Fast Reconfiguration of Distribution Systems Considering Loss Minimization

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***Abstract*—This paper addresses the problem of finding the state of switching devices (open or closed) in primary distribution networks so that the total loss is minimum. Radiality and capacity constraints are taken into account. This optimization problem is a mixed-integer nonlinear optimization problem, in which the integer variables represent the state of the switches, and the continuous variables represent the current flowing through the branches. The standard Newton method (with second derivatives) is used to compute branch currents at each stage within the integer search, which, in turn, is implemented as a simple best-first search. Although a best-first search cannot normally guarantee the optimality of the solution, the high quality of the suboptimal solutions found, together with the high processing speed, make this approach very attractive for real-size distribution systems. Results from the application of the proposed methodology to a 1128-branch, 129-switch, real-world distribution system are presented and discussed.**

***Index Terms*—Loss minimization, power distribution, system reconfiguration.**

NOMENCLATURE

p	Load bus index.
I_p	Current injected at load bus p (p.u.).
c_{jk}	Capacity factor of branch jk (p.u.).
r_{jk}	Electrical resistance of branch jk (p.u.).
I_{jk}	Electrical current through branch jk (A).
i_{jk}	Electrical current through branch jk (p.u.).
Ω_B	Set of all branches in the electrical system.
Ω_{Bp}	Set of all branches connected to load bus p.
n_{CS}	Necessary number of closed switches in a proper radial network.

I. INTRODUCTION

OPTIMIZATION problems where some (or all) of the variables must assume integer values are particularly difficult to deal with [1]. In the present case, an integer variable (binary, in particular) is associated with each switch in the network, and it can be assigned either a value 0 (zero) or 1, meaning open switch or closed switch, respectively. The number of possible network states grows exponentially with the number of switches (2^n), making rudimentary techniques such as exhaustive search totally unsuitable (for a 129-switch system, the number of possible states is over 10^{38}). Therefore, more sophisticated search techniques are required for applications aimed at real-world problems.

The distribution system reconfiguration problem has been studied over the past thirty years through various approaches [2]–[9]. More recently, modern techniques such as constrained decision problems (CDPs) [10] and genetic algorithms (GAs) [11] have also been employed. Despite the fact that neither CDP nor GA guarantees that the optimal solution will be achieved, they do provide high-quality suboptimal solutions. Both techniques suffer, to a certain extent, from scaling the problem size from a few switches up to 100 switches or more, typically found on realistic distribution systems.

This paper proposes a methodology for reconfiguring distribution networks considering loss minimization as the primary objective to be achieved. Since primary distribution feeders normally operate in radial configuration (each load point supplied by one parent branch only), the methodology also enforces the radiality of the final solution. This is an important feature because, as will be seen in the next section, the radiality constraint is extremely difficult to implement through analytical expressions. Branch capacity constraints are also incorporated into the methodology, allowing the suppression of possible overloads in the final solution. The core of the proposed approach is a best-first search for establishing the state of all switches (open/closed) combined with the standard Newton method with second derivatives for computing branch currents. As with CDP and GAs, this approach does not guarantee that the optimal solution will be found, but it does provide high-quality suboptimal solutions and, above all, has shown very good performance when applied to realistic distribution networks.

The paper is organized as follows. Section III presents the most important aspects of the proposed methodology, including the computation of branch currents through the Newton method, the search procedure toward the final solution, and the enforcement of radiality and capacity constraints. Section IV reports the application of the methodology in three electrical systems, including a 96-branch, 28-switch validation system and a 1128-branch, 129-switch distribution system. Finally, Section V presents the conclusion of the paper and discusses some directions for further development.

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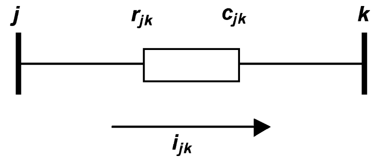

Fig. 1. Branch representation.

II. Methodology

This section presents the most relevant aspects concerning the proposed methodology. A detailed discussion on the radiality constraint is included in order to give a clear understanding of its high complexity as well as to point out the simplicity of the search procedure used for defining the state of switches. It is also shown how two potential difficulties associated with the standard Newton method, namely, the existence of local minima and the singularity of the Hessian matrix, do not affect the solution of the loss minimization problem.

A. Branch Modeling

Fig. 1 shows the representation of branches used in this paper.

The capacity factor c_{jk}, which is constant for each branch, is defined as

$$c_{jk} = \frac{I_{\text{maxbranch}}}{I_{\text{base}}} \tag{1}$$

where $I_{\text{maxbranch}}$ is the maximum current of branch jk (A), and I_{base} is the base current (A) of the per-unit system adopted. Also, the branch current in p.u. is defined as the actual branch current in ampère as a fraction of its maximum current

$$i_{jk} = \frac{I_{jk}}{I_{\text{maxbranch}}}. \tag{2}$$

From (1) and (2), it follows that the branch current in p.u. of the per-unit system is $c_{jk} \cdot i_{jk}$. These definitions allow the capacity constraint for branch jk to be expressed simply as

$$i_{jk}^2 \leq 1. \tag{3}$$

B. Computation of Branch Currents

In this paper, all switches are initially set to the state "closed." Through a step-by-step procedure, one switch is opened at each step (the criterion for choosing the switch to be opened will be presented later). This constitutes a "destructive strategy" because the electrical network evolves from the initial fully meshed state to a guaranteed radial configuration after the opening of the last switch.

At each step, branch currents are computed to minimize total loss in the current network configuration. To this end, the constrained optimization problem is converted into an unconstrained problem through the Method of Exterior Penalties [12], by which penalty terms associated with the constraints are added to the original objective function. The extra "cost" added by the penalty terms drives the solution to a point where the original objective function is minimized, and all constraints are satisfied.

Equation (4) shows the objective function defined for the loss minimization problem for a given switch profile

$$\min\ E(\widetilde{i}) = L \cdot \sum_{jk\in\Omega_B} r_{jk} \cdot c_{jk}^2 \cdot i_{jk}^2 + K \cdot \sum_p \left(\sum_{jk\in\Omega_{Bp}} c_{jk} \cdot i_{jk} + I_p \right)^2. \tag{4}$$

In this equation, $\widetilde{i}$ represents the vector of all branch currents, whose value is to be determined. $E(\widetilde{i})$ represents the function to be minimized, which takes into account the total loss and the penalty terms. Parameters L and K allow the relative weight of the loss term and the penalty term to be controlled and are easily determined upon experimentation. The second term in (4) represents the composite contribution of Kirchhoff's Current Law (KCL) applied to all load busses p; it is squared because both negative and positive current mismatches have to be made equal to zero. For clarity purposes, the capacity constraint is not being introduced here; it will be included later in this section.

The capacity factor of branch jk in (4) is negative when referring to the current *leaving* bus j and is positive when referring to the current *entering* bus k. Hence, the current sign (negative or positive) in KCL is transferred from the current i_{jk} to the constant capacity factor c_{jk}. All terms in KCL equations now have a positive sign, which greatly simplifies the computation of derivatives in the Newton method.

Problem formulation (4) is solved through the standard Newton method [12]. The update rule for branch currents is given by

$$\widetilde{i}^{(k+1)} = \widetilde{i}^{(k)} - \left[\nabla^2 E(\widetilde{i}^{(k)})\right]^{-1} \cdot \nabla E(\widetilde{i}^{(k)}) \tag{5}$$

where

$$\nabla E(\widetilde{i}^{(k)}) = \begin{bmatrix} \frac{\partial E}{\partial i_1} & \frac{\partial E}{\partial i_2} & \cdots & \frac{\partial E}{\partial i_n} \end{bmatrix}^t$$

is the gradient vector

$$\nabla^2 E(\widetilde{i}^{(k)}) = \begin{bmatrix} \frac{\partial^2 E}{\partial i_1^2} & \frac{\partial^2 E}{\partial i_2 \partial i_1} & \cdots & \frac{\partial^2 E}{\partial i_n \partial i_1} \\ \frac{\partial^2 E}{\partial i_1 \partial i_2} & \frac{\partial^2 E}{\partial i_2^2} & \cdots & \frac{\partial^2 E}{\partial i_n \partial i_2} \\ \cdots & \cdots & \cdots & \cdots \\ \frac{\partial^2 E}{\partial i_1 \partial i_n} & \frac{\partial^2 E}{\partial i_2 \partial i_n} & \cdots & \frac{\partial^2 E}{\partial i_n^2} \end{bmatrix}$$

is the Hessian matrix, and k is the iteration count.

Equation (5) arises from the expansion of function $E(\widetilde{i})$ around an operating point using first- and second-order terms and then minimizing the approximation error. The equation is applied iteratively until the difference between the last two instances of the branch current vector is less than or equal to a prespecified tolerance.

It should be noted that formulation (4) is quadratic. This makes the standard Newton method a prime candidate for solving the problem, since it can be shown that convergence in the Newton method is achieved in just one iteration when the problem is quadratic [12] (note that the Hessian matrix is constant in such case).

As the penalty terms in (4) arise from the application of KCL to all load nodes, it follows that the Hessian matrix reflects the

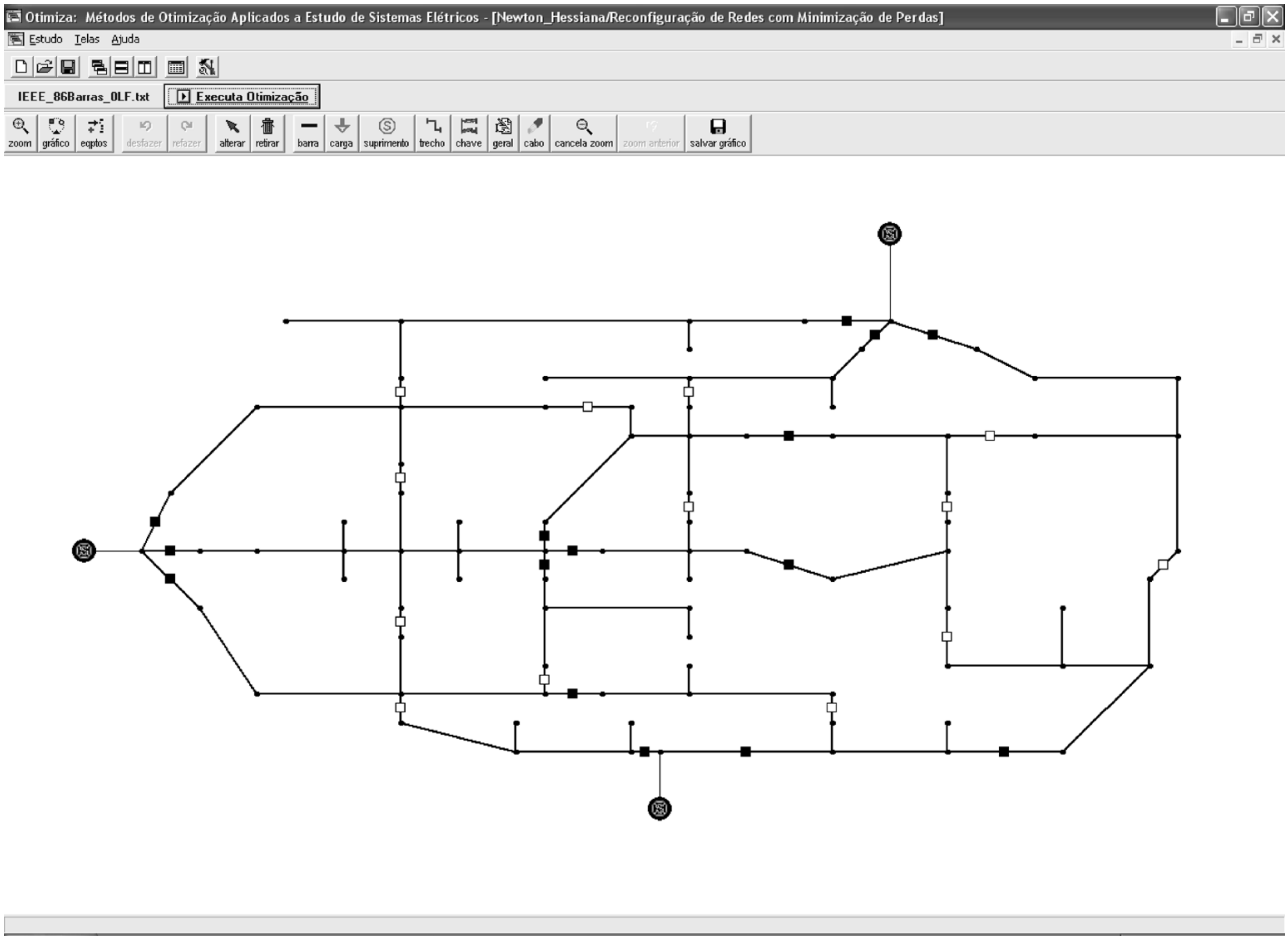

Fig. 2. Ninety-six-branch, 28-switch validation system.

topology of the electrical network. For this reason, the Hessian matrix is sparse, a feature that is fully exploited in the computational implementation.

C. Radiality Constraint

A radial network containing n_L load nodes must have exactly n_L conducting branches (nonswitch branches plus branches with closed switch), regardless of the number of supply nodes. This property allows easy computation of the number of closed switches (n_{CS})

$$n_{CS} = n_L - n_{jk} \tag{6}$$

where n_{jk} indicates the number of nonswitch branches.

Equation (6) is frequently used to enforce the radiality constraint in distribution system optimization problems. Unfortunately, this expression is a necessary but not sufficient condition for radiality. This will be illustrated through a 96-branch validation system, which is shown in Fig. 2.

The number of load busses in this system is 83, the number of nonswitch branches is 68, and the number of switches is 28. From (6), the number of switches that must remain closed is $n_{CS} = 83 - 68 = 15$. Consequently, the number of switches that must remain open is $28 - 15 = 13$.

TABLE I
ANALYSIS OF THE SOLUTION SPACE FOR THE 96-BRANCH SYSTEM

Item	Value
Total number of solutions	$2^{28} = 268{,}435{,}456$
Number of feasible solutions	853,158
Optimal solution loss (kW)	1,251.55
Worst solution loss (kW)	70,211.84
Average loss - all feasible solutions (kW)	10,592.21
Average loss - 1000 best feasible solutions (kW)	1,541.14
Number of solutions with 13 open switches and 15 closed switches	C_{15}^{28} 37,442,160
Processing time (2.66 GHz processor)	7 hours approx.

An exhaustive search procedure was implemented in order to assess the robustness of (6) and also to determine the optimal and suboptimal solutions for validation purposes. Table I summarizes the analysis of the system of Fig. 2 through exhaustive search.

In this table, "feasible solution" refers to a network state with neither disconnected busses nor meshes (i.e., a proper radial network). All feasible solutions (853 158) possess 13 open switches and 15 closed switches, as established by the necessary condition (6), but they account for only 2.3% of all solutions with 13 open switches and 15 closed switches (37 442 160). This means that the remaining solutions (36 589 002, or 97.7%) correspond to networks with disconnected busses and meshes. From this analysis, it becomes clear that enforcing the radiality constraint through (6) is inadequate, and therefore, other alternatives should be sought.

D. Best-First Search

It would be highly desirable if the radiality constraint could be expressed in analytical form, and moreover, it had readily obtainable derivatives. Such a function would be easily integrated into formulation (4) for solution through the Newton method. It turns out, however, that this function is very difficult to obtain, because a topological analysis (search) has to be performed on the network to determine the radiality/nonradiality of a given switch profile.

In this paper, a simple best-first search procedure was devised for guiding the switch opening procedure. Initially, all switches are set to the "closed" state, so the network operates in meshed mode. Equation (5) is solved to determine the current in all branches and the total loss [which is actually the minimum loss due to problem formulation (4)].

Next, a candidate switch is selected for opening. The criterion for selecting a switch is precisely the least increase in total loss that the switch opening would cause (two different methods for estimating that increase were developed; both are discussed below). It should be noted that a switch opening will never imply a decrease in total loss, because an additional constraint on flow distribution is imposed through the switch opening (and there cannot be a better solution with an additional constraint).

Once a switch has been selected for opening, a topological analysis is carried out to determine whether or not this opening will produce a network with disconnected busses. If the opening does not imply a disconnected network, the switch is effectively opened, and a new step is initiated (whereby the next switch to be opened will be selected); otherwise, the selected switch is abandoned, and the next one in the sorted list of candidate switches is selected for analysis.

The process stops when exactly n_{CS} switches are still closed ($n_{CS} = 15$ in the system of Fig. 2). At this point, the necessary condition (6) is satisfied, and because no switch opening leading to a disconnected network was performed, there are no disconnected busses in the solution (and, therefore, there are no meshes either).

1) Loss Increase Estimation—Method A: At each step during the switch opening procedure, the candidate switches (i.e., all remaining closed switches) are analyzed and sorted in ascending order of the loss increase that their opening would produce. Method A provides a fast, approximate estimation of the loss increase through the quadratic approximation of the loss function around the current operating point

$$E(\widetilde{i}+\Delta\widetilde{i})-E(\widetilde{i}) = \left(\Delta\widetilde{i}^t \cdot \nabla E\right)+\left(\frac{1}{2}\Delta\widetilde{i}^t \cdot \nabla^2 E \cdot \Delta\widetilde{i}\right) \quad (7)$$

where $E(\widetilde{i})$ is the total loss for the current switch state, and $\Delta\widetilde{i}$ indicates the vector of current changes due to the opening of a candidate switch. In order to compute the whole vector $\Delta\widetilde{i}$, a full load-flow calculation should be performed, but it is assumed instead that only the branch mn, whose switch is being opened, will experience a change in current. Therefore, all elements in vector $\Delta\widetilde{i}$ will be zero, except the element mn: Its value will be the difference between zero (the final current after the switch opening) and the initial current (before the opening).

With this approximation, the vector-vector and matrix-vector multiplications in (7) become a simple scalar operation (see Appendix)

$$E(\widetilde{i}+\Delta\widetilde{i})-E(\widetilde{i}) = (-i_{mn} \cdot G_{mn})+\left(\frac{1}{2}i_{mn}^2 \cdot H_{mn,mn}\right) \quad (8)$$

where i_{mn} is the current through branch mn before its opening, G_{mn} is the corresponding element of the gradient vector, and $H_{mn,mn}$ is the corresponding diagonal element of the Hessian matrix. Estimation of the loss increase using (8) is very fast since both the gradient vector and the Hessian matrix are available at no extra computational cost because of the last load-flow calculation.

2) Loss Increase Estimation—Method B: This method provides an exact value for the loss increase caused by the opening of a given switch. The first n candidate switches, already ordered by the minimum loss increase criterion (8), are opened one at a time. For each temporarily open switch, a load-flow calculation is carried out through (5), thus yielding the exact value of the loss increase. The switch with the minimum loss increase is selected for permanent opening, provided that its operation does not imply a disconnected network.

It should be noted that parameter n above is set in advance by the user. It can vary from 1 (in which case, the algorithm behaves as in Method A) to the number of candidate switches at the current step. This parameter provides a convenient control over the compromise between the quality of the solution and processing speed (the higher this parameter, the more likely that a better solution will be found, at the expense of greater processing time). However, fixing this parameter in advance is a difficult task. In the application cases presented in the next section, its value was established through experimentation.

It should also be pointed out that in Method B, the approximate loss increase obtained through (8) is only used for presorting the candidate switches and not for selecting the best solution at that step. If the user specifies a sufficiently large value for parameter n (greater than or equal to the number of candidate switches), then the local best solution will be guaranteed because the exact value of the loss increase is obtained through a series of exact load-flow calculations and not through approximation (8).

E. Capacity Constraint

Problem formulation (4) was modified to incorporate capacity constraints in some or all branches, thus avoiding solutions with low values of loss but with unacceptably high overloads. During the iterative solution of (4), each branch is checked against overload according to (3). If a branch jk is found to be overloaded, the following extra penalty term is added to (4) to redirect the solution to a nonoverload solution

$$C \cdot f(i_{jk}) \tag{9}$$

where C is a parameter that allows controlling the relative weight of the capacity constraint in the full objective function, and function $f(.)$ is given by

$$f(i_{jk}) = \begin{cases} (i_{jk}+1)^2, & \text{if } i_{jk} < -1 \\ 0, & \text{if } -1 \le i_{jk} \le +1 \\ (i_{jk}-1)^2, & \text{if } i_{jk} > +1. \end{cases} \tag{10}$$

By using the extra penalty terms (9) and (10), the problem formulation retains its quadratic nature.

F. Search Strategy Improvement

A simple but efficient improvement was incorporated into the basic search described earlier. If, during the topological analysis of a candidate switch to be opened at a given step, it is found that the opening of that switch leads to a disconnected network, then the switch opening is blocked in all subsequent steps (the topological analysis for the switch will not be repeated unnecessarily). This is a valid measure because of the destructive nature of the procedure (i.e., switches are always opened and never closed): If a switch opening implies a disconnected network at a given step, its opening in any future step will certainly lead to the same situation. Significant processing time savings with this improvement were obtained for the real-world distribution system, because in this case, the topological analysis is responsible for a considerable fraction of the total processing time.

G. Problem Convexity and Hessian Matrix Singularity

It is well known that the most important drawbacks of the standard Newton method are 1) local minima in the objective function and 2) Hessian matrix singularity. As with all derivative-based methods, the existence of local minima can make the standard Newton method converge to rather poor solutions. The existence of local minima is associated with the convexity of the problem under consideration. If a quadratic problem is convex, then it does not have local minima—only the desired global minimum. The convexity of a given quadratic problem can be determined by analyzing its Hessian matrix. If the Hessian matrix is positive definite (all eigenvalues strictly positive), then the problem is convex, and the global minimum is unique. If the Hessian matrix is positive semi-definite (at least one eigenvalue equal to zero and the remaining eigenvalues positive), then the function does not have local minima, but the global minimum is not unique. The computation of the eigenvalues of the Hessian matrix was incorporated into the computational implementation to verify the validity of the solutions found by the algorithm.

Several modifications to the standard Newton method have been developed to overcome the drawbacks mentioned above [12]. For instance, the stochastic gradient is aimed at nonconvex problems and the Marquardt–Levenberg regularization avoids Hessian matrix singularities. These two techniques were incorporated into the proposed methodology, but none of them were necessary since the Hessian matrix was always found to be positive definite (and, hence, nonsingular).

1. Initialization: all switches are closed.
2. Stopping condition: if exactly n_{CS} switches (6) are still closed, the search is terminated. The current system state yields the desired solution.
3. Loadflow calculation with loss minimization (5). During the iterative procedure, branches are checked for overloads and penalty terms given by (9) and (10) are added if necessary, thus avoiding overloads.
4. A candidate switch (i.e., a closed switch) is selected for opening, using either Method A or Method B.
5. Connectivity check: if the opening of the selected switch in Step 4 implies a disconnected network, return to Step 4 for selecting the following switch in the ordered list of candidate switches; otherwise go to Step 6.
6. The switch selected in Step 4 is effectively opened.
7. Return to Step 2.

Fig. 3. Overall procedure.

H. Summary

The block diagram in Fig. 3 summarizes the solution procedure for the proposed approach.

III. Results

The methodology described in the preceding section was implemented as a computational program, which also includes an easy-to-use graphical user interface and a customizable interface for accessing corporate databases from electrical utilities. This section presents and discusses results from the application of the methodology to three electrical systems (Electrical System 1, 2, and 3).

A. Electrical System 1

This electrical system is a modification of the 37-branch test system used by Baran and Wu [5]. In that work, the branch-exchange approach sought to open a conducting branch (either having or not having a closed switch) in exchange for the closing of an open switch (tie line). On the other hand, the methodology developed in the present paper requires that only switch-equipped branches can have their state open/closed changed. For this reason, additional closed switches were defined in the Baran and Wu test system for all branches except branches 0–1, 4–5, 16–17, 20–21, 23–24, 26–27, and 31–32. These branches were excluded because of the 32-bit limit for integer variables imposed by the programming environment. This limit only applies to the exhaustive search implementation, which was used in this case (if every branch had a switch, there would be 2^{37} possible solutions, which is beyond the limit). The total number of switches is now 30 (25 closed switches plus five tie lines), thus yielding 2^{30} possible solutions.

Table II shows some relevant results obtained for Electrical System 1, obtained from both exhaustive search and the proposed methodology.

The solution index in Table II is the relative position that the solution takes in the ordered list of solutions determined by exhaustive search (the optimal solution has index 1). The proposed

TABLE II
RESULTS FOR ELECTRICAL SYSTEM 1 ([1] INDICATES VALUE FROM EXHAUSTIVE SEARCH; [2] INDICATES VALUE FROM THE PROPOSED APPROACH)

Parameter	Value
Total number of solutions [1]	2^{30} = 1,073,741,824
Number of feasible solutions [1]	22,262
Optimal solution loss (kW) [1]	139.10
Worst solution loss (kW) [1]	1,290.81
Average loss - all feasible solutions (kW) [1]	410.47
Number of solutions with 5 open switches and 25 closed switches [1]	C_{25}^{30} = 142,506
Optimal solution - open switches [1]	6-7 8-9 13-14 24-28 30-31
Initial configuration loss (before optimization) (kW) [2]	194.53
Solution index (after optimization) [2]	5
Loss (after optimization) (kW) [2]	142.03
Loss difference w.r.t. the optimal solution	2.93 kW / 2.1%
Processing time (s) (2.66 GHz) [2]	0.01
Incorrectly opened switches (w.r.t. the optimal solution)	17-32 27-28
Incorrectly closed switches (w.r.t. the optimal solution)	24-28 30-31

TABLE III
RESULTS FOR ELECTRICAL SYSTEM 2

Parameter	Method A	Method B
Optimal solution loss (exhaustive search) (kW)	1,251.55	
Solution index	19	2
Loss (kW)	1,320.10	1,253.24
Loss difference w.r.t. the optimal solution	68.55 kW / 5.5%	1.69 kW / 0.1%
N° of loadflow calculations	14	66
Processing time (s) (2.66 GHz)	0.16	0.18
Incorrectly opened switches (w.r.t. the optimal solution)	23 - 26 38 - 64	23 - 26
Incorrectly closed switches (w.r.t. the optimal solution)	20 - 36 23 - 25	20 - 36

approach found Solution 5 regardless of the method used for estimating the loss increase at each step (Method A or Method B, as discussed in the preceding section). This solution means a loss 2.1% higher than the optimal solution loss.

B. Electrical System 2

This electrical system, shown in Fig. 2, contains 83 load busses, three supply busses, 68 nonswitch branches, and 28 switch branches.

As discussed earlier, a radial solution in this system must have 13 open switches and 15 closed switches. The best-first procedure consists of 14 steps, one for the initial load-flow calculation (all switches closed) plus 13 opening operations.

Table III shows some relevant results obtained for Electrical System 2.

The number of load-flow calculations (Method B) in Table III was set to four. No better solution was found with greater values for this parameter.

Method A yielded the nineteenth solution (with a difference of 68.55 kW with respect to the optimal solution), whereas Method B produced the second solution (difference of 1.69 kW). Due to the relatively small size of the system, the processing time in both cases is approximately the same.

In Solution 2, a wrong switch opening (switch 23–26) occurred in step 12. In that step, the switch that should have been opened (20–36) ranked fourth in the list of ordered switches after all load-flow calculations were performed, and so it was not selected for opening. This underlines the main limitation of the best-first search: As it makes decisions based solely on information available locally (i.e., the current step), it cannot forecast that a potentially wrong decision is being made. On the other hand, this situation tends to occur in steps near the end, when the differences in terms of total loss among all switching alternatives are relatively low. For this reason, the suboptimal solutions obtained with this methodology tend to be of high quality, very close to the optimal solution.

C. Electrical System 3

Fig. 4 shows a real-world distribution system, referred to as Electrical System 3. It consists of five primary feeders with mixed urban–rural features, totaling 1107 load busses, five supply busses, 999 nonswitch branches, and 129 switch branches. A radial solution for this system must have 108 closed switches (= 1,107 − 999) and, therefore, 21 open switches.

Table IV shows some relevant results obtained for Electrical System 3. In this initial analysis, no capacity constraint was enforced. The normal radial operating state of the system was known in advance, thus allowing the computation of the corresponding total loss and its comparison with total loss after optimization.

It can be seen from Table IV that a significant reduction in total loss was achieved through Methods A and B (21.1% and 29.4%, respectively). Also, both solutions are quite similar (switches that were operated by only one of the methods appear in bold characters in the table).

As for the processing time, Method B is significantly more time consuming, as expected. In this case, the number of load-flow calculations at each step was set to 15 (again, no better solution was obtained with a greater number).

Finally, a further analysis was carried out, taking into account the capacity constraint. The maximum outcoming current at feeder F5 was reduced from 400 to 250 A to force the program to find another solution. This analysis used only Method B; the results are presented in Table V.

Table V shows that the solution in this case is almost the same as in Table IV—one less switch was opened and one less closed. The total loss increased as expected, from 129.74 to 136.38 kW.

Table VI shows detailed current values for the five feeders in all Method B cases. It becomes apparent that the capacity constraint caused a load group to be transferred from feeder F5 to feeder F4.

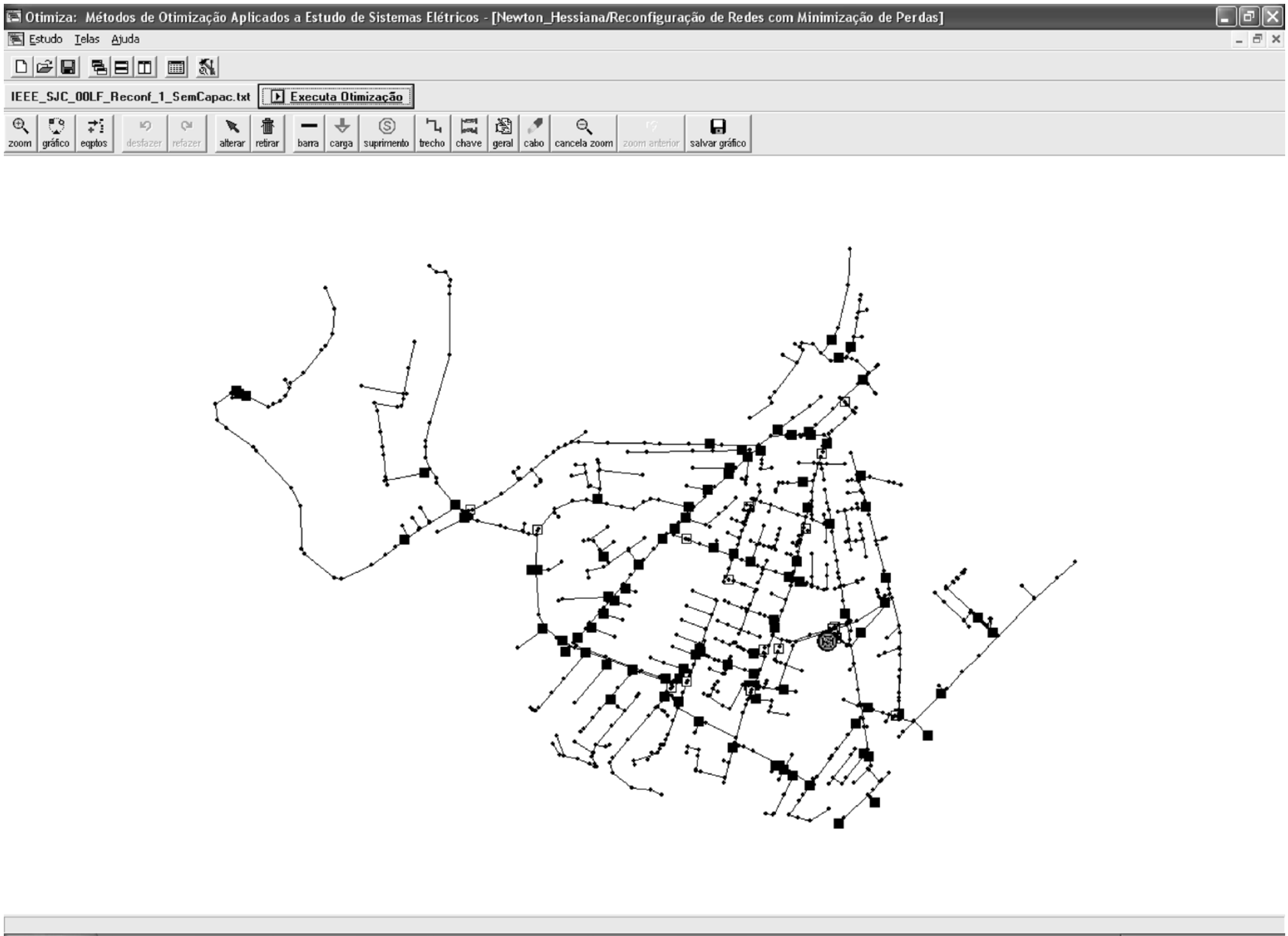

Fig. 4. Electrical System 3: 1128 branches, 129 switches.

TABLE IV
RESULTS FOR ELECTRICAL SYSTEM 3 (NO CAPACITY CONSTRAINT)

Parameter	Method A	Method B
Loss for initial state (kW)	183.83	
Loss after optimization (kW)	144.96	129.74
Loss reduction by optimization (% of initial state loss)	21.1	29.4
N° of loadflow calculations	22	330
Processing time (s) (2.66 GHz)	4.98	32.92
Switches opened by optimization (w.r.t. initial state)	3625-3634 3736-3779 **3750-3751** **4275-4303** 4320-4354 4394-4400 4457-4539 4677-4678	3625-3634 3736-3779 **4034-4231** 4320-4354 4394-4400 4457-4539 4677-4678
Switches closed by optimization (w.r.t. initial state)	3584-4690 3600-3781 3616-3791 3641-4399 3724-4631 **3847-4387** 4024-4363 4313-4348	3584-4690 3600-3781 3616-3791 3641-4399 3724-4631 4024-4363 4313-4348

TABLE V
RESULTS FOR ELECTRICAL SYSTEM 3 (CAPACITY CONSTRAINT ENFORCED + METHOD B)

Parameter	Value
Loss for initial state (kW)	183.83
Loss after optimization (kW)	136.38
Loss reduction by optimization (% of initial state loss)	25.8
N° of loadflow calculations	330
Processing time (s) (2.66 GHz)	42.54
Switches opened by optimization (w.r.t. initial state)	3625-3634 3736-3779 4320-4354 4394-4400 4457-4539 4677-4678
Switches closed by optimization (w.r.t. initial state)	3584-4690 3600-3781 3616-3791 3641-4399 3724-4631 4313-4348

IV. CONCLUSION

This paper has presented a methodology for reconfiguring distribution systems considering loss minimization. This

TABLE VI
FEEDER CURRENTS FOR ELECTRICAL SYSTEM 3 (METHOD B)

Feeder		Initial state	Optimization WITHOUT capacity constraint	Optimization WITH capacity constraint
F1	Feeder outcoming current (A) →	163.18	221.40	221.40
F2		136.72	253.47	253.47
F3		79.47	79.47	79.47
F4		252.94	38.25	252.94
F5		267.98	307.72	93.02
	Loss (kW) →	183.83	129.74	136.38

problem is of a combinatorial nature, which means that considerable difficulties arise when solving problems with a large number of decision variables.

The technique for determining the value of the integer variables (best-first search) does not guarantee that the global minimum will be achieved, but on the other hand, the search procedure is extremely fast. As the suboptimal solutions that can be found are of high quality, the proposed approach exhibits considerable potential for routine use by electrical utilities. Application to a real-size distribution system allowed a saving of over 20% in total loss with just a few seconds of processing time and a low number of switch operations, which shows its potential for online applications.

The standard Newton method was used to compute the branch current profile at each step along the integer search. This method is particularly well suited to the reconfiguration problem because of the quadratic formulation adopted: Only one iteration is required to solve the load-flow computation. The main drawback of the standard Newton method, namely, local minima, has not posed any difficulty since the quadratic optimization problem appears to be convex and, therefore, free of local minima. Further research is being developed to determine under which conditions convexity occurs.

Within the framework of quadratic formulations, other objective functions, such as voltage profile optimization, can be formulated as well. Another aspect that is worth of looking into is the determination of the number of load-flow calculations in Method B, possibly related to network size. These lines are also the subject of ongoing research.

APPENDIX

This appendix shows how (8) is derived from (7):

$$
\begin{aligned}
&\left(\Delta\tilde{i}^t \cdot \nabla E\right) + \left(\frac{1}{2}\Delta\tilde{i}^t \cdot \nabla^2 E \cdot \Delta\tilde{i}\right) \\
&\quad = [0 \quad \ldots \quad -i_{mn} \quad \ldots \quad 0] \\
&\qquad \cdot \begin{bmatrix} * \\ \ldots \\ G_{mn} \\ \ldots \\ * \end{bmatrix} + \frac{1}{2} \cdot [0 \quad \ldots \quad -i_{mn} \quad \ldots \quad 0] \\
&\qquad \cdot \begin{bmatrix} * & \ldots & * & \ldots & * \\ \ldots & \ldots & \ldots & \ldots & \ldots \\ * & \ldots & H_{mn,mn} & \ldots & * \\ \ldots & \ldots & \ldots & \ldots & \ldots \\ * & \ldots & * & \ldots & * \end{bmatrix} \cdot \begin{bmatrix} 0 \\ \ldots \\ -i_{mn} \\ \ldots \\ 0 \end{bmatrix} \\
&\quad = (-i_{mn} \cdot G_{mn}) + \left(\frac{1}{2} i_{mn}^2 \cdot H_{mn,mn}\right).
\end{aligned}
$$

REFERENCES

[1] F. S. Hillier and G. J. Lieberman, *Introduction to Operations Research*. New York: McGraw-Hill, 1997.

[2] A. Merlin and H. Back, "Search for a minimal-loss operating spanning tree configuration in an urban power distribution system," in *Proc. 5th Power Syst. Comput. Conf.*, vol. 1, Cambridge, U.K., 1975, pp. 1–18.

[3] S. Cinvalar, J. J. Grainger, H. Yin, and S. S. H. Lee, "Distribution feeder reconfiguration for loss reduction," *IEEE Trans. Power Del.*, vol. 3, no. 3, pp. 1217–1223, Jul. 1988.

[4] C. C. Liu, S. J. Lee, and K. Vu, "Loss minimization of distribution feeders: Optimality and algorithms," *IEEE Trans. Power Del.*, vol. 4, no. 2, pp. 1281–1289, Apr. 1989.

[5] M. E. Baran and F. F. Wu, "Network reconfiguration in distribution systems for loss reduction and load balancing," *IEEE Trans. Power Del.*, vol. 4, no. 2, pp. 1401–1407, Apr. 1989.

[6] S. K. Goswami and S. K. Basu, "A new algorithm for the reconfiguration of distribution feeders for loss minimization," *IEEE Trans. Power Del.*, vol. 7, no. 3, pp. 1484–1491, Jul. 1992.

[7] C. S. Chen and M. Y. Cho, "Energy loss reduction by critical switches," *IEEE Trans. Power Del.*, vol. 8, no. 3, pp. 1246–1253, Jul. 1993.

[8] R. Cherkaoui, A. Bart, and A. J. Germond, "Optimal configuration of electrical distribution networks using heuristic methods," in *Proc. 11th Power Syst. Comput. Conf.*, vol. 1, Avignon, France, 1993, pp. 147–154.

[9] H. C. Chang and C. C. Kuo, "Network reconfiguration in distribution systems using simulated annealing," *Elect. Power Syst. Res.*, vol. 29, pp. 227–238, 1994.

[10] C. C. B. de Oliveira and N. Kagan, "Heuristic model for the selection and allocation of shunt capacitors and voltage regulators in electrical power distribution systems," in *Proc. Int. Conf. Intell. Syst. Appl. Power Syst.*, Rio de Janeiro, Brazil, 1999, pp. 175–179.

[11] N. Kagan and C. C. B. de Oliveira, "Fuzzy decision model for the reconfiguration of distribution networks using genetic algorithms," in *Proc. 13th Power Syst. Comput. Conf.*, Trondheim, Norway, 1999.

[12] A. Cichocki and R. Unbehauen, *Neural Networks for Optimization and Signal Processing*. New York: Wiley, 1993.

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