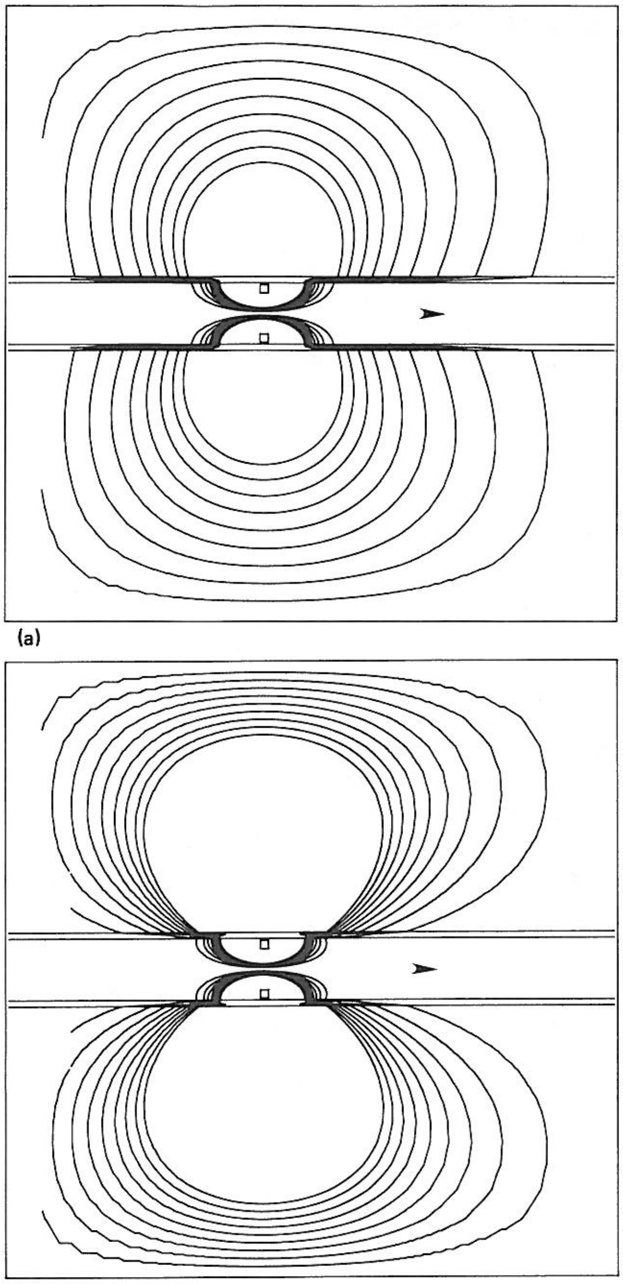

FIG. 1. Flux distribution for a moving coil at 40 Hz. (a) $v = 0$ m/s, (b) $v = 10$ m/s.

mesh used in eddy current computations is usually relatively fine in order to take the skin depth into consideration.

In addition to calculation of impedance, the field distribution in space was calculated to see any oscillatory effects that may occur. Figure 1 shows the flux distribution for a coil at 40 Hz at zero velocity and at 10 m/s. None of them displays any noticeable numerical anomaly. (The flux lines plotted are only the outer, weak lines.) To demonstrate how severe the oscillations can be, the field of the same coil at a velocity of 100 m/s was also calculated. This is shown in Fig. 2. It shows spurious flux lines that do not seem physically possible.

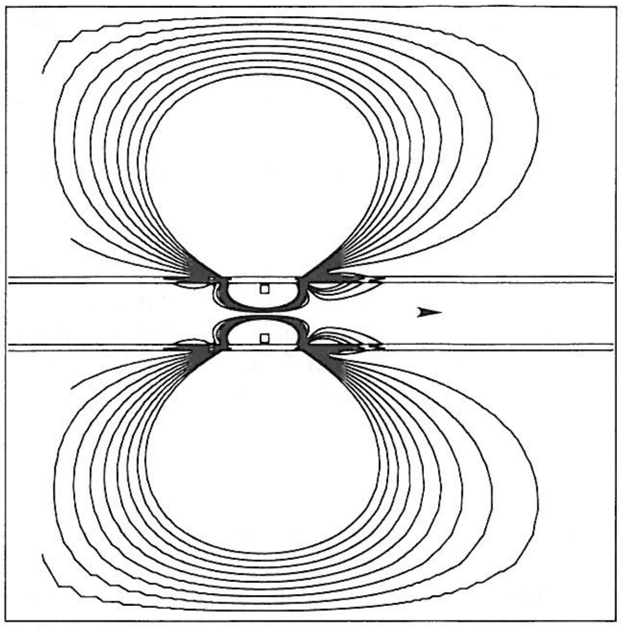

FIG. 2. Flux distribution for a coil at 40 Hz, at 100 m/s without upwinding. The nonphysical behavior is due to the large convection term.

CONCLUSIONS

The formulation presented here represents a simple way of taking into account velocity effects in finite element modeling of electromagnetic fields. In particular, the importance of such a model for eddy current nondestructive testing applications was demonstrated. Velocity effects are relatively small for most applications but may be quite significant in NDT where testing velocities are relatively high. Their correct computation is important for correct interpretation of results, especially with magnetic materials.

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[2]P. E. Allaire, *Basics of the Finite Element Method* (Brown, Dubuque, 1985).

[3]T. J. R. Hughes, Int. J. Numer. Methods Eng. **12**, 1359 (1978).

[4]G. W. Sutton and A. Sherman, *Engineering Magnetodynamics* (McGraw-Hill, New York, 1965).

[5]J. C. Sabonnadierre, in *Finite Elements in Electrical and Magnetic Field Problems*, edited by M. V. K. Chari and P. P. Silvester (Wiley, Chichester, 1980), p. 109.

[6]M. V. K. Chari, in *Finite Elements in Electrical and Magnetic Field Problems*, edited by M. V. K. Chari and P. P. Silvester (Wiley, Chichester, 1980), p. 87.

[7]R. Wait and A. R. Mitchel, *Finite Element Analysis and Applications* (Wiley, Chichester, 1985).