Theoretical Models For the Remote Field Effect

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Abstract: The Remote Field Effect is used extensively for testing of thick tubular products where deep penetration into (primarily ferromagnetic) materials is required [1]. The testing apparatus consists of two coils, one generating a magnetic field, the other is used as a pickup coil. The distance between the two coils is normally large (of the order of two coil diameters). The operating frequency is low, normally below 100 Hz. The method has been assumed in the past to rely on a special effect, and therefore the name Remote Field Effect. One of the most remarkable aspect of the method is its equal sensitivity to inner and outer defects in thick ferromagnetic tubes. This effect could not easily be explained by direct induction and therefore, many models have been proposed. The common model used is that of a wave propagating from the inside of the material to the outside and then back again [2]. We propose a theoretical model that shows the effect to be merely that of induction at large distances [3-5]. The field equations are solved directly using first an analytic Bessel function approach and then using an integral approach. Both of these models confirm the basic results associated with the remote field effect and also include the velocity of the coils. The models described here are general and applicable to other induction applications.

I. THEORETICAL MODELS

We establish a system of coordinates which is moving with the coils. In this system $\mathbf{B}=\mathbf{B}'$ and $\mathbf{E}=\mathbf{E}'+\mathbf{v}\times\mathbf{B}$, where $\mathbf{B}'$, $\mathbf{E}'$ are the fields in a stationary reference frame. Introducing a vector magnetic potential $\mathbf{A}$ with Coulomb gauge, namely, $\nabla\times\mathbf{A}=\mathbf{B}$, $\nabla\cdot\mathbf{A}=0$, into Maxwell's equation, we obtain the governing equations. These are written here in cylindrical coordinates since we consider the problem shown in Fig. 1. The general problem to solve is

$$\left(\frac{1}{\rho}\frac{\partial}{\partial\rho}\left(\rho\frac{\partial}{\partial\rho}\right)+\frac{\partial^2}{\partial z^2}-\mu\sigma v\frac{\partial}{\partial z}-\mu\sigma\frac{\partial}{\partial t}-\frac{1}{\rho^2}\right)A=-\mu_0 J \qquad (1)$$

where we will assume that the source is in free space. To solve the problem the following Green's function is considered

$$\Delta G=-\mu_0\delta(\rho-\rho_0)\,\delta(z-z_0) \qquad (2)$$

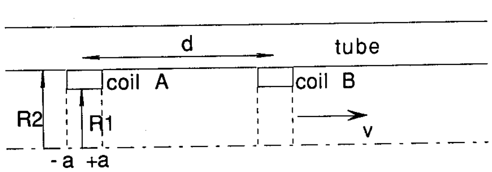

Fig. 1. General geometry.

With J uniformly distributed in a domain Ω (e.g. the coil), the following solution is obtained.

$$A=\int_\Omega JG(\rho-\rho_0,z-z_0)d\rho_0 dz_0 \qquad (3)$$

where both A and G are bounded at $\rho=\rho_0$ and $\rho=\infty$. The Green's function is found by relatively standard means. For this problem, the Green's function is found through the Fourier transform

$$G=\frac{\mu_0\rho_0 e^{i\omega t}}{2\pi}\int_{-\infty}^{\infty}[FI_1(\chi\rho)+HK_1(\chi\rho)]e^{i\xi(z-z_0)}d\xi \qquad (4)$$

where ξ is the transformed z coordinate and

$$A=\frac{1}{2\pi}\int_{-\infty}^{+\infty}\tilde{A}e^{iz\xi}\,d\xi$$

For a coil extending from $\rho=R_1$ to $\rho=R_2$ and $z=-a$ to $z=+a$, the solution is

$$A=Je^{i\omega t}\int_{R_1}^{R_2}d\rho_0\int_{-a}^{+a}dz_0\frac{1}{2\pi}\int_{-\infty}^{\infty}\mu_0\rho_0[FI_1(\chi\rho)+HK_1(\chi\rho)]e^{i\xi(z-z_0)}d\xi \qquad (5)$$

where F and H are functions defined in the various domains of the problem in terms of modified Bessel functions. These are given as

$$F=\begin{cases}K_1(\xi\rho_0)+\dfrac{P_1}{Q}I_1(\xi\rho_0)\\ \dfrac{P_1}{Q}I_1(\xi\rho_0)\\ \dfrac{P_2}{Q}I_1(\xi\rho_0)\\ 0\end{cases}\qquad H=\begin{cases}0 & \rho_0\geq\rho\\ I_1(\xi\rho_0) & R_1\geq\rho>\rho_0\\ \dfrac{P_3}{Q}I_1(\xi\rho_0) & R_2\geq\rho>R_1\\ \dfrac{P_4}{Q}I_1(\xi\rho_0) & \rho>R_2\end{cases}$$

$$\chi=\begin{cases}\zeta & R_2\geq\rho>R_1\\ \xi & otherwise\end{cases}$$

and

$$\zeta^2=\xi^2+i\mu\sigma(\xi v+\omega)$$

Terms P_1, P_2, P_3, P_4, and Q can be found in [5].

This solution allows calculation of the magnetic vector potential for any coil or combination of coils and from that the magnetic flux density or induced voltages in coils may be calculated as

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$$B_\rho=-\frac{\partial A}{\partial z} = -i2\mu_0 J e^{i\omega t}\frac{1}{2p}\int_{-\infty}^{\infty}[\phi I_1(\chi\rho)+\varphi K_1(\chi\rho)]\sin(\xi a)e^{i\xi z}d\xi \quad (6)$$

$$B_z=\rho\frac{\partial(\rho A)}{\partial \rho} = 2\mu_0 J e^{i\omega t}\frac{1}{2\pi}\int_{-\infty}^{\infty}\eta[\phi I_0(\chi\rho)-\varphi K_0(\chi\rho)]\frac{\sin(\xi a)}{\xi}e^{i\xi z}d\xi \quad (7)$$

where

$$\eta = \begin{cases} \zeta & R_2 \geq \rho > R_1 \\ |\xi| & otherwise \end{cases}$$

To calculate the induced voltage V in an identical coil at $z=z_0$, we write

$$V_s=-\frac{\partial}{\partial t}\int_s \mathbf{B}.\mathbf{d}\mathbf{s}=-i\omega\int_0^d \rho d\rho\int_0^{2\pi} B_z d\theta = -i2\pi\omega\int_0^d B_z\rho d\rho \quad (8)$$

for a loop with radius d inside the tube. Performing integration over the coil area, the total induced voltage in the coil is obtained.

The integrations necessary in Eqs. (5) through (8) are performed numerically but we consider this model to be exact.
The method can also be used to calculate fields and induced voltages due to axisymmetric defects by assuming the defects to contain a current which is equal and opposite in direction to the induced current in the tube without defects.

The second model we use is somewhat simpler and more numerical in nature. Because of that, it also allows incorporation of defects in the material so that their effect on the signal may be studied. In this model, the tube in Fig. 1 is divided into finite size concentric rings as shown in Figure 2. Only a finite number of rings is considered (only those close to and interacting with the coils). Inside the nth ring we have

$$\frac{\mathbf{J}}{\sigma}=\mathbf{E} = \mathbf{E}' + \mathbf{v}\times\mathbf{B} = -\nabla V - \frac{\partial \mathbf{A}}{\partial t} + \nabla\times\nabla\times\mathbf{A} \quad (9)$$

By superposition, the magnetic vector potential is obtained by adding the contributions of all rings and that of the coil which we denote by A_e. This now becomes

$$-\nabla V = \frac{\mathbf{J}_k}{\sigma} + \left(\frac{\partial}{\partial t} - \mathbf{v}.\frac{\partial}{\partial z}\right)\left(\sum_{j=1}^{n}\mathbf{A}_j + \mathbf{A}_e\right) \quad (10)$$

The solution for the magnetic vector potential is

$$A_j = \frac{\mu_0}{4\pi}\int_{V_j}\frac{J_j}{r}dV_j = \frac{\mu_0}{4\pi}\int_{S_j}\int_{l_j}\frac{dl_j}{r}\,.\,J_j dS_j$$

Integrating Eq. (10) along a closed line in the *k-th* ring we have

$$\int_{l_k} -\nabla V\, dl_k = 0; \qquad J = \frac{i}{S}; \qquad \int_{l_k}\frac{1}{\sigma}J_k dl_k = R_k I_k$$

$$\int_{l_k} A_j\, dl_k = \int_{S_j}\left[\frac{\mu}{4\pi}\int_{l_k}\int_{l_j}\frac{dl_k dl_j}{r_{kj}}\right]J_j dS_j = \Phi_{kj}$$

$$N_{kj} = \frac{\mu}{4\pi}\int_{l_k}\int_{l_j}\frac{dl_k dl_j}{r_{kj}}$$

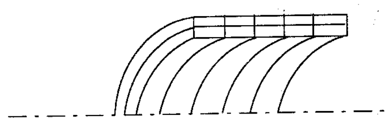

Fig. 2. Division of the tube and coils in Fig. 1 into concentric rings of varying cross section.

The integration of the vector potential A_j in the *k-th* ring, due to the contribution of the *j-th* rings at a distance r_{kj} from the *k-th* ring, represents the flux Φ_{kj} generated by the *j-th* element and linked with a line of the k-th ring. N_{kj} is the Neumann integral of the *k-th* and *j-th* rings, $l_k=2\pi r_k$, $l_j=2\pi r_j$ are the lengths of the *k-th* and *j-th* rings, the total flux $\Phi_{tot\ kj}$ due to the *j-th* ring and linked with the *k-th* ring is equal to the flux due to the *k-th* ring and linked to the *j-th* ring, and is

$$\Phi_{tot\ kj} = \int_{S_k}\Phi_{kj}\frac{J_k}{i_k}dS_k = \frac{1}{S_k}\int_{S_k}\int_{S_j}N_{kj}J_j dS_j dS_k = M_{kj}i_j$$

where M_{kj} is the mutual inductance coefficient between the *k-th* and *j-th* element, S_j and S_k are the *j-th* and *k-th* element cross-sections, therefore Eq. (10) becomes

$$0 = R_k i_k + \sum_{j=1}^{n} M_{kj}\frac{di_j}{dt} - \sum_{j=1}^{n} v i_j\frac{dM_{kj}}{dz} + M_{ke}\frac{di_e}{dt}$$

Writing Eq. (10) for the n rings in the system results in the following linear complex system of equations

$$[I] = -([M] + [R])^{-1}[M_e]\,[I_e] \qquad (11)$$

where $I_e=I_{emax}\sin(2\pi ft+\phi)$.

The various matrices are given as

$$[R] = [R_r] + [R_v]$$

$$[M] = \begin{bmatrix} j2\pi fL_1 & j2\pi fM_{12} & \cdots & j2\pi fM_{1n} \\ & j2\pi fL_2 & \cdots & \cdots \\ & & \vdots & \vdots \\ & & & j2\pi fL_n \end{bmatrix} \qquad [M_e] = \begin{bmatrix} j2\pi fM_{1e} \\ j2\pi fM_{2e} \\ \vdots \\ j2\pi fM_{ne} \end{bmatrix}$$

$$[R_v] = \begin{bmatrix} 0 & v\frac{\partial M_{12}}{dz} & v\frac{\partial M_{13}}{dz}\cdots & v\frac{\partial M_{1n}}{dz} \\ v\frac{\partial M_{12}}{dz} & 0 & v\frac{\partial M_{23}}{dz}\cdots & v\frac{\partial M_{2n}}{dz} \\ \vdots & \vdots & \vdots\ \cdots & \vdots \\ v\frac{\partial M_{1n}}{dz} & v\frac{\partial M_{2n}}{dz} & \cdots\ \cdots & 0 \end{bmatrix}$$

$$[R_r] = \begin{bmatrix} R_1 & 0 & \cdots & 0 \\ 0 & R_2 & \cdots & 0 \\ \vdots & \vdots & & \vdots \\ 0 & 0 & \cdots & R_n \end{bmatrix}$$

Hence, the integral approach modifies the problem solution in the solution of an equivalent electric network which branches are formed by self and mutual inductance coefficients and resistances of the rings in which the tube is subdivided.

The calculation of the inductive coefficients is a crucial point for the solution accuracy but, for a non ferromagnetic material analytic formulas are available in the literature for accurate computation while for ferromagnetic materials, the analytic solution described in the first part of this work can be used.

From the magnetic vector potential, the induced voltage in the pickup coil is calculated from Eq. (10) by setting $J=0$. Similarly, the magnetic flux density can be calculated by calculating the derivatives of A as in Eqs. (6) and (7).

This model can used be for electromagnetic field analysis of the remote field eddy current technique for tubes with axisymmetric defects by assuming that defects are composed of rings with zero induced current.

The equivalent network is shown in Fig. 3. The number of branches remains as for the case without defects. A defect simply means that the branches (rings) in the defect region are switched off.

II. RESULTS

As examples of the type of results calculable by this method, the induced voltage and phase are plotted as a function of defect position in Figs. 4 and 5, for a distance between coils of 1.5 times the coil diameter. Dotted lines are for an internal axisymmetric defect, solid line for an identical external defect. The coil moves at a velocity of 5 m/s. Distance is shown with respect to location of defect (zero distance is at the defect). The results here were calculated using the geometry in Fig. 1 for a nonferromagnetic tube. It is obvious that sensitivity is roughly equal for both defects and thus the usefulness of the remote field effect. The methods presented here are equally applicable to calculations in ferromagnetic materials and show that the remote field effect is a simple induction effect but one that only shows up at low field intensities (far from the source).

CONCLUSIONS

The theoretical models described here were developed for the purpose of understanding the remote field phenomenon but are general methods of solution for axisymmetric geometries. A number of other aspects of testing including frequency variations and infinitely thick tubes were also analyzed but are not reported here. The methods are currently being extended to include transient excitation and ferromagnetic materials.

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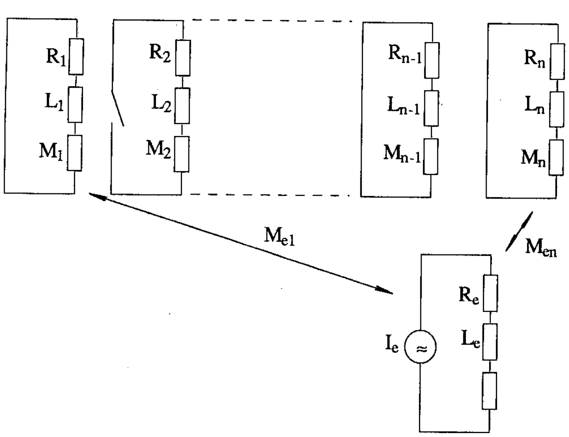

Fig. 3. Equivalent network of the integral model.

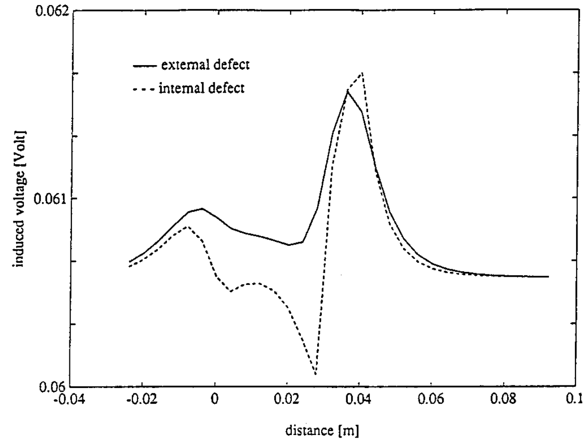

Fig. 4. Induced voltage due to an axisymmetric defect.

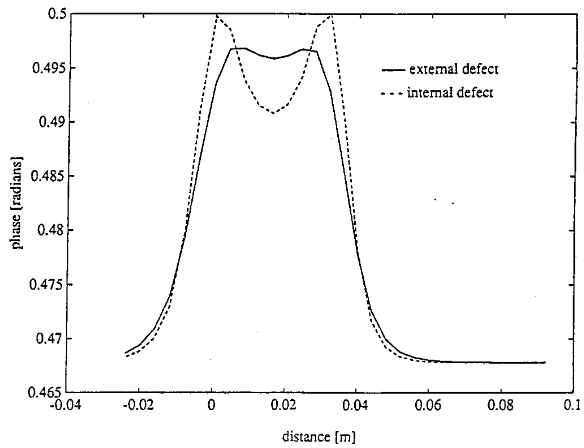

Fig. 5. Phase of induced voltage due to an an axisymmetric defect.

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