A Variable Local Relaxation Technique in Non-linear Problems

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Abstract **- Application of under and over-relaxation is a very useful technique in solution of non-linear problems. However, the choice of the relaxation factor is problem dependent and difficult to estimate. In this work we propose a new method in which the relaxation factor is chosen automatically by the computer code, based on the dynamics of the convergence process. In addition, the local application of relaxation factors is introduced, so that different unknowns have different relaxation factors. Two examples are presented to demonstrate the proposed method's efficiency.**

I. INTRODUCTION

First, we establish how under- and over-relaxation are defined in this work. In the following equation:

$$a(i+1) = a(i+1) + R\left[a(i+1) - a(i)\right] \qquad (1)$$

a represents the unknown vector potential at a node or on an edge as will be shown in examples. The index *(i+1)* and *(i)* represent iterations "*i+1*" and "*i*" created during the convergence process. The term in square brackets is the change in the unknown between two successive iterations. If *R* is zero, no relaxation is applied; under-relaxation is obtained for "*-1<R<0*" and over-relaxation for "*0<R<1*".

It is well known that, in many cases, the use of relaxation factors to accelerate convergence is very efficient [1,2]. In fact, some applications require relaxation to converge. However, it is difficult to determine an optimal value for relaxation, mainly because it depends on the case to be solved. For instance, in scalar potential applications, over-relaxation gives good acceleration towards the solution. On the other hand, the use of vector potential very often requires under-relaxation [3]. For the scalar potential this is because in the curve $\mu(H^2)$, H always increases regardless of the saturation level. In using the vector potential with reluctivity curve of the form $\nu(B^2)$, the change in B after saturation is quite small for large changes in ν. As a result, the convergence process can be unstable, and under-relaxation can be very useful and even necessary.

In this work we propose a method of self-adjustment of the relaxation factor, which takes into account the dynamics of the ongoing convergence. Also, because different unknowns behave differently, we will introduce the application of a local relaxation factor, in which each unknown has its own relaxation factor, which is continually adjusted during the process.

II. DESCRIPTION OF THE METHOD

The basic concept of this method is to allocate one value of R to each unknown. Thus, R becomes an array with the same dimension as the unknown vector rather than a single value. In this way, each value of R depends on the behavior of the unknown with which R is associated.

The method follows these rules:
a) set for each unknown an initial R_0 and define the increment dR
b) for each unknown calculate the following two differences:

$$\Delta(i-1) = a(i-1) - a(i-2) \qquad (2)$$

$$\Delta(i) = a(i) - a(i-1) \qquad (3)$$

c) calculate the product $\Delta(i) * \Delta(i-1)$";
d) if the product is negative, oscillation in convergence occurred: in this case set:

$$a(i) = a(i-1), \qquad \text{and} \qquad R = R_{osc} \qquad (4)$$

e) if the product is positive, there is no oscillation and use:

$$R = R + dR \qquad (5)$$

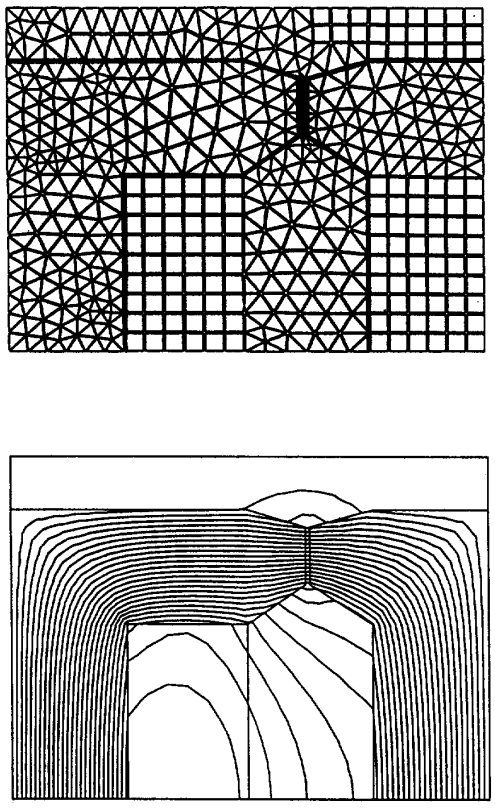
R_{osc} is the value affected if the unknown had oscillated. Normally it is smaller than R_o. As soon as an oscillation is detected, the value for the corresponding unknown is set to its previous value and R is taken as R_{osc}, forcing a restart with a stronger under-relaxation. If there is no oscillation R increases slowly towards over-relaxation. Thus, the unknowns use different relaxation levels, determined by their different behavior. Typical values are: R_o=−0.6, dR=0.05 and R_{osc}=−0.7. These values are a matter of choice and depend on application. A value of R_o=−0.7 is sufficiently small for most applications. If convergence is good, larger values can be used. A set of parameters can usually be used for a whole class of problems.

III. EXAMPLES

To present some results, two examples using different geometries and formulations are shown. The first is related to a 2D vector potential application using nodal finite elements and the Newton-Raphson method. The second example is a 3D vector potential case, solved with hexahedral edge elements and the successive approximation method for non-linearity.

A. Example 1

The structure and mesh are shown in Fig. 1a and the field plot in Fig. 1b. where the iron parts use the B(H) curve of TEAM problem number 20 [4]. The magnetic structure is coupled with the external electrical circuit fed by a voltage source. The coupled problem has its electrical and magnetic parts solved simultaneously, giving, as results, the potential on the nodes and the current in the coil [5]. This structure has been chosen because convergence is difficult to obtain. Among other reasons for this, the iron regions close to the airgap have narrow sections and there is a coupling with the external electric circuit. Also, because of the inductive character of the circuit, currents established in the coil vary from small to large values, covering many different regions on the B(H) curve. Gauss elimination is used to solve the matrix system.



a. b.

Fig. 1. a. Example 1: FE mesh. b. field plot.

Among many numerical results we will give those directly related to the work presented here. The results in Table 1 were obtained with a voltage pulse of 30 Volts applied to the coil. We call the present method Variable Local Relaxation (VLR) and the classical relaxation method Fixed Relaxation (FR) .

Table 1. Number of iterations for various time steps for the fixed relaxation (FR) and variable local relaxation (VLR) methods. NR=Newton-Raphson iterations. SA=Successive approximation.

Time step(s)	FR	VLR
0.001	31 (NR)	21 (NR)
0.002	27 (NR)	25 (NR)
0.003	24 (NR)	19 (NR)
0.004	21 (NR)	22 (NR)
0.005	40 (SA)	26 (NR)

The number of iterations with VLR is somewhat smaller than for the classical FR. However, the most important remark concerns the fact that for t=0.005 seconds, Newton-Raphson converges only for VLR while the FR method does not convergence. Therefore, the method of successive approximations was employed to solve the non-linearity, with slightly less accurate results as expected. For this code, we have been using R_0=–0.83 for successive approximation method, when necessary, R_0=–0.1 for the Newton-Raphson method, R_{osc}=–0.75 and dR=0.05. As complementary results, we remark that for V=10 Volts, both, FR and VLR gave very good results, but for V=100 Volts, with very strong saturation, only VLR with the Newton-Raphson method converged.

B. Example 2

With the purpose of showing the flexibility of the method, it was also applied to a 3D vector potential problem. As mentioned before, TEAM problem number 20 is solved using hexahedral edge finite elements and a modified ICCG solver [4,6]. Fig. 2 shows the structure which is made of a magnetic circuit, a coil and an iron pole under a magnetic force. Fig. 3 shows the field plot in a quarter of the geometry. Since the calculations were performed on a personal computer (486-DX-66), the mesh is quite coarse. The results presented here are for the successive approximation method. For the FR method, the initial value of R is "–0.78", which, in our experience, gave the best convergence. For instance, for R_0=–0.83 convergence is obtained but much slower than for R_0=–0.78, even though these numbers seem to be close; for R_0=–0.70, strong oscillations were observed, and the process diverged. For VLR, we choose R_0=–0.60, R_{osc}=–0.85 and dR=0.05. As a result, with the FR method, convergence was reached very slowly and by the 40th iteration the average relative error was close to 10^{-3}. As is often the case with under-relaxation methods, the closer we get to convergence, the smaller the ratio of convergence becomes. Quicker convergence is obtained sing VLR. In Table 2, the same relative error was reached by the 22th iteration. Complete convergence (our criterion was a relative error smaller than 10^{-4} for all nodes) was obtained by the 36th iteration, giving an average error close to 0.3×10^{-5}.

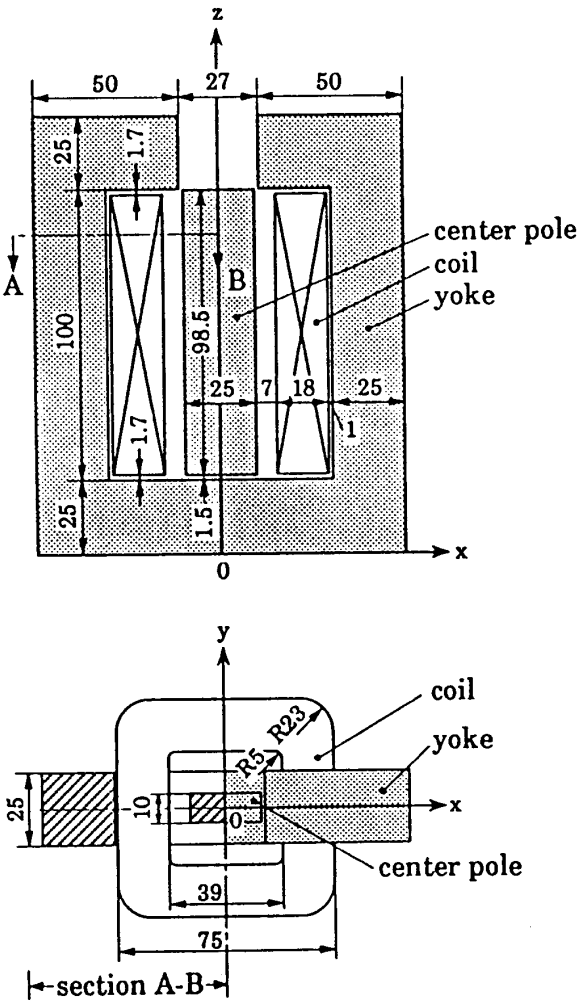


Fig. 2. Problem structure (TEAM Problem 20)

An interesting fact is noted in Table 2 with the VLR method: as the iterations are performed and the error diminishes, the average R, from the 10th to the 15th iteration decreased, contrarily to what happened before. This indicates that many unknowns had oscillated during these iterations. Their respective values of R were set as R_{osc}, producing an average R smaller than what we

should expect if there were no oscillations. This is the reason why the error continued to decrease.

Table 2. Errors for various iterations.

Iteration	Average R	Error,VLR	Error, FR
1	-0.60	0.43e-1	0.23e-1
5	-0.48	0.82e-2	0.74e-2
10	-0.28	0.20e-2	0.31e-2
15	-0.39	0.68e-3	0.16e-2
20	-0.29	0.22e-3	0.88e-3
25	-0.22	0.32e-4	0.50e-3
30	-0.21	0.11e-4	0.29e-3
36	-0.27	0.31e-5	0.16e-3
40	------	---------	0.11e-3

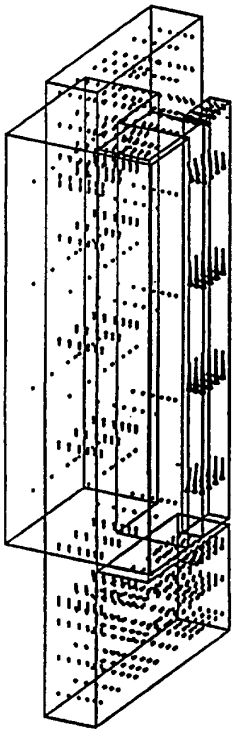


Fig. 3. Example 2: field plot.

One concern that arose during implementation of the proposed method was that the results are not modified. In the problem presented above, the final values of B, obtained by derivation of the vector potential did not differ between methods by more than 10^{-4}. This difference could also be attributed to differences in convergence.

IV. CONCLUSION

A local variable relaxation method was presented in this paper. With this technique the convergence process adjusts the relaxation factors of the unknowns automatically, taking into account their oscillations and their different behavior dynamically. In fact this method is not completely automatic, since initial parameters have to be set. However, when good initial values are chosen, the code will adjust the relaxation factors according to the case under calculation. We noticed that for problems exhibiting easy convergence the new method has little advantage compared to classical methods, but in some critical cases, where these methods fail to converge, the proposed method was able to solve them successfully.

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