Virtual Elements: A Smoothing Technique to Provide Continuity in FEM Discretized Results

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Abstract - A method of providing continuity for discretized results obtained in FEM calculations is presented. This consists in defining a very fine mesh in the post-processor section and calculating derivative quantities from the smoothed potential values. The fine mesh thus defined is called a virtual mesh. Noticeable improvements in values of fields and forces are obtained by this technique.

I. INTRODUCTION

The FEM provides results that are highly dependent on mesh quality. If the values of potentials are relatively consistent in different meshes, their derivative quantities (fields and forces) are susceptible to large variations if the mesh changes from a fine to a coarse mesh. In this paper a method of providing continuity for discretized results obtained in FEM calculations is presented. Two techniques are used for this purpose. The first is generation of virtual finite elements, by which is meant a set of elements generated in certain regions of the solution domain in the post-processing step of calculation. The second technique is smoothing of potentials obtained directly from the FEM, which will be applied to the virtual elements in order to calculate derivative quantities therefore improving accuracy of results. The combination of the two techniques to provide continuity of results as well as to improve the results of a relatively poor mesh is presented.
In order to show the efficiency of this technique two examples are given: the first is intended to show the improvement in field values and the second shows the improvement in force values obtained by the Maxwell tensor.

II. VIRTUAL ELEMENTS

The dashed lines in Fig. 1 indicate the "real mesh", generated in the pre-processor and used in the solver.

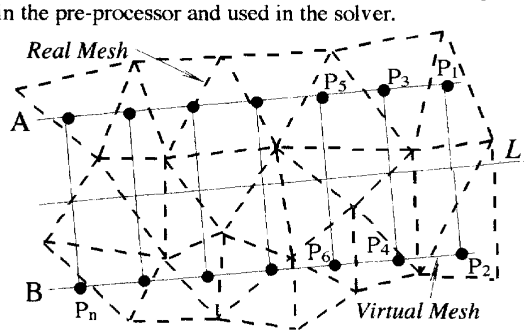

Fig. 1. Virtual elements and the real mesh.

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Suppose we are interested in physical results in the region delimited by lines A and B, placed close to line L, selected by the user and crossing only in air. It would be very useful if this region could be discretized using a highly refined mesh. Instead, it is possible to construct a "virtual mesh" similar to that shown in solid lines in Fig. 1. The method used to construct the virtual mesh is as follows:

- using the real mesh elements, by interpolation on potential FE functions, obtain the potentials at P_1, P_2,....P_n.
- with the coordinates of P_1, P_2,....P_n and the interpolated potentials, all data required are available

Using this technique a mesh of any density is defined in the post-processing step, without increasing computation time and memory allocation either for mesh generator or solver.

III. THE SMOOTHING TECHNIQUE

If a virtual element is completely inside the real element, the field provided by the virtual element will be identical to the field calculated in the real mesh, in spite of the fact that the virtual element could be much smaller than the real element. To improve the result, it is necessary to apply a correction to the potential values.

Normally, the potential values of two different meshes are quite close. The difficulty with poor meshes appears when the derivatives of potentials (for example: fields and forces) are needed [1,2]. Therefore, smoothing potential curves means that relatively small changes in the original curve are introduced. This provides assurance that the results of the calculation are not violated. For this reason smoothing is performed on potentials instead of fields.

Using only lines crossing through air (or airgaps) small discontinuities in A are generated due to discretization. These values can be modified slightly to obtain a closer representation to the physical results expected from such electromagnetic phenomena.

Among many methods, the least squares method was the most suitable for our purpose. This method yields a polynomial that provides the best fit to a given data by minimizing the difference between the given data and those obtained from the approximated curve [3]. A direct application of the method is not suitable for this problem, since the number of points required to correctly describe the potential may be very large (between 200 and 600 points). Furthermore, a single high order polynomial is not convenient since high order polynomials yield curves with strange shapes, without any relationship with the potential curves. The solution to this difficulty is in local application of the least squares method. Fig. 2 presents the technique by which this is accomplished.

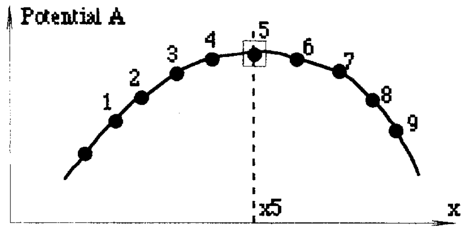


Fig. 2.

Suppose we wish to calculate the smoothed value at point 5. We consider two neighboring points on each side of point 5. These are points 3, 4, and 6, 7. Using the least squares method we obtain the coefficients of the polynomial curve F(x) from these points. Afterwards we calculate the new value for the potential at point 5 as F(x5). For point 6 we consider points 4 through 8 and so on.

Note also that it is possible to use different degrees for the polynomial F(x). According to several tests performed we obtained best results with first order polynomials and using four neighboring points (on each side of the smoothed point).

This procedure is applied for lines A and B of Figure 1. Thus we calculate the suitable potential values for points P1, P2,.., Pn. With these, the improved values of the fields for the virtual mesh are obtained.

Two methods were used to analyze results of the virtual elements and to compare these with the results of the real mesh. These are:

a) calculation of flux errors: two points on line L having potentials A_1 and A_2, give the flux crossing the line segment between these points as "A_1-A_2". It is also possible to calculate this flux using the sum of elementary fluxes given by the product of B.l along the segment. Two values of flux are calculated: using B from the real mesh and B from the virtual elements, which are then compared to the difference "A_1-A_2". It is expected that the integrated flux is close to this difference, showing consistency of the field values with the potentials that generate them. The flux errors for both, real and virtual elements were evaluated.

b) mesh comparisons: the second method is the comparison of results for several meshes for the same problem.

IV. EXAMPLE 1: CALCULATION OF FIELDS

Figs. 3a and 3b show a domain with an airgap between two iron pieces. The aim of this example is to observe the shape of the field and flux errors. Two meshes were used: the first mesh (mesh *A*, Fig. 3a), uses rectangular elements, with *144* nodes in the airgap region. This type of FE provides excellent accuracy and results match experimental values[1]. The second mesh (mesh *B*, Fig. 3b), is coarser, with *37* nodes in the airgap region. To avoid confusion we will use the following notations: VM for virtual mesh and RM for real mesh.

The potential curves obtained from the finite element solution with and without smoothing are visually indistinguishable although they are not identical. Subtle differences can cause large changes in derived quantities. Fig. 4a shows the field perpendicular to line *L* of mesh *A*, with its expected discontinuities, and the field for the VM, with lower discontinuities. The smoothed curve is displaced upwards to simplify visualization. Fig. 4b shows the equivalent curves for mesh *B*. The superposition of these curves shows that for the RM, the difference in fields are much larger compared to the VM fields. Flux errors for the two meshes are:

A - Real	*A* - Virtual	*B* - Real	*B* - Virtual
$0.107E^{-3}$	$0.113E^{-4}$	$0.8622E^{-3}$	$0.530E^{-5}$

The flux errors are significantly lower with the VM

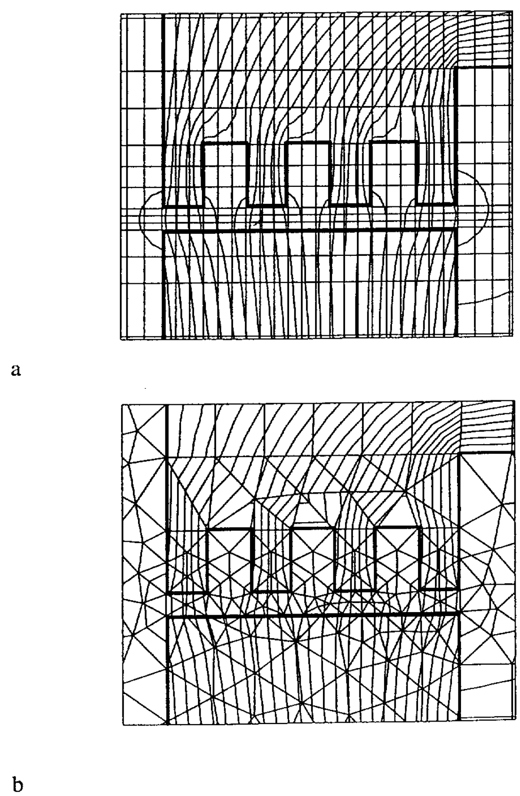
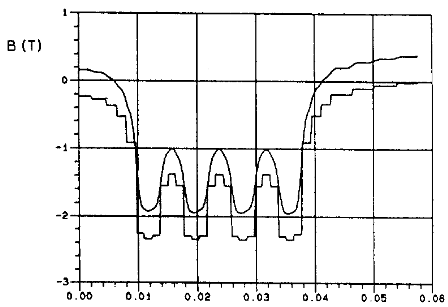


Fig. 3. Mesh and solution. a. Dense mesh, b. Coarse mesh.

V. EXAMPLE 2: CALCULATION OF FORCES

In the case, shown in Fig. 5, there are 2 parts of iron, with the bottom part displaced to the right. It is well known that the attraction forces can be calculated with good accuracy using the Maxwell tensor. However the restoring forces are difficult to evaluate by any method. In this example we will be concerned with flux errors, field shapes and especially forces.

Three meshes were used in this study: mesh A (Fig. 6a) is the densest, with 67 nodes in the airgap region: mesh B (Fig. 6b), has 20 nodes in the airgap, and mesh C, the coarsest, has only 13 nodes in this region (Fig. 6c).



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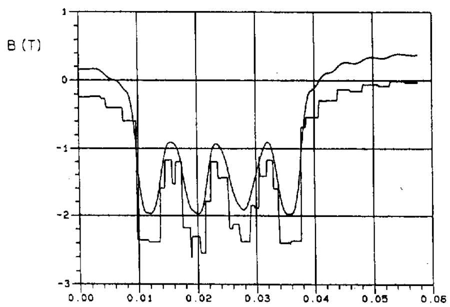


b

Fig. 4. Field plot. a. For mesh A, b. For mesh B.

Figures 7a, 7b and 7c show the y component of the field (perpendicular to airgap) obtained for the three meshes; the field calculated by the VM is displaced upwards; note that even for the mesh C, the agreement with the results from mesh A is quite good, considering the small number of elements. This is not very surprising, since the fields in this direction are normally quite accurate.

Figures 8a, 8b and 8c show the x component of the field (parallel to the airgap). In these figures there is no displacement in the smoothed field, since it is easy to distinguish between the curves visually. We note relatively good agreement between VM curves for mesh A and B although the fields for the RM differ more. The field curve for mesh C is quite different from the others, for both RM and VM, which is a consequence of the coarseness of this mesh.
Forces are calculated for 3 lines in the airgap: one in the middle, the second and the third close to each of the two iron pieces. Based on the numerical results, the following remarks are appropriate:
a) The errors for crossing flux are always smaller with VM (3 to 10 times smaller), depending on case.
b) Attraction force: results for virtual mesh agree well. The values for VM are slightly smaller that values for RM (about 3%). However the variation of force for the three lines is smaller with VM.

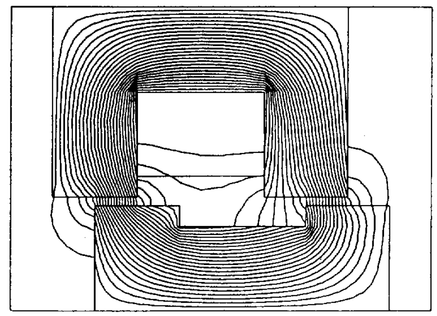
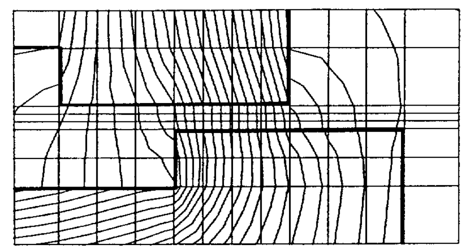


Fig. 5. Field plot as used for evaluation of forces.



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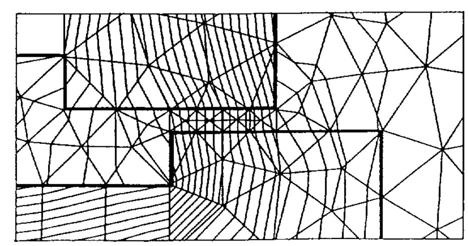

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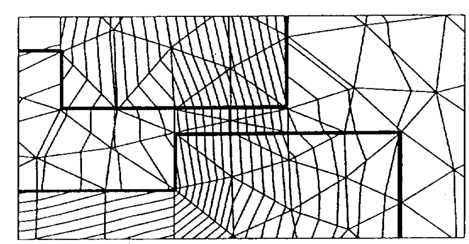
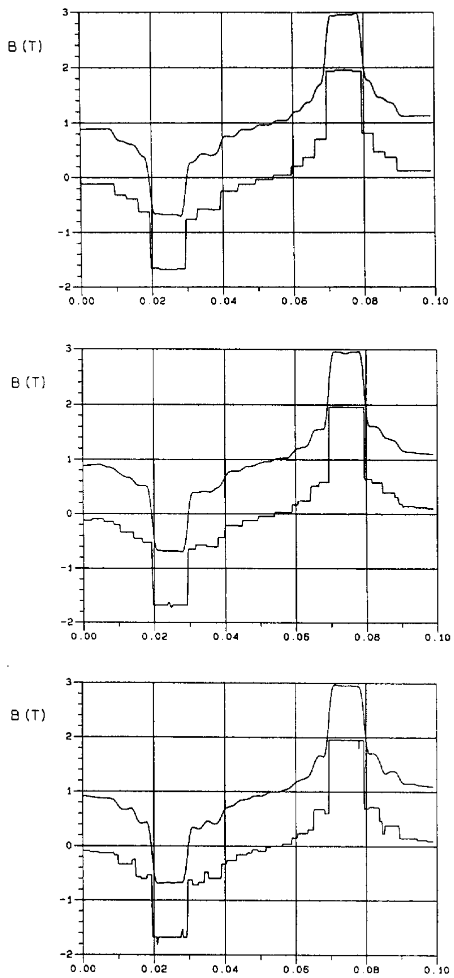


c

Fig. 6. a. Fine mesh, b. Medium mesh and c. Coarse mesh.

c) Restoring force: it is well known that accuracy for this force is normally poor in thin airgaps. This is due to the fact that restoring forces depend on the product "$h_x.h_y$"; if the value of h_x is somewhat affected by the mesh, the restoring force will be affected as well. Note also that the attraction forces depend on "$h_y^2-h_x^2$" and as hx is normally much smaller than h_y the value of this force is less affected by poor accuracy of h_x. In spite of the above, VM provide some improvement in this area. For mesh A, results for VM and RM differ by 1.2% and the variation of force on the three lines is very small (less than 2% for both types of mesh). For mesh B values of force are the same for both VM and

RM and 6.5% bigger than values of mesh A. However, the variation of force for the three lines is 0.8% with VM against 6.9% for RM. Mesh C, is too coarse to provide good results; however, for VM the average force is 23.6% larger than the correct value of mesh A and 28.1% with the RM. Variation among the three lines is much smaller for VM (0.56%) than for RM (11.0%).



a

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Fig. 7. Perpendicular fields in gap. a. Fine mesh, b. Medium mesh and c. Coarse mesh.

VI. CONCLUSIONS

A new technique to provide continuity to discretized FEM results has been presented. Using this procedure, a very dense mesh of virtual elements is generated in the post-processing step, by calculating values of magnetic fields from a smoothed set of vector potentials. This smoothing of potentials provides the required continuity for fields. Different degrees of the least squared method and number of neighboring points for local application of the method were tested and an efficient set of these variables was obtained. Two critical examples related to the shape of the field and calculation of force have shown a noticeable improvement of these quantities when VM is applied. A strict rule, based on the calculation of crossing magnetic flux error was employed to test the accuracy of the results. In all cases, fields and forces calculated by the virtual elements are more accurate than the discretized results obtained directly from the ***FEM.***

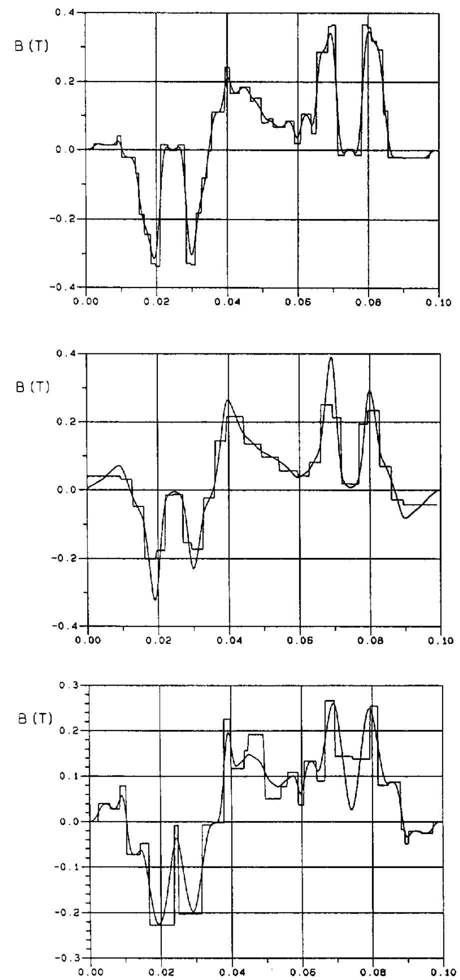

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Fig. 8. Tangential field. a. Fine mesh, b. Medium mesh and c. Coarse mesh

REFERENCES

[1] N.Sadowski, Y. Lefèvre, M. Lajoie-Mazenc, J. Cros, "Finite element torque calculation in electrical machines while considering movement", IEEE Trans. Magn., Vol 28, No. 2, March 1992, pp 1410-1413.

[2] N. Ida, J.P.A. Bastos, "Electromagnetics and calculation of fields", Springer-Verlag, New York, 1992.

[3] Y. Jaluria, "Computer Methods for Engineering", Allyn and Bacon, Inc, Boston, 1988, chapter 6, pp 259-283.