Computation of Fusing Currents in Composite Conductors

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Fusing currents in composite cables are calculated by coupling the non-linear heat equation with the calculation of the joules heating in a non-linear finite element process that includes conduction and radiation effects. The method is verified using analytical formulas available for simple copper cables. The method is useful in multi-material composite cables as well as in conductors of arbitrary cross section such as traces on printed circuit boards where analytic formulas are not available. It is also useful for the calculation of temperatures and temperature distributions in power lines or any other current-carrying conductor. For short fusing times, the effects of conduction and radiation are negligible, simplifying the solution.

Index Terms— **Coupled problems, finite element analysis, fusing currents.**

I. Introduction

AN IMPORTANT specification of wires and cables is the fusing current—the current at which the conductor melts. It is usually provided with the time needed to fuse the conductor, the latter is typically between 0.25 and 30 s, but it can be as short as 0.05 s (three cycles). Obviously, fusing current depends on time, that is, the current needed to fuse the conductor in, say, 0.5 s is much higher that the current needed to fuse it in 30 s. The typical need is for calculation of fusing currents in a specific time and in specific types of conductors [1]. Knowledge of the maximum current that can be carried by a conductor and its relation to temperature has been studied in power systems [1], [2] but, equally, in electronic circuits and in particular in microelectronics where bond wires are particularly susceptible to fusing [3], [4]. Because of the importance of fusing currents, *ad hoc* relations have been developed for the most common conductors and cables. Attempts to estimate fusing currents go back to the end of the 19th century [5] when the first approximate fusing formula was introduced. Formulas for fusing currents for short times have also been developed for wires for various applications [2]–[5]. These are quite accurate but only apply to some types of conductors (solid, cylindrical conductors, and clad cylindrical conductors) and do not provide any useful information in composite cables—cables made of more than one type of material in conductor bundles or in conductors of other cross sections. In many cases, the fusing currents are determined experimentally at considerable cost and are often available in tables from manufacturers for specific applications or for specific conductors. In addition to analytic and experimental methods, numerical approaches to calculation of fusing currents were developed. Some are simple and approximate [6], whereas others are specific to an application [7], [8].

A. Analytic Formulas

Almost all work on fusing currents rely on analytic formulas, starting with Preece's simple relation for the fusing current of a conductor [5]

$$I_f = ad^{3/2} \qquad [A] \tag{1}$$

where a is a constant that depends on the material and d is the diameter of the conductor in inches. As archaic as this formula is, it has been modified to allow calculation of fusing time [9]

$$t = 0.233\sqrt{A} = 0.05918\sqrt{B} \qquad [s] \tag{2}$$

where t is in seconds, A is the cross-sectional area in square mils (1 mil = 0.0254 mm), and B is the cross-sectional area in mm^2. An improved formula is due to Onderdonk [10] and has been in use for many decades [2]

$$I_f = A\sqrt{\frac{\log_{10}\left(\frac{T_m - T_a}{234 + T_a} + 1\right)}{33t}} \qquad [A] \tag{3}$$

where I_f is the fusing current in amperes, T_m is the melting temperature of the material, T_a is the ambient temperature, both in °C, t is the fusing time in seconds, and A is the area in circular mils (1 circular mil = 0.000506 mm^2). Attempts to improve on these formulas and include clad conductors as well, results in the formula due to Sverak [1]. In its general form, it is rather complex but for practical computation, [2] offers a simplified form

$$I_f = \frac{A}{K\sqrt{t}} \qquad [A] \tag{4}$$

where A is the cross-sectional area in mm^2 and K is a tabulated constant that depends on the material properties (resistivity, temperature coefficient of resistivity, density, and thermal conductivity) and on the fusing and ambient temperature. The results used for comparison in this paper are based either on (3) or (4).

B. Numerical Models

Numerical models have been developed primarily for computation of fusing currents in bond wires for microcircuits and for printed circuit traces, but also for grounding conductors. These were based on the finite difference method [11] or the finite element method [7] to consider conditions around the

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conductors or using the heat equation along the conductor (1-D solution) [6] to obtain temperature distributions but also fusing currents in what can be called an approximate method. None of these methods, however, allow for solutions in composite media and arbitrary conductors.

II. Current Model

The current model generalizes the computation of fusing currents and the calculation of temperature rise in conductors of arbitrary composition and profiles. The model couples the non-linear heat equation with the Joule heating in conductors considering the non-linear properties of heat capacity and of the electric conductivity as well as the exact conductors configuration including twisting of cables. The method can also model non-linear thermal conductivity, radiation, and conduction and can either accept constant current or constant voltage sources. The results show that some parameters are less important for calculation of fusing current than others. Whereas heat capacity and electric conductivity have drastic effects on fusing currents, thermal conduction, and radiation can usually be neglected because of the short duration involved in these calculations, confirming long held principles that have been used in literally all analytical formulas used to estimate fusing currents [1], [2], [5], [6].

The heat equation governing fusing in conductors is

$$\rho C_p(T)\frac{\partial T}{\partial t} - \nabla \cdot k(T)\nabla T = \rho \mathbf{J} \cdot \frac{\mathbf{J}}{\sigma_0}\left(1 + \alpha\left(T - T_0\right)\right) \quad (5)$$

where ρ is the density of the medium, C_p is the heat capacity, T is the temperature, k is the thermal conductivity, σ_0 is the electric conductivity at reference temperature T_0, α is the temperature coefficient of resistance, and $\mathbf{J}$ is the current density. Both the heat capacity and electric conductivity are functions of temperature as is, in general, the heat conductivity. The latter is taken here as constant since it has little effect on the solution, probably because the amount of heat lost through conduction during the short fusing time is negligible. There are two mechanisms by which heat is lost. The first is through conduction to structures to which the conductor is physically attached. We assume that these are at room temperature and are infinite sinks. To do so, we apply Dirichlet boundary conditions ($T = T_0$) on these boundaries. The second term of loss is through radiation into air across free-standing boundaries, governed by the Stefan–Boltzman law

$$k\frac{\partial T}{\partial n} = \sigma\varepsilon\left(T^4 - T_\infty^4\right) \quad (6)$$

where σ is the Stefan–Boltzman constant, ε is the emissivity of the surface, and T_∞ is the external temperature. Although emissivity may also be temperature dependent, because of the short fusing times, it is taken here as a constant. The outer surface of the conductor may either be insulated, or the Stephan–Boltzman law in (6) may be used as a boundary condition. The initial condition is taken as the surrounding temperature. The current density at the contact is calculated from the total imposed current, the cross-sectional area of the various conductors and their conductivity and is used as a Neumann boundary condition. The boundary current densities applied on copper and steel cross-sectional areas are

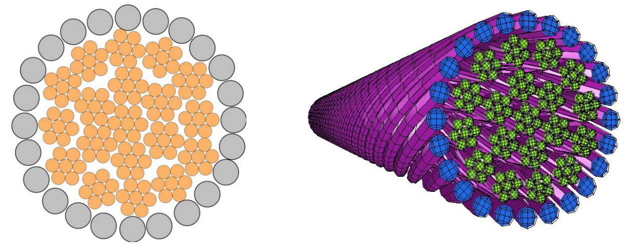

Fig. 1. (a) Cross-sectional view of the cable. (b) FEM mesh.

$$J_c = I_t\frac{\sigma_c}{\sigma_c A_c + \sigma_s A_s}, \qquad J_s = I_t\frac{\sigma_s}{\sigma_c A_c + \sigma_s A_s} \quad (7)$$

where I_t is the total current applied, J, σ, and A are the current density, conductivity, and cross-sectional area, respectively, whereas subscripts c and s indicate copper and steel. The use of current density allows the calculation of power dissipated per unit volume as J^2/s independent of the length of conductors.

The model is solved using a finite element process (in ELMER) with an explicit time-marching method, a preconditioned stabilized biconjugate gradient iterative solver with incomplete LU preconditioning and the Newton iteration for the non-linear process.

III. Result

The model used as an example is shown in Fig. 1. It consists of 19 bundles of seven strands of copper wires, 0.7874 mm in diameter, surrounded by 24 steel wires, each 1.524 mm in diameter. The cable (strand bundles and steel wires) is twisted with pitch of 116 mm. The FEM mesh shown in Fig. 1 was obtained using Salome [12] because it allows extrusion and twisting, but also because it supports quadrangle surface and hexagon volume elements. It consists of 157 000 prism elements [Fig. 1(b)] with 200 175 nodes, 194 680 surfaces and 355 448 edges. The final mesh only models the solid conductor with the net effect that heat conduction across wires is not considered. In effect, the numerical experiments have shown that during the short fusing time neither radiation nor conduction contribute to the fusing current or fusing time hence the simplified mesh. Nevertheless, the formulation can consider all effects and hence it can be applied for the calculation of steady state temperature in current-carrying conductors. To obtain the fusing current for a specific fusing time, the total current I_t in (3) is set at some small value and the solution in time obtained. If, at the end of the set time, the temperature of the conductors is below melting point, the current is increased, and the solution repeated. The current that produces the melting temperature is taken as the fusing current. Fig. 2 shows the temperature distribution in the cross section of the composite cable as a function of time. These were prepared with the Elmer postprocessor [13]. This particular sequence is for a 747 ms fusing time. The solution steps in time in steps of 74.7 ms, using a second-order backward difference formula and a Newton non-linear iteration with a convergence

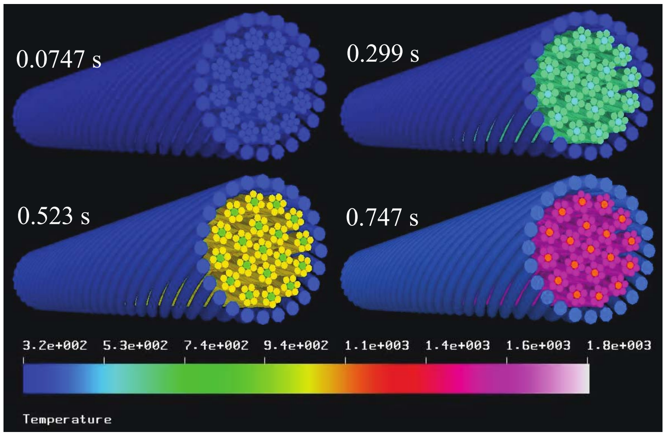

Fig. 2. Time sequence of temperature in the cable. Note the higher temperature of the inner strands. The temperatures are in K.

TABLE I

COMPARISON OF RESULTS, 3/0 ROPE-LAY

Time to fusing	Computed	Analytic	Measured
3s (no steel wires)	13,670A	14,000A [2]	
30s (no steel wires)	3,990A	3,990A [1]	
747ms	24,925A		25,100A
258ms	42,395A		42,638A

tolerance of 10^{-7}. The sequence shows that the outer, steel wires remain relatively cool because of the relatively low current density in them. As time progresses, the copper strands heat up but heating is not even. The inner strand in each seven conductor bundle remains cooler than the outer strands simply because the inner conductors are straight and shorter and hence have a lower resistance, whereas the outer conductors are twisted around the inner conductor and are longer. For the same current density, this necessarily translates in lower power density in the inner conductors. Table I compares the results obtained with the present model with data available from the IEEE standards, theoretical values for copper wires and experimental data for composite cables.

Because the model above does not consider heat conduction between strands, it is only suitable for short-duration transient calculations. If the conduction across strands and into air must be considered, the model must be modified in one of two ways. The obvious but complex solution is to mesh the air between the strands and assign the heat conductivity of air to the added elements. This is quite complex in a geometry like the one in Fig. 1, especially since for round twisted wire bundles the outer surface is not easily defined. It is of course possible to mesh air around the outer surface as well, but then the imposition of radiation condition on the outer boundary of the conductors cannot be imposed. Another slight issue is the meshing between the strands that necessarily will lead to a very large number of elements and possibly to elements with large aspect ratios.

An alternative is to model the conductors as equivalent solids with the same total copper and steel cross section but neglecting the twisting of the strands. This is shown in Fig. 3 for the same cable shown in Fig. 1. Since copper only occupies ~60% of the space, the inner solid conductor leaves a gap between itself and the outer steel layer. The gap can be meshed if conduction through air is to be considered (as would be the case for steady-state calculations or for long transients) or it may be left out if this conduction is insignificant (as it would be for fusing current calculations). The outer boundary is assumed to radiate. The model only requires 7168 hexahedral elements and hence the computational effort in this case is negligible compared with the model in Fig. 1. This method approximates the geometry and therefore introduces some error in the calculations, but it is simple and very economical in computer resources. Table II shows the computed results using this method for a bundle of copper conductors equivalent in cross section to a 3/0 rope-lay as in Fig. 1 but without the steel conductors and compares them with the approximate analytical values in [2]. In this case, the issue of conduction between layers is irrelevant since the outer layer in Fig. 3 is removed and the only heat loss is through radiation from the outer surface. It should be noted that these are inferior to the results in Table I as expected from this simpler method. Note also that at shorter fusing times the results improve, as was also observed in Table I. The model is much simpler and sufficiently accurate in many cases, especially at short fusing times.

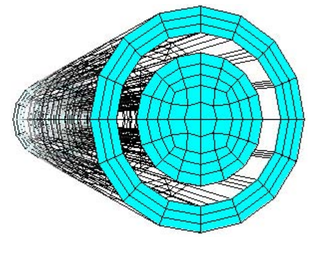

Fig. 3. Simplified model of the composite cable. For fusing current calculations, the air between the conductor is not meshed and there is no conduction between the inner and outer conductors.

TABLE II

COMPARISON OF RESULTS FOR THE SIMPLIFIED MODEL FOR A 3/0 COPPER ROPE-LAY

Time to fusing	Computed	Analytic	error
30s	4,150A	3,990A [1]	8.77%
3s	12,884A	13,720A [2]	6.1%
747ms	24,610A	27,500A[2]	1.56%

A. Errors

The maximum error observed is ~2.35% with respect to the analytical solution in [2] (Table I, first row). It should be noted as well that the computed results tend to underestimate the fusing current in the cable. There are a number of sources for these errors. One source lies in the way we applied the current density at the ends of the cable. The current density is divided between the copper and steel based on the conductivity of the steel and copper. As the copper heats up much more than the steel, its resistance increases and hence in practice, the total current in the copper does not remain constant as we assumed but rather decreases with temperature, whereas the total current in steel increases since its temperature is much lower than that of copper. As the current in copper is reduced with temperature, it will take either a longer time to fuse or, for a fixed time, it will require a larger current to fuse. The results support this observation even in the case of a single material (copper) since the strands are not heated uniformly (see Fig. 2) and hence different strands have different resistance and conductivity. However, since the current density is used as a boundary condition in the solution it is not possible in the present model to make it temperature dependent. Second, we neglected radiation from internal surfaces (copper strands) as well as heat conduction between strands based on the fact that fusing currents are applied for short periods. In addition, we assumed that the fixed end temperature is constant and does not affect the temperature distribution in the cable but in experiments this will have some influence. Finally, we note that the analytical formulas are themselves approximate and assume constant material properties. Perhaps the largest error, especially with respect to experimental results lies in the values of material properties. There is some variability in published values from different sources, the available data are limited especially at higher temperatures, and hence there is an element of uncertainty in the results although it is difficult to asses exactly the effect of this variability.

IV. Conclusion

This paper discussed the important problem of fusing currents in cables. The calculation of fusing currents is a more accurate and more universal alternative to tables and approximate analytical formulas. In particular, in complex, composite conductors it allows simulation of the complexity of the structure such as the twisting of the wires. The method is general and the user has the option of selecting the levels of approximation that suits the nature of the problem. Accurate fusing currents are obtained that compare well with other available methods.

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References

[1] J. G. Sverak, "Sizing of ground conductors against fusing," *IEEE Trans. Power App. Syst.*, vol. PAS-100, no. 1, pp. 51–59, Jan. 1981.
[2] *IEEE Guide for Safety in AC Substation Grounding*, IEEE Standard 80-2000, 2000.
[3] G. T. Nöbauer and H. Moser, "Analytical approach to temperature evaluation in bonding wires and calculation of allowable current," *IEEE Trans. Adv. Packag.*, vol. 23, no. 3, pp. 426–435, Aug. 2000.
[4] M. Coxon, C. Kershner, and D. McEligot, "Transient current capacities of bond wires in hybrid microcircuits," *IEEE Trans. Compon., Hybrids, Manuf. Technol.*, vol. 9, no. 3, pp. 279–285, Sep. 1986.
[5] W. H. Preece, "On the heating effects of electric currents," *Proc. Roy. Soc. London*, vol. 43, no. 2, p. 280, 1887.
[6] K. C.-Y. Chen *et al.*, "Conductor fusing and gapping for bond wires," *Prog. Electromagn. Res. M*, vol. 31, pp. 199–214, Jun. 2013.
[7] A. Mallik and R. Stout, "Simulation methods for predicting fusing current and time for encapsulated wire bonds," *IEEE Trans. Electron. Packag. Manuf.*, vol. 33, no. 4, pp. 255–264, Oct. 2010.
[8] H. P. Iskanderian and W. J. Horvath, "Determination of the temperature rise and the maximum safe current through multiconductor electric cables," *J. Appl. Phys.*, vol. 17, no. 4, pp. 255–262, 1946.
[9] D. Brooks, "Fusing current: When traces melt without a trace," in *Printed Circuit Design*. San Francisco, CA, USA: Miller Freeman, Dec. 1998.
[10] AIEE Committee Report, "Application guide on methods of substation grounding [includes discussion]," *Trans. Amer. Inst. Elect. Eng. Power App. Syst. III*, vol. 73, no. 1, pp. 271–277, Jan. 1954.
[11] X. Guo, H. Cao, and C. Qu, "Numerical simulation of temperature distribution of three-wire cables with combination coordinates," in *Proc. IEEE Int. Conf. Ind. Technol.*, Apr. 2008, pp. 1–5.
[12] *SALOME*. [Online]. Available: http://www.salome-platform.org, accessed Mar. 2014.
[13] *Elmer*. [Online]. Available: http://www.csc.fi/web/elmer, accessed Mar. 2014.